LINEAR ORDINARY DIFFERENTIAL EQUATI(OF FIRST ORDER :-> (ODE)

A First order differential equation of the form.

$$a_1(x) \frac{dy}{dx} + a_0(x) y = g(x) \longrightarrow ①$$

is said to be linear differential equation independent variable.

If eq① can be written in the form

$$\Rightarrow \frac{dy}{dx} + \frac{a_0(x)}{a_1(x)} y = \frac{g(x)}{a_1(x)}$$

$$\Rightarrow \frac{dy}{dx} + P(x) y = \phi x \quad \because \phi x = \frac{g(x)}{a_1(x}, \quad P(x) = \frac{a_0(}{a_1(}$$

The general solution of first order linear ODE is

$$y(x) = \frac{1}{\mu(x)} \int \mu(x) \, \phi(x) \, dx$$

where $\mu(x) = e^{\int P(x) dx}$

Q NO 1:-> $\frac{dy}{dx} - 3y = 0$

<u>Solution</u> :-> $\frac{dy}{dx} + P(x) y = \phi x$

$\Rightarrow P(x) = -3 \;, \quad \phi x = 0$

where we have the formula

$$y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x) dx$$

Now putt the values in formula.

(2)

$\Rightarrow \mu(x) = e^{\int -3dx}$

$\Rightarrow \mu(x) = e^{-3x} + c$

$\Rightarrow y(x) = \frac{1}{e^{-3x}+c} \int (e^{-3x}+c) \cdot 0\, dx$

$\Rightarrow y(x) = \frac{1}{e^{-3x}+c} (0+c)$

$y(x) = \frac{c}{e^{-3x}+c}$

$\Rightarrow \left\{ y(x) = \frac{c}{e^{-3x}} \right\}$ Answer.

Q NO 2:⇒ $\frac{dy}{dx} + 4xy = x$

Solution:⇒ $\frac{dy}{dx} + P(x)y = \phi x$

$\Rightarrow P(x) = 4x$ and $\phi x = x$

⇒ Formula $y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\, dx$

$\Rightarrow \mu(x) = e^{\int 4x\, dx}$

$\Rightarrow \mu(x) = e^{\cancel{4}x^2/\cancel{2}}$

$\Rightarrow \mu(x) = e^{2x^2}$

Now putt the values in formula

$\Rightarrow y(x) = \frac{1}{e^{2x^2}} \int e^{2x^2} \cdot x\, dx$

P.T.O

$\Rightarrow$ let $z = 2x^2$

$\Rightarrow \frac{dz}{dx} = 4x^2$

$\Rightarrow dz = 4x\, dx$

$\Rightarrow y(x) = \frac{1}{e^{2x^2}} \cdot \frac{1}{4} \int e^{2x^2} \cdot 4x\, dx$

$\Rightarrow y(x) = \frac{1}{e^{2x^2}} \cdot \frac{1}{4} \int e^{z}\, dz$

$\Rightarrow y(x) = \frac{1}{4e^{2x^2}} (e^{z}+c)$

where $z = 2x^2$

Q NO 3:⇒ $(x^2-9)\frac{dy}{dx} + xy = 0$

Solution:⇒ $\frac{dy}{dx} + P(x)\, y = \phi(x) \longrightarrow (2)$

$\Rightarrow \frac{dy}{dx} + \frac{x}{(x^2-9)} y = 0 \longrightarrow (3)$

Compare (3) & (2)

$\Rightarrow P(x) = \frac{x}{(x^2-9)}, \quad \phi = 0$

Formula: $y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\, dx$

$\Rightarrow \mu(x) = e^{\int P(x)\, dx} \longrightarrow (4)$

P.T.O

④

$$\Rightarrow \mu(x) = e^{\frac{1}{2}\ln(x^2-9)}$$

$$\Rightarrow \mu(x) = \cancel{e}^{\cancel{\ln}(x^2-9)^{1/2}}$$ $\because$ Log three formulas

e and ln are inverse function will be cancel.

1) $N\log M = \ln M^N$

2) $\log MN = \ln M + \ln N$

3) $\ln \frac{M}{N} = \ln M - \ln N$

$$\Rightarrow y(x) = \frac{1}{\mu(x)} \int \mu(x)\,\phi(x)\,dx$$

$$\Rightarrow y(x) = \frac{1}{(x^2-9)^{1/2}} \int (x^2-9)^{1/2} \cdot 0\,dx$$

$$\Rightarrow y(x) = \frac{1}{(x^2-9)^{1/2}} (0+c)$$

$$\Rightarrow \left\{ y(x) = \frac{c}{(x^2-9)^{1/2}} \right\}$$ Answer

—*—*—*—*—

Q.No4:-> $\frac{dy}{dx} + y = e^{3x}$

solution :-> $\frac{dy}{dx} + (x)y = \phi x$

$$\Rightarrow p(x) = 1 \quad \& \quad \phi(x) = e^{3x}$$

$$\Rightarrow \mu(x) = e^{\int p(x)\,dx}$$

$$\Rightarrow \mu(x) = e^{\int 1\,dx}$$

$$\Rightarrow \mu(x) = e^{x}$$

Formula :-> $\frac{dy}{dx} = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\,dx$

P.T.O

⑤

Now Putt the values in formula.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^x} \int e^{x} \cdot e^{3x}\,dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^x} \int e^{4x}\,dx$$

$$\Rightarrow \left\{ \frac{dy}{dx} = \frac{1}{e^x} \left[\frac{e^{4x}}{4} + c \right] \right\}$$ Answer

