$x^1+1=0$ Linear equation

$x^2+x+2=0$ quadratic equation

$y = x^2+8$ Algebraic equation

$\frac{dy}{dx} = 2x$ Differential equation

$y = x^2+8$

$y' = \frac{dy}{dx} = 2x$

$\frac{d^2y}{dx^2} = 2$

solve

$\frac{dy}{dx} = 2x$

solution :-

separate variables

$dy = 2x \cdot dx$

$\int dy = \int 2x \, dx$

$y = 2\int x^1 \, dx$

$y = 2 \frac{x^{1+1}}{1+1} + c$

formula

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$y = \frac{2x^2}{2} + c$

(2)

4/9/018

$\boxed{y = x^2 + c}$

Solve

$$\frac{dy}{dx} = \sin 3x$$

solution:

seprate variables

$$dy = \sin 3x \, dx$$

$$\int dy = \int \sin 3x \, dx$$

formula

$$\int \sin x \, dx = \frac{-\cos x}{\frac{d}{dx}(x)} + c$$

$$y = \frac{-\cos(3x)}{\frac{d}{dx}(3x)} + c$$

$$\boxed{y = \frac{-\cos 3x}{3} + c}$$

solve

$$\frac{dy}{dx} = (x+1)^2$$

solution:

Firstly we have to expnd RHS

$$\frac{dy}{dx} = x^2 + 2x + 1$$

(3)

4/9/018

seprate variables

$$dy = x^2 + 2x + 1 \; dx$$

$$\int dy = \int x^2 \, dx + 2\int x \, dx + \int 1 \cdot dx$$

$$y = \frac{x^3}{3} + \frac{\cancel{2} x^2}{\cancel{2}} + x + c$$

$$\boxed{y = \frac{x^3}{3} + x^2 + x + c}$$

5/9/018

Question:

$$dy - (y-1)^2 dx = 0$$

solution:

$$dy - (y-1)^2 dx = 0$$

$$\Rightarrow -dy = -(y-1)^2 dx$$

$$\Rightarrow dx = \frac{\cancel{-}dy}{\cancel{-}(y-1)^2}$$

$$\Rightarrow dx = \frac{dy}{(y-1)^2}$$

$$\Rightarrow dx = dy \, (y-1)^{-2}$$

(4)

5/9/018

Taking integration.

$$\int (y-1)^{-2}\, dy = \int dx$$

$$\frac{(y-1)^{-2+1}}{-2+1} = x + c$$

$$x + c = \frac{(y-1)^{-1}}{-1}$$

$$x + c = -(y-1)^{-1}$$

$$\boxed{x + c = (y-1)}$$

Question:- $x \frac{dy}{dx} \quad 4y$

solution:

$$x\, dy = 4y\, dx$$

$$\frac{4\, dx}{x} = \frac{dy}{y}$$

$$\frac{dy}{y} = \frac{4\, dx}{x}$$

Taking integration

$$\int \frac{1}{y}\, dy = 4 \int \frac{1}{x}\, dx$$

(5)

5/9/018

$$\int \frac{1}{x}\, dx = \ln x + c$$

Formula

$$\int \frac{1}{x}\, dx = \ln x + c$$

$$\int \frac{1}{y}\, dy = \ln y + c$$

$$\boxed{\ln y = 4 \ln x + c}$$

Question:

$$\frac{dy}{dx} + 2x^2 y = 0$$

solution:

$$dy = -2x y^2\, dx$$

$$\frac{dy}{y^2} = -2x\, dx$$

Taking integration.

$$\int y^{-2}\, dy = -2\int x\, dx$$

$$\frac{y^{-2+1}}{-2+1} = -2\, \frac{x^{1+1}}{1+1} + C$$

$$\frac{y^{-1}}{-1} = -\cancel{2}\, \frac{x^2}{\cancel{2}} + C$$

$$-y^{-1} = -x^2 + C$$

(6) 6/9/2018

$$-\frac{1}{y} = -x^2 + c$$

$$\boxed{\frac{1}{y} = x^2 + c}$$

Questions:-

$$\frac{dy}{dx} = e^{3x+2y}$$

solution:-

$$\frac{dy}{dx} = e^{3x} \cdot e^{2y}$$

$$\frac{dy}{e^{2y}} = e^{3x}\, dx$$

apply integration

$$\int e^{-2y}\, dy = \int e^{3x}\, dx$$

$$\boxed{\frac{e^{-2y}}{-2} = \frac{e^{3x}}{3} + c}$$

Formula

$$\int e^{x}\, dx = \frac{e^{x}}{\frac{d}{dx}(x)}$$

(7) 6/9/2018

Questions:-

$$e^{x}y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$$

solution:-

$$e^{x}y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$$

$$e^{x}y \frac{dy}{dx} = e^{-y}(1 + e^{-2x})$$

$$\frac{e^{x}y\, dy}{e^{-y}} = (1+e^{-2x})\, dx$$

$$y\, dy \cdot e^{y} = \frac{(1+e^{-2x})\, dx}{e^{x}}$$

$$\int y \cdot e^{y}\, dy = \int (1+e^{-2x}) \cdot e^{-x}\, dx$$

applying integration.