—*—*—*—*—*—

Q.No5:-> $\frac{(x^2y' + xy)}{x^2} = \frac{1}{x^2}$

Solution:-> $\frac{(x^2y' + xy)}{x^2} = \frac{1}{x^2}$

$$\Rightarrow \frac{\cancel{x^2}y'}{\cancel{x^2}} + \frac{\cancel{x}y}{x^{\cancel{2}}} = \frac{1}{x^2}$$

$$\Rightarrow y' + \frac{y}{x} = \frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y = 1/x^2$$

$$\Rightarrow \frac{dy}{dx} + P(x)y = \phi x$$

$\mu(x) = ?\ \int p(x)\,dx$

$\mu(x) = e^{\int \frac{1}{x}dx}$

$= e^{\ln x}$

$= \cancel{e}^{\cancel{\ln} x}$

$\mu(x) = x$

$$\Rightarrow P(x) = \frac{1}{x} \quad \& \quad \phi = 1/x^2$$

Formula:-> $y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\,dx$

P.T.O

⑥

$\Rightarrow y(x) = \frac{1}{x}\int x \cdot \frac{1}{x^2}\,dx$

$\Rightarrow y(x) = \frac{1}{x}\int \frac{1}{x}\,dx$

$\Rightarrow \{y(x) = \frac{1}{x}[\ln x + c]\}$ Answer

Q NO 6:-> $3\frac{dy}{dx} + 12y = 4$

Solution:-> $3\frac{dy}{dx} + 12y = 4$

Divide and multiply both side by 3

$\Rightarrow \frac{\cancel{3}\frac{dy}{dx}}{\cancel{3}} + \frac{\cancel{12}^{4}y}{\cancel{3}} = \frac{4}{3}$

$\Rightarrow \frac{dy}{dx} + 4y = 4/3$

$\Rightarrow \frac{dy}{dx} + P(x)y = \phi(x)$

$P(x) = 4$ & $\phi(x) = 4/3$

Formula:-> $y(x) = \frac{1}{\mu(x)}\int \mu(x)\cdot\phi(x)\,dx$

$\Rightarrow \mu(x) = e^{\int P(x)dx}$

$\Rightarrow \mu(x) = e^{\int 4\,dx}$

$\Rightarrow \mu(x) = e^{4x}$

P.T.O

Put the values in formula

$\Rightarrow y(x) = \frac{1}{e^{4x}}\int e^{4x}\cdot 4/3\,dx$

$\Rightarrow \{y(x) = \frac{4}{3e^{4x}}\left(\frac{e^{4x}}{4} + c\right)\}$ Answer

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Q NO 7:-> $\frac{dy}{dx} + 3x^2y = x^2 \longrightarrow ①$

Solution:-> $\frac{dy}{dx} + P(x)y = \phi x \longrightarrow ②$

Compare ① and ②

$P(x) = 3x^2$ & $\phi(x) = x^2$

Now we have to find $\mu(x)$ for formula

$\Rightarrow \mu(x) = e^{\int P(x)dx}$

$\Rightarrow \quad = e^{\int 3x^2\,dx}$

$= e^{3\int x^2\,dx}$

$= e^{\cancel{3}\frac{x^3}{\cancel{3}}}$

$\Rightarrow \mu(x) = e^{x^3}$

Now Put the values in formula

$\Rightarrow y(x) = \frac{1}{e^{x^3}}\frac{1}{3}\int e^{x^3}\cdot 3x^2\,dx$ Let $x^3 = z$

$\Rightarrow y(x) = \frac{1}{3e^{x^3}}\int e^{z}\,dz$

P.T.O

⑤

$$\Rightarrow \left\{ y(x) = \frac{1}{3e^{x^3}} (e^z + c) \right\} \text{ Answer.}$$

where $z = x^3$

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Q.no8:-> $y' + 2xy = x^3$

Solution:-> $\frac{dy}{dx} + p(x)y = \phi x$

$\Rightarrow p(x) = 2x$ & $\phi(x) = x^3$

Now we find $\mu(x)$ for formula.

$$\Rightarrow \mu(x) = e^{\int p(x)dx}$$

$$\Rightarrow \mu(x) = e^{\int 2x\,dx}$$

$$\Rightarrow \mu(x) = e^{\cancel{2}x^2/\cancel{2}}$$

$$\mu(x) = e^{x^2}$$

Formula:-> $y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)$

Now put the values in formula

$$\Rightarrow y(x) = \frac{1}{e^{x^2}} \int \mu(x) \cdot \phi(x)dx$$

let $x^2 = z$

$$\Rightarrow y(x) = \frac{1}{e^{x^2}} \frac{1}{2} \int x^2 e^{x^2} 2x\,dx$$

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$$\Rightarrow y(x) = \frac{1}{e^{2x}} \frac{1}{2} \int z e^z dz$$

$$\Rightarrow y(x) = \frac{1}{e^{x^2}} \left[\frac{1}{2} \left(z \int e^z dz - \int \frac{d}{dz}(z) \int e^z dz\, dz \right) \right]$$

$$\Rightarrow y(x) = \frac{1}{e^{x^2}} \left[\frac{1}{2} \left(z e^z - \int e^z dz \right) \right]$$

$$\Rightarrow y(x) = \frac{1}{e^{x^2}} \left[\frac{1}{2} \left(z e^z - e^z + 2 \right) \right]$$

$$\Rightarrow y(x) = \frac{1}{e^{x^2}} \left[\frac{z \cdot e^z}{2} - \frac{e^z}{2} + \frac{c}{2} \right]$$

Put $z = e^{x^2}$

$$\Rightarrow y(x) = \frac{x^2 \cancel{e^{x^2}}}{2\cancel{e^{x^2}}} - \frac{\cancel{e^{x^2}}}{2\cancel{e^{x^2}}} + \frac{c}{2e^{x^2}}.$$

$$\Rightarrow \left\{ y(x) = \frac{x^2}{2} - \frac{1}{2} + \frac{c}{2e^{x^2}} \right\} \text{ Answer.}$$

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Q.no9:-> $y' - 2y = x^2 + 5$

Solution:-> $\frac{dy}{dx} - 2y = x^2 + 5$

$$\Rightarrow \frac{dy}{dx} + p(x)y = \phi(x)$$

$p(x) = -2$ & $\phi(x) = x^2 + 5$

P.T.O

(10) $\Rightarrow \mu(x) = e^{\int P(x)dx}$

$\Rightarrow \mu(x) = e^{\int -2dx}$

$\Rightarrow \mu(x) = e^{-2\int 1 dx}$

$\Rightarrow \mu(x) = e^{-2x}$

$\Rightarrow \mu(x) = e^{-2x}$

Putt the values in formula

$\Rightarrow y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x) dx$

$\Rightarrow y(x) = \frac{1}{e^{-2x}} \int e^{-2x} \cdot (x^2+5) dx$

$\Rightarrow y(x) = \frac{1}{e^{-2x}} \int (x^2 e^{-2x} + 5e^{-2x}) dx$

$= \frac{1}{e^{-2x}} \left[\int x^2 e^{-2x} dx + \int 5e^{-2x} dx \right.$

integration by parts for $\int x^2 e^{-2x} dx$

d/dx	$\int$
$+ x^2$	e^{-2x}
$- 2x$	$-\frac{1}{2} e^{-2x}$
$+ 2$	$\frac{1}{4} e^{-2x}$
$- 0$	$-\frac{1}{8} e^{-2x}$

P.T.O

$= \frac{1}{e^{-2x}} \left[-\frac{x^2 e^{-2x}}{2} - \frac{2xe^{-2x}}{4} - \frac{2e^{-2x}}{8} - \frac{5e^{-2x}}{2} + c \right]$

$= \frac{-x^2 \cancel{e^{-2x}}}{2\cancel{e^{-2x}}} - \frac{x\cancel{e^{-2x}}}{2\cancel{e^{-2x}}} - \frac{\cancel{e^{-2x}}}{4\cancel{e^{-2x}}} - \frac{5\cancel{e^{-2x}}}{2\cancel{e^{-2x}}} +$

$y(x) = \left\{ -\frac{x^2}{2} - \frac{x}{2} - \frac{1}{4} - \frac{5}{2} + \frac{c}{e^{-2x}} \right\}$

Answer

Q no 10:-> $x \frac{dy}{dx} - y = x^3 \sin x$

Solution :-> $\frac{dy}{dx} - \frac{1}{x} y = x^2 \sin x$

$\Rightarrow \frac{dy}{dx} + P(x)y = \phi(x)$

$\Rightarrow P(x) = -\frac{1}{x}$ & $\phi(x) = x^2 \sin x$

$\Rightarrow \mu(x) = e^{\int P(x)dx}$

$\Rightarrow \mu(x) = e^{\int -\frac{1}{x}}$

$\Rightarrow \mu(x) = e^{-\ln x}$

$\Rightarrow \boxed{\mu(x) = -x}$

Now Putt the values in formula

P.T.O

(12)

$$\Rightarrow y(x) = \frac{1}{\mu(x)}\int \mu(x)\cdot\phi(x)\,dx.$$

$$\Rightarrow y(x) = \frac{1}{-x}\int -x\cdot x^2\sin x\,dx$$

$$= \frac{1}{-x}\int -x^3\sin x\,dx$$

d/dx	$\int$
$+\ -x^3$	$\sin x$
$-\ -3x^2$	$-\cos x$
$+\ -6x$	$-\sin x$
$-\ -6$	$\cos x$
0	$\sin x$

$$\Rightarrow y(x) = \frac{1}{-x}\left(x^3\cos x - 3x^2\sin x - 6x\cos x\right.$$
$$\left. 6\sin x + C\right)$$

$$\Rightarrow y(x) = \left(\frac{\cancel{x^3}\cos x}{-\cancel{x}} - \frac{3\cancel{x^2}\sin x}{-\cancel{x}} - \frac{6\cancel{x}\cos x}{-\cancel{x}}\right.$$
$$\left.\frac{6\cancel{x}\sin x}{-\cancel{x}} + \frac{C}{-x}\right)$$

$$\Rightarrow y(x) = \left(-x^2\cos x - 3\sin x + 6\cos x - \frac{6\sin}{x}\right.$$
$$\left. - \frac{C}{x}\right) \quad \text{Answer.}$$

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(13)

Q No 11 :-> $\frac{dy}{dx} + \frac{2}{x}y = 3/x$

Let $p(x) = 2/x$ and $\phi(x) = 3/x$

$$\mu(x) = e^{\int p(x)\,dx}$$

$$\mu(x) = e^{\int 2/x\,dx}$$

$$\mu(x) = e^{2\ln x}$$

$$\mu(x) = x^2$$

Now Putt the values in formula

$$y(x) = \frac{1}{\mu(x)}\int \mu(x)\cdot\phi(x)\,dx.$$

$$= \frac{1}{x^2}\int \cancel{x^2}\cdot\frac{3}{\cancel{x}}\,dx$$

$$= \frac{1}{x^2}3\int x\,dx$$

$$= \frac{1}{x^2}3\left[\frac{x^2}{2} + c\right]$$

$$= \frac{1}{x^2}\left[\frac{3x^2}{2} + 3c\right]$$

$$= \frac{3\cancel{x^2}}{2\cancel{x^2}} + \frac{3c}{x^2}$$

$$y(x) = \frac{3}{2} + \frac{3c}{x^2} \quad \text{Answer}$$

Q No 12:-> $\frac{dy}{dx} + \frac{4}{x} y = x - 1$

$P(x) = \frac{4}{x}$ and $\phi(x) = x - 1$

$\mu(x) = ?$

$\Rightarrow \mu(x) = e^{\int p(x)\,dx}$

$= e^{\int \frac{4}{x}\,dx}$

$= e^{4 \ln x} = e^{\ln x^4}$

$\Rightarrow \mu(x) = x^4$

Now putt the values in formula.