using integration By parts

putting values in formula

Formula

$$\int f \cdot g\, dx = f \cdot \int g - \int \frac{d}{dx}(f) \cdot \int g\, dx$$

(8)

6/9/018

$$\Rightarrow y\int e^y - \int \frac{d}{dy}(y)\cdot \int e^y\,dy = \frac{e^{-x}}{-1} + \frac{e^{-3x}}{-3}$$

$$y\cdot e^y - \int 1\cdot e^y\,dy = \frac{e^{-x}}{-1} + \frac{e^{-3x}}{-3}$$

$$\boxed{y\cdot e^y - e^y = -e^{-x} - \frac{e^{-3x}}{3} + C}$$

Questions:-

$$\int x^3 e^{2x}$$

Solution:-

By applaying integration By parts.

$$\Rightarrow x^3\int e^{2x} - \int \frac{d}{dx}(x^3)\int e^{2x}\,dx$$

$$\Rightarrow x^3\frac{e^{2x}}{2} - \int 3x^2\cdot\frac{e^{2x}}{2}\,dx$$

again applaying integration By part

$$\Rightarrow \frac{x^3e^{2x}}{2} - \left[3x^2\cdot\int\frac{e^{2x}}{2} - \int\frac{d}{dx}(3x^2)\cdot\int\frac{e^{2x}}{2}\,dx\right]$$

$$\Rightarrow \frac{x^3e^{2x}}{2} - 3x^2\frac{e^{2x}}{4} + \int 6x\cdot\frac{e^{2x}}{4}\,dx$$

again applaying integration by parts

$$\Rightarrow \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \left[6x\cdot\frac{e^{2x}}{8} - \int 6\int\frac{e^{2x}}{4}\,dx\right]$$

(9)

11/9/018

$$\Rightarrow \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{6xe^{2x}}{8} - \int 6\cdot\frac{e^{2x}}{8}\,dx$$

$$\Rightarrow \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} - \frac{6xe^{2x}}{8} - \frac{6e^{2x}}{16} + C$$

$$\Rightarrow \boxed{\frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3xe^{2x}}{4} - \frac{3e^{2x}}{8}}$$

or

Question:- $\int x^3e^{2x}$

	d/dx	$\int$
		e^{2x}
+	x^3	$\frac{e^{2x}}{2}$
−	$3x^2$	$\frac{e^{2x}}{4}$
+	$6x$	$\frac{e^{2x}}{8}$
−	6	$\frac{e^{2x}}{16}$
	0	

$$\Rightarrow \boxed{\frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{6xe^{2x}}{8} - \frac{6e^{2x}}{16} + C}$$

(10)

11/9/018

Homogenous Differential equation :-

Equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$

is said to be homogenous differential equation of $f(x,y)$ and $g(x,y)$ are homogenous functions of same degree.

eg

$\frac{x-y}{x+y}$ $\quad$ $\frac{xy+y^2}{x^2+y^2}$

$\frac{x^2+y^2}{x^2-y^2}$

They are always with out const.

Question :-

$(x-y)\,dx + (x+y)\,dy = 0$

$(x+y)\,dy = -1(x-y)\,dx$

$$\frac{dy}{dx} = \frac{(y-x)}{(x+y)} \rightarrow (1)$$

put $y = vx \rightarrow (2)$

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

(11)

11/9/018

By using products rules

$\Rightarrow v \frac{d}{dx}(x) + x \frac{d}{dx}(v)$

$\Rightarrow v \cdot 1 + \frac{x\,dv}{dx}$

$\Rightarrow v + \frac{x\,dv}{dx} \rightarrow (3)$

put eq (2) & (3) in (1)

$$\Rightarrow v + \frac{x\,dv}{dx} = \frac{vx - x}{x + vx}$$

$$\Rightarrow v + \frac{x\,dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(v+1)}$$

$$\Rightarrow v + \frac{x\,dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow \frac{x\,dv}{dx} = \frac{v-1}{v+1} - v$$

Taking LCM

$$\Rightarrow \frac{x\,dv}{dx} = \frac{(v-1) - v - v^2}{(1+v)}$$

$$\Rightarrow \frac{x\,dv}{dx} = \frac{\cancel{v} - 1 - \cancel{v} - v^2}{(1+v)}$$

12

11/9/018

$$\frac{x\,dv}{dx} = \frac{-1-v^2}{1+v}$$

$$\frac{x\,dv}{dx} = \frac{-(1+v^2)}{1+v}$$

in shifting whole term the term will invert

$$\left(\frac{1+v}{1+v^2}\right) dv = -\frac{1}{x}\,dx$$

$$\left(\frac{1}{1+v^2}\right) dv + \left(\frac{v}{1+v^2}\right) dv = -\frac{1}{x}\,dx$$