$y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\, dx.$

$= \frac{1}{x^4} \int x^4 \cdot (x-1)\, dx$

$= \frac{1}{x^4} \int x^5 - x^4\, dx$

$= \frac{1}{x^4} \left[\frac{x^6}{6} - \frac{x^5}{5} + c \right]$

$= \left[\frac{x^6}{6x^4} - \frac{x^5}{5x^4} + \frac{c}{x^4} \right]$

$y(x) = \left\{ \frac{x^2}{6} - \frac{x}{5} + \frac{c}{x^4} \right\}$ Answer

Q No 13:-> $x^2 y' + x(x+2) y = e^x$

$\Rightarrow x^2 \frac{dy}{dx} + x(x+2) y = e^x$

$\Rightarrow \frac{\cancel{x^2}}{\cancel{x^2}} \frac{dy}{dx} + \frac{x(x+2)y}{x^2} = \frac{e^x}{x^2}$

$\Rightarrow \frac{dy}{dx} + \left(\frac{\cancel{x^2}}{\cancel{x^2}} + \frac{2x}{x^2} \right) y = \frac{e^x}{x^2}$

$\Rightarrow \frac{dy}{dx} + \left(1 + \frac{2}{x} \right) y = \frac{e^x}{x^2}$

$\Rightarrow p(x) = 1 + \frac{2}{x}$ and $\phi(x) = \frac{e^x}{x^2}$

$\mu(x) = ?$

$\Rightarrow \mu(x) = e^{\int p(x)\,dx}$

$= e^{\int 1 + \frac{2}{x}\,dx}$

$= e^{x + 2\ln x}$

$= e^x \cdot e^{2\ln x}$ ∵ log formula.

$= e^x e^{\ln x^2}$

$\Rightarrow \mu(x) = x^2 e^x$

Now formula.

$y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\, dx$

P.T.O

Putt the values in formula.

$$y(x) = \frac{1}{x^2e^x}\int \cancel{x^2}e^x \cdot \frac{e^x}{\cancel{x^2}}\, dx$$

$$y(x) = \frac{1}{x^2e^x}\int e^x \cdot e^x\, dx$$

$$= \frac{1}{x^2e^x}\int 2e^x\, dx$$

$$= \frac{1}{x^2e^x}\left[\frac{e^{2x}}{2} + c\right]$$

$$= \frac{1}{x^2\cancel{e^x}} \cdot \frac{\cancel{e^x}\cdot e^x}{2} + \frac{c}{x^2e^x}$$

$$\left\{ y(x) = \frac{e^x}{2x^2} + \frac{c}{x^2e^x} \right\}$$ Answer.

—*—*—*—*—*—

Q No14:-> $xy' + (1+x)y = e^{-x}\sin 2x$

Solution:-> $y' + \left(\frac{1+x}{x}\right)y = \frac{e^{-x}}{x}\sin 2x$

$P(x) = \frac{(1+x)}{x}$ and $\phi(x) = \frac{e^{-x}}{x}\sin 2x$.

$\mu(x) = ?$

$\mu(x) = e^{\int P(x)dx}$.

$= e^{\int\left(\frac{1+x}{x}\right)dx}$.

P.T.O

$$\mu(x) = e^{\ln x + x}$$

$$\mu(x) = \cancel{e}^{\cancel{\ln} x} \cdot e^x$$

$$\mu(x) = xe^x$$

Formula:-> $y(x) = \frac{1}{\mu(x)}\int \mu(x)\cdot \phi(x)\,dx$

Putt the values in formula

$$\Rightarrow y(x) = \frac{1}{xe^x}\int \cancel{x}\cancel{e^x}\cdot \frac{\cancel{e^{-x}}}{\cancel{x}}\sin 2x\, dx$$

$$= \frac{1}{xe^x}\int \sin 2x\, dx$$

$\because \sin 2x = \frac{-\cos 2x}{2}$

$$= \frac{1}{xe^x} - \frac{\cos 2x}{2} + c$$

$$\left\{ y(x) = -\frac{\cos 2x}{2xe^x} + \frac{c}{xe^x} \right\}$$ Answer.

—*—*—*—*—*—

"Nobody seems to appreciate the love I give"

Fawad Ahmad

(18)

Q no 15: $\cos x \frac{dy}{dx} + \sin(x)\, y = 1$

Solution: $\frac{dy}{dx} + \left(\frac{\sin x}{\cos x}\right) y = \frac{1}{\cos x}$.

$\Rightarrow \frac{dy}{dx} + (\tan x)\, y = \sec x$

$p(x) = \tan x$ and $\phi(x) = \sec x$.

$\mu(x) = ?$

$\Rightarrow \mu(x) = e^{\int p(x)dx}$

$= e^{\int \tan x\, dx}$.

$= e^{\int \frac{\sin x}{\cos x} dx}$

$= e^{\int \frac{\sin x}{\cos x} dx}$.

$= e^{-\int \frac{du}{u}}$

$= e^{-\ln(\cos x)}$.

$= e^{\ln(\cos x)^{-1}}$

$= e^{\ln\left(\frac{1}{\cos x}\right)}$

$= \cancel{e}^{\cancel{\ln}(\sec x)}$

$\boxed{\mu(x) = \sec x}$

$\because \tan x = \frac{\sin x}{\cos x}$.

$\sec x = \frac{1}{\cos x}$

$u = \cos x$

$du = -\sin x dx$

$-du = \sin x dx$

P.T.O

Formula: $y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\, dx$

Putt the values in formula

$y(x) = \frac{1}{\sec x} \int \sec x \cdot \sec x\, dx$

$= \frac{1}{\sec x} \int \sec^2 x\, dx$

$= \frac{1}{\sec x} (\tan x + c)$

$= \cos x \cdot (\tan x + c)$

$= \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + \cos x c$

$\because \int \sec^2 x = \tan x$

$\cos x = \frac{1}{\sec x}$

$\{y(x) = \sin x + \cos x c\}$ Answer.

"My Talent"

Not Sleeping at Night.

Fawad Ahmad

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ODE REDUCIBLE TO LINEAR FORM

Q NO 1:→ $\sec^2 y \frac{dy}{dx} + \tan y = x.$

Chain Rules

$\frac{dv}{dx} = \frac{dv}{dy} \cdot \frac{dy}{dx}$

$\frac{dv}{dx} = \frac{d}{dy}(\tan y) \cdot \frac{dy}{dx}$

$= \sec^2 y \cdot \frac{dy}{dx}$

$\frac{dv}{dx} = \sec^2 y \cdot \frac{dy}{dx}$

Solution:→ Let $v = \tan y$

$\frac{dv}{dx} =$

$\frac{dv}{dx} = \sec^2 y \cdot \frac{dy}{dx}.$

$\frac{dv}{dx} + v = x.$

$P(x) = 1, \quad \phi(x) = x$

$\mu(x) = ?$

$\mu(x) = e^{\int P(x)\,dx}$

$\mu(x) = e^{\int 1\,dx}$

$\boxed{\mu(x) = e^{x}}$

$v(x) = \frac{1}{e^x}\int e^x \cdot x\,dx$

$= \frac{1}{e^x}\left[x \cdot e^x - \int e^x\,dx\right]$

$= \frac{1}{e^x}\left[x e^x - e^x + c\right]$ Put $v(x) = \tan y$

$\left\{\tan y = \frac{1}{e^x}\left[x e^x - e^x + c\right]\right\}$ Answer

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Q NO 2:→ $y^3 \frac{dy}{dx} + \frac{1}{2x} y^4 = x.$

Chain Rules

$\frac{dv}{dx} = \frac{dv}{dy} \cdot \frac{dy}{dx}$

$\frac{dv}{dx} = \frac{d}{dy}(y^4) \cdot \frac{dy}{dx}$

$\frac{dv}{dx} = 4y^3 \cdot \frac{dy}{dx}$

Solution:→ Let $v = y^4$

$\Rightarrow \frac{dv}{dx} = 4y^3 \cdot \frac{dy}{dx}$

$\Rightarrow \frac{1}{4}\frac{dv}{dx} = y^3 \frac{dy}{dx}$

$\Rightarrow \frac{1}{4}\frac{dv}{dx} + \frac{1}{2x} v = x$

$\Rightarrow \times$ by 4

$\Rightarrow \frac{dv}{dx} + \frac{2}{x} v = 4x.$

$P(x) = \frac{2}{x}$ and $\phi(x) = 4x.$

$\mu(x) = ?$

$\Rightarrow \mu(x) = e^{\int P(x)\,dx}.$

$\Rightarrow \quad = e^{\int \frac{2}{x}\,dx}.$

$= e^{2\int \frac{1}{x}\,dx}$

$= e^{2\ln x}$ ∵ Applying log formula

$= e^{\ln x^2}$

$\boxed{\mu(x) = x^2}$

P.T.O

(12)

Formula:→ $y(x) = \frac{1}{\mu(x)} \int \mu(x) \cdot \phi(x)\, dx$.

Putt the values in formula.

$y(x) = \frac{1}{x^2} \int x^2 \cdot 4x\, dx$.

$= \frac{1}{x^2} \int 4x^3 dx$.

$= \frac{1}{x^2} \left(4 \frac{x^4}{4} dx \right)$

$= \frac{1}{x^2} (x^4 dx)$

$= \frac{x^4}{x^2} + \frac{c}{x^2}$

$\left\{ (y(x) = x^2 + \frac{c}{x^2} \right\}$ Answer.

—*—*—*—*—*—

"I Will Win
Not Immediately
but definitely

Fawad Ahmad

(12)

HIGHER ORDER LINEAR (ODE) :→

$$a \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} \cdots\cdots$$

$$a_{n-1} \frac{dy}{dx} + a_n y = f(x)$$

Case1:→ IF Real & Distinct.

$y = C_1 e^{m_1 x} + C_1 e^{m_2 x}$ | $m_1, m_2 \ldots m_n$ are roots.

$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$

Case2:→ Real & Equal.

$y = (c_1 + c_2 x) e^{mx}$

$y = (c_1 + c_2 x + c_3 x^2) e^{mx}$.

Case3:→ Imignary & Distinct

$\alpha + i\beta$

$y = e^{\alpha x} (c_1 \cos\beta x + C_2 \sin\beta x)$

Case4:→ Imignary & Equal.

$y = e^{\alpha x} ((c_1 + c_2 x) \cos\beta x + (c_3 + c_4 x) \sin\beta x)$

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Q no 01:-> $9\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$

Solution:-> $(9D^2 - 12D + 4)\,y = 0$

Characteristics Equation

$9D^2 - 12D + 4 = 0$

$a = 9,\ b = -12$ and $c = 4$

$\Rightarrow D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow D = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$

$\Rightarrow D = \frac{12 \pm \sqrt{144 - 144}}{18}$

$\Rightarrow D = \frac{12 \pm \sqrt{0}}{18} \Rightarrow \frac{12 \pm 0}{18},$

$\Rightarrow D = \frac{\cancel{12} + 0}{\cancel{18}}, \frac{\cancel{12} - 0}{\cancel{18}}$

$\Rightarrow D = \frac{2}{3}, \frac{2}{3}$

$\{y = (c_1 + c_2 x)\,e^{2/3\,x}\}$ Answer.

Q no 02:-> $2y'' - 5y' - 3y = 0$

Solution:-> $(2D^2 - 5D - 3)\,y = 0$

Characteristics Equation

$2D^2 - 5D - 3 = 0$

$a = 2,\ b = -5$ & $c = -3$

$\Rightarrow D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Put the values.

$\Rightarrow D = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$\Rightarrow D = \frac{5 \pm \sqrt{25 + 24}}{4}$

$\Rightarrow D = \frac{5 \pm \sqrt{49}}{4}$

$\Rightarrow D = \frac{5 \pm 7}{4}$

$\Rightarrow D = \frac{5 + 7}{4}, \frac{5 - 7}{4}$

$\Rightarrow D = \frac{12}{4}, -2/4$

$\Rightarrow D = 3, -1/2$

$\{y = c_1 e^{-\frac{1}{2}x} + c_2 e^{3x}$

Answer.

Q No3:-> $y''' - 4y'' - 5y' = 0$

Solution :-> $(D^3 - 4D^2 - 5D)y = 0$

Characteristic Equation

$D^3 - 4D^2 - 5D = 0$

$D(D^2 - 4D - 5) = 0$

$D = 0, \quad D^2 - 4D - 5 = 0$ By Factorization

$D = 5, -1$

$y = c_1 e^{0x} + c_2 e^{5x} + c_3 e^{-x}$.

$\{ y = c_1 + c_2 e^{5x} + c_3 e^{-x} \}$ Answer

Q No4:-> $y''' - y = 0$

$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(D^3 - 1)y = 0$

$D^3 - 1 = 0$

$(D-1)(D^2 + D + 1) = 0$

$D = 1 \quad D^2 + D + 1 = 0$

$a = 1, \quad b = 1 \quad \& \quad c = 1$

P.T.O

$\Rightarrow D = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow D = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow D = \dfrac{-1 \pm \sqrt{-3}}{2}$

$\Rightarrow D = \dfrac{-1 \pm i\sqrt{3}}{2}$

$\Rightarrow D = \dfrac{-1}{2} + \dfrac{i\sqrt{3}}{2}, \quad \dfrac{-1}{2} - \dfrac{i\sqrt{3}}{2}$

$\alpha = \dfrac{-1}{2}, \quad \beta = \dfrac{\sqrt{3}}{2}$

$y = c_1 e^{x} + e^{\alpha x}(c_2 \cos \beta x + c_3 \sin \beta x)$

$\{ y = c_1 e^{x} + e^{-\frac{1}{2}x}\left(c_2 \cos \frac{\sqrt{3}}{2}x + c_3 \sin \frac{\sqrt{3}}{2}x\right)$

Answer

-> Morning : Tired
-> Afternoon : Dying For a rest
-> Night : Can't sleep
"Student Life"

Fawad Ahmad

(25)

Q No 5:-> $y^{iv} - 2y'' + y = 0$

Solution:-> $(D^4 - 2D^2 + 1)y = 0$

Characteristic Equation

$\Rightarrow D^4 - 2D^2 + 1 = 0$

$\Rightarrow D^4 - D^2 - D^2 + 1 = 0$

$\Rightarrow D^2(D^2 - 1) - 1(D^2 - 1) = 0$

$\Rightarrow (D^2 - 1)(D^2 - 1) = 0$

$\sqrt{D^2} = \sqrt{1}$, $\sqrt{D^2} = \sqrt{1}$

$D = \pm 1$, $D = \pm 1$

$\{y = (c_1 + c_2 x)e^{x} + (c_3 + c_4 x)e^{-x}\}$ Answer

Q No 6:-> $y^{iv} - 7y'' - 18y = 0$

Solution:-> $(D^4 - 7D^2 - 18)y = 0$

Characteristic Equation

$\Rightarrow (D^4 - 7D^2 - 18) = 0$

$\Rightarrow D^4 - 9D^2 + 2D^2 - 18 = 0$

$\Rightarrow D^2(D^2 - 9) + 2(D^2 - 9) = 0$

$\Rightarrow (D^2 + 2)(D^2 - 9) = 0$

P.T.O

(2

$\Rightarrow (D-3)(D+3)(D^2+2) = 0$

$\Rightarrow (D-3) = 0$ and $(D+3) = 0$ and $(D^2+2) = 0$

$\Rightarrow D = 3$, $D = -3$ and $D^2 = -2$

$\Rightarrow D = 3$, $D = -3$ and $D = \pm\sqrt{-2}$

$\Rightarrow D = 3$, $D = -3$ $\alpha = 0$ $\beta = \sqrt{2}$

As General Solution is

$\Rightarrow y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + e^{\alpha x}(C_3 \cos\beta x +$
$C_4 \sin\beta x)$

$\{y = C_1 e^{3x} + C_2 e^{-3x} + C_3 \cos\sqrt{2}x + C_4 \sin\sqrt{2}x$

Answer

Q No 7:-> $16y^{iv} + 24y'' + 9y = 0$

Solution:-> $(16D^{iv} + 24D'' + 9)y = 0$

Characteristics Equation

$\Rightarrow (16D^{iv} + 24D'' + 9) = 0$

$\Rightarrow 16D^4 + 24D^2 + 9 = 0$

$\Rightarrow 4D^2(4D^2 + 3) + 3(4D^2 + 3) = 0$

$\Rightarrow (4D^2 + 3)(4D^2 + 3) = 0$ P.T.O

$\Rightarrow 4D^2+3=0$ and $4D^2+3=0$

$\Rightarrow D^2 = \frac{-3}{4}$ and $D^2 = \frac{-3}{4}$

$\Rightarrow \sqrt{D^2} = \sqrt{\frac{-3}{4}}$ and $\sqrt{D^2} = \sqrt{\frac{-3}{4}}$

$\Rightarrow D = \sqrt{\frac{-3}{4}}$ and $D = \sqrt{\frac{-3}{4}}$

$\Rightarrow D = -\frac{\sqrt{3}}{2}i, \frac{\sqrt{3}}{2}i$ and $D = -\frac{\sqrt{3}}{2}i, \frac{\sqrt{3}}{2}$

$\because \alpha = 0$ and $\beta = \frac{\sqrt{3}}{2}$

$y = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x + C_3\cos\beta x + C_4\sin\beta x)$

$\therefore \left\{ y = C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x + C_3\cos\frac{\sqrt{3}}{2}x + C_4\sin\frac{\sqrt{3}}{2}x \right\}$ Answer.

" Every One has weakness but remember that I'm not everyone.

Fawad Ahmad

(30)

CAUCHY EULER EQUATION Formulas

There are two types of equation

(i) with constant coefficient:

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

Formulas:

(1) Real and Equal:

$m_1 = m_2$ and real

$y(x) = (c_1 + xc_2)e^{m_1x}$

(ii) Real and not equal

$m_1 \neq m_2$ and real

$y(x) = c_1e^{m_1x} + c_2e^{m_2x}$

(iii) Imaginary

$m = \alpha + i\beta$

$y(x) = e^{\alpha x}[c_1\cos\beta x + c_2\sin\beta x]$

(2) With variable coefficient:

$$a(x)\frac{d^2y}{dx^2} + b(x)\frac{dy}{dx} + c(x)y = 0$$

(i) Real and equal:

... and real

$$y(x) = (c_1 + c_2 \ln x)\, x^{m}$$

(ii) Real and Not equal.

$m_1 \neq m_2$ and real

$$y(x) = c_1 x^{m_1} + c_2 x^{m_2}$$

(iii) Imaginary:

$m = \alpha + i\beta$

$$y(x) = x^{\alpha}\,[c_1\cos(\beta \ln x) + c_2\sin(\beta \ln x)$$

CAUCHY EULER EQUATION:→

A 2nd order ODE of the form

$$ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$$

is called cauchy - Euler equation.

Q.No1:→ $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$

Solution:→ $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$

let $y = x^m$

$y' = mx^{m-1}$ → 1st derivative

$y'' = m(m-1)x^{m-2}$ → 2nd derivative

$\Rightarrow x^2 m(m-1)x^{m-2} - 2x\,m x^{m-1} - 4x^m = 0$

$\Rightarrow m(m-1)x^m - 2mx^m - 4x^m = 0$

$\Rightarrow x^m(m(m-1) - 2m - 4) = 0$

$\Rightarrow (m^2 - m - 2m - 4)x^m = 0$

$\Rightarrow (m^2 - 3m - 4)x^m = 0$

characteristics Equation

$m^2 - 3m - 4 = 0$

P.T.O

$\Rightarrow m^2 - 4m + m - 4 = 0$

$\Rightarrow m(m-4) + 1(m-4) = 0$

$\Rightarrow m = 4 \;,\; m = -1$

$\{ y = c_1 x^4 + c_2 x^{-1} \}$ Answer

Q no 02:- $4x^2 \dfrac{d^2y}{dx^2} + \dfrac{8x\,dy}{dx} + y = 0$

Solution:- $4x^2 \dfrac{d^2y}{dx^2} + 8x \dfrac{dy}{dx} + y = 0$

Let $y = x^m$

$\Rightarrow y' = mx^{m-1}$

$\Rightarrow y'' = m(m-1)x^{m-2}$

$\Rightarrow 4x^2 m(m-1)x^{m-2} + 8x\,m x^{m-1} + x^m = 0$

$\Rightarrow x^m(4m(m-1) + 8m + 1) = 0$

$\Rightarrow x^m(4m^2 + 4m + 1) = 0$

$\Rightarrow 4m^2 + 4m + 1 = (2m+1)^2$

$\Rightarrow x^m(2m+1)^2 = 0$

$\Rightarrow (2m+1)^2 = 0$

$\Rightarrow (2m+1)(2m+1) = 0$

$\Rightarrow m = -1/2 \;,\; m = -1/2$

$y(x) = (c_1 + \ln x\, c_2)x^{-1/2}$ Answer

Q no 03:- $4x^2y'' + 17y = 0$

Solution:- $4x^2y'' + 17y = 0$

Let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow 4x^2 m(m-1)x^{m-2} + 17x^m = 0$

$\Rightarrow x^m(4m(m-1) + 17) = 0$

$\Rightarrow x^m(4m^2 - 4m + 17) = 0$

$\Rightarrow 4m^2 - 4m + 17 = 0$

$\Rightarrow a = 4,\; b = -4$ and $c = 17$

$\Rightarrow m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow m = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(17)}}{2a}$

$\Rightarrow m = \dfrac{4 \pm \sqrt{16 - 272}}{2(4)}$

P.T.O

$\Rightarrow \quad m = \dfrac{4 \pm \sqrt{16-272}}{8}$

$\Rightarrow \quad m = \dfrac{4 \pm 16i}{8}$

$\Rightarrow \quad m = \dfrac{1}{2} + 2i$

$m = \dfrac{1}{2} + 2i \;, \quad \dfrac{1}{2} - 2i$

$\alpha = \dfrac{1}{2} \quad \beta = 2$

$y(x) = x^{1/2}(c_1 \cos 2 \ln x + c_2 \sin 2 \ln x)$

Ans

Q NO 4:→ $x^2y'' - 2y = 0$

:→ $x^2y'' - 2y = 0$

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow x^2(m(m-1)x^{m-2}) - 2x^m = 0$

$\Rightarrow (m^2 - m)x^m - 2x^m = 0$

$\Rightarrow x^m(m^2 - m - 2) = 0$

$\Rightarrow m^2 + m - 2m - 2 = 0$

$\Rightarrow m(m+1) - 2(m+1) = 0$

$\Rightarrow (m+1)(m-2) = 0$

$m + 1 = 0$ and $m - 2 = 0$

$m_1 = -1$ and $m_2 = 2$

$y = c_1x^{m_1} + c_2x^{m_2}$

$\{y = c_1x^{-1} + c_2x^2\}$ Answer.

Q NO 5:→ $4x^2y'' + y = 0$

Solution:→ $4x^2y'' + y = 0$

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow 4x^2(m(m-1)x^{m-2}) + x^m = 0$

$\Rightarrow (4m^2 - 4m)x^m + x^m = 0$

$\Rightarrow (4m^2 - 4m + 1)x^m = 0$

$\Rightarrow 4m^2 - 4m + 1 = 0$

$\Rightarrow 4m^2 - 2m - 2m + 1 = 0$

$\Rightarrow 2m(2m-1) - 1(2m-1) = 0$

$\Rightarrow (2m-1)(2m-1) = 0$

$2m - 1 = 0$ and $2m - 1 = 0$

$m_1 = \dfrac{1}{2}$ and $m_2 = \dfrac{1}{2}$ P.T.O

$\{y = (c_1 + c_2 \ln x) x^{1/2}\}$ Answer

QNO6:-> $x y'' - 3y' = 0$

Solution:-> $x y'' - 3y' = 0$

let $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

$\Rightarrow x m(m-1) x^{m-2} - 3 m x^{m-1} = 0$

$\Rightarrow (m^2 - m) x^{m-1} - 3 m x^{m-1} = 0$

$\Rightarrow x^m (m^2 - m - 3m) = 0$

$\Rightarrow m^2 - m - 3m = 0$

$\Rightarrow m^2 - 4m = 0$

$\Rightarrow m(m-4) = 0$

$m = 0$ and $m - 4 = 0$

$m = 0$ and $m = 4$

$y = C_1 x^{m_1} + C_2 x^{m_2}$

$\{y = C_1 + C_2 x^4\}$ Answer.

دل کے رشتے قسمت سے ملتے ہیں
ورنہ ملاقات تو ہزاروں سے ہوتی ہے

QNO7:-> $x^2 y'' + x y' + 4y = 0$

Solution:-> $x^2 y'' + x y' + 4y = 0$

let $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

$\Rightarrow x^2 m(m-1) x^{m-2} + x m x^{m-1} + 4 x^m = 0$

$\Rightarrow (m^2 - m) x^m + m x^m + 4 x^m = 0$

$\Rightarrow x^m (m^2 - m + m + 4) = 0$

$\Rightarrow x^m (m^2 + 4) = 0$

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm i2 = \alpha \pm i\beta$

$\alpha = 0, \beta = 2$

$y(x) = x^0 (c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x))$ $\because x^0 =$

$y(x) = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)$ Answer

Q NO8:→ $x^2y'' + 5xy' + 3y = 0$

Solution:→ $x^2y'' + 5xy' + 3y = 0$

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow x^2 m(m-1)x^{m-2} + 5x\, mx^{m-1} + 3x^m = 0$

$\Rightarrow m(m-1)x^m + 5mx^m + 3x^m = 0$

$\Rightarrow (m^2 - m)x^m + 5mx^m + 3x^m = 0$

$\Rightarrow (m^2 - m + 5m + 3)x^m = 0$

$\Rightarrow (m^2 + 4m + 3)x^m = 0$

$\Rightarrow (m^2 + 4m + 3) = 0$

$\Rightarrow (m^2 + m + 3m + 3) = 0$

$\Rightarrow m(m+1) + 3(m+1) = 0$

$\Rightarrow (m+1)(m+3) = 0$

$m_1 = -1$ and $m_2 = -3$

$y = C_1x^{m_1} + C_2x^{m_2}$

$\{y = C_1x^{-1} + C_2x^{-3}\}$ Answer.

Q NO9:→ $x^2y'' - 3xy' - 2y = 0$

Solution:→ $x^2y'' - 3xy' - 2y = 0$

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow x^2 m(m-1)x^{m-2} - 3x\, mx^{m-1} - 2x^m = 0$

$\Rightarrow m(m-1)x^m - 3mx^m - 2x^m = 0$

$\Rightarrow (m^2 - m)x^m - 3mx^m - 2x^m = 0$

$\Rightarrow (m^2 - m - 3m - 2)x^m = 0$

$\Rightarrow m^2 - 4m - 2 = 0$

$\Rightarrow a = 1$, $b = -4$ and $c = -2$

$\Rightarrow m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow m = \dfrac{4 \pm \sqrt{16 - 4(1)(-2)}}{2}$

$\Rightarrow m = \dfrac{4 \pm \sqrt{16 + 8}}{2}$

$\Rightarrow m = \dfrac{4 \pm \sqrt{4 \times 6}}{2}$

$\Rightarrow m = \dfrac{\cancel{4} \pm 2\sqrt{6}}{\cancel{2}}$

$\Rightarrow m = 2 \pm \sqrt{6}$

D.T.O

$\Rightarrow m_1 = 2+\sqrt{6}$ and $m_2 = 2-\sqrt{6}$

$y = C_1 x^{m_1} + C_2 x^{m_2}$

$y = C_1 x^{(2+\sqrt{6})} + C_2 x^{(2-\sqrt{6})}$

Q No 10:-> $x^2y'' + 3xy' - 4y = 0$

Solution :-> $x^2y'' + 3xy' - 4y = 0$

Let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow x^2 m(m-1)x^{m-2} + 3x\,m x^{m-1} - 4x^m = 0$

$\Rightarrow m(m-1)x^m + 3mx^m - 4x^m = 0$

$\Rightarrow (m^2-m)x^m + 3mx^m - 4x^m = 0$

$\Rightarrow (m^2 - m + 3m - 4)x^m = 0$

$\Rightarrow (m^2 + 2m - 4)x^m = 0$

$\Rightarrow (m^2 + 2m - 4) = 0$

$a = 1$, $b = 2$ and $c = -4$

$\Rightarrow m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

P.T.O

$\Rightarrow m = \dfrac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{4+16}}{2}$

$= \dfrac{-2 \pm \sqrt{20}}{2}$

$= \dfrac{-2 \pm \sqrt{4\times 5}}{2}$

$= -1 \pm \sqrt{5}$

$= -1+\sqrt{5}\ ,\ -1-\sqrt{5}$

$y = C_1 x^{m_1} + C_2 x^{m_2}$

$y = C_1 x^{(-1+\sqrt{5})} + C_2 x^{(-1-\sqrt{5})}$ } Answer

The End

Recomended book

Advanced Engineering Mathematics 6th Edition

by Dennis G. Zill