Now taking integration
multiplying and dividing by 2 to fit to rule

$$\int\left(\frac{1}{1+v^2}\right) dv + \frac{1}{2}\int\frac{2v}{1+v^2}\,dv = -\int\frac{1}{x}\,dx$$

$$\Rightarrow \tan^{-1}v + \frac{1}{2}\ln(1+v^2) = -\ln x + C$$

where $v = y/x$

Formulas used

1) $\int\frac{f'(x)}{f(x)}\,dx = \ln f(x)+c$

2) $\int\frac{1}{1+x^2}\,dx = \tan^{-1}x + c$

13

13/09/018

Question:-

$$(y^2+2xy)\,dx + x^2\,dy = 0$$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = \frac{-(y^2+2xy)}{x^2} \longrightarrow (i)$$

put $y = vx$ in LHS $\longrightarrow$ (ii)

$$\frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

applying products rules.

$$= v\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(v)$$

$$= v\cdot(1) + \frac{x\,dv}{dx}$$

$$= v + \frac{x\,dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = v + \frac{x\,dv}{dx} \longrightarrow (iii)$$

Now put (ii) & (iii) in (i)

$$v + \frac{x\,dv}{dx} = -\frac{((vx)^2 + 2x(vx))}{x^2}$$

$$= -\frac{(v^2x^2 + 2x^2v)}{x^2}$$

$$= -\frac{\cancel{x^2}(v^2+2v)}{\cancel{x^2}}$$

$$v + \frac{x\,dv}{dx} = (-v^2 - 2v)$$

$$\frac{x\,dv}{dx} = -v^2 - 2v - v$$

(14)

13/9/018

$$\frac{x\,dv}{dx} = -v^2-3v$$

$$x\,dv = (-v^2-3v)\ dx$$

$$x\,dv = -(v^2+3v)\ dx$$

$$\frac{dv}{v^2+3v} = -\frac{dx}{x}$$

Taking integration.

$$\Rightarrow \int \frac{1}{v^2+3v}\,dv = -\int \frac{1}{x}\,dx$$

multiplying & divide LHS by 3

$$\Rightarrow \frac{1}{3}\int \frac{1\times 3}{v^2+3v}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{3}\int \frac{3}{v(v+3)}\,dv = -\int \frac{1}{x}\,dx.$$

Adding and subtracting v from LHS

$$\Rightarrow \frac{1}{3}\int \frac{v+3-v}{v(v+3)}\,dv = -\int \frac{1}{x}\,dx.$$

$$\Rightarrow \frac{1}{3}\int \frac{(v+3)}{v(v+3)}\,dv - \frac{v}{v(v+3)}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{3}\int \frac{1}{v}\,dv - \frac{1}{3}\int \frac{1}{v+3}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{3}\ln v - \frac{1}{3}\ln v+3 = -\ln x + c$$

where $v = y/x$

(15)

13/09/018

Question:-

$$(x^2-3y^2)\,dx + 2xy\,dy = 0$$

$$2xy\,dy = -(x^2-3y^2)\,dx$$

$$\frac{dy}{dx} = \frac{-(x^2-3y^2)}{2xy} \longrightarrow (i)$$

put $y = vx$ in LHS $\rightarrow$ (ii)

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

applying product rules.

$$\Rightarrow \frac{d}{dx}(y) = v\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(v)$$

$$\Rightarrow \frac{d}{dx}(y) = v\cdot(1) + \frac{x\,dv}{dx}.$$

$$\Rightarrow \frac{d}{dx}(y) = v + \frac{x\,dv}{dx} \longrightarrow (iii)$$

put (ii) & (iii) in (i)

$$\Rightarrow v + \frac{x\,dv}{dx} = \frac{-(x^2-3(vx)^2)}{2x(vx)}$$

$$= \frac{-(x^2-3v x^2)}{2x^2 v}$$

$$= \frac{-x^2(1-3v)}{x(2v)}$$

13/9/018

$$v + \frac{x\,dv}{dx} = \frac{-1 + 3v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{-1 + 3v^2}{2v} - v$$

Now taking LCM

$$\frac{x\,dv}{dx} = \frac{-1 + 3v^2 - 2v^2}{2v}$$

$$x\,dv = \left(\frac{-1+3v^2-2v^2}{2v}\right) dx$$

$$\frac{2v\,dv}{-1+v^2} = \frac{1}{x}\,dx$$

$$\left(\frac{2v}{v^2-1}\right) dv = \frac{1}{x}\,dx$$

Now taking integration

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{1}{x}\,dx$$

$$\ln(v^2-1) = \ln x + c$$

where $v = y/x$

Questions:-

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2} \longrightarrow (i)$$

put $y = vx$ in LHS of (i) $\longrightarrow$ (ii)

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \longrightarrow (iii)$$

put (ii) & (iii) in eq (i)

$$v + \frac{x\,dv}{dx} = \frac{x^2 + x(vx) + (vx)^2}{x^2}$$

$$= \frac{x^2 + vx^2 + v^2x^2}{x^2}$$

$$= \frac{\cancel{x^2}(1 + v + v^2)}{\cancel{x^2}}$$

$$v + \frac{x\,dv}{dx} = 1 + v + v^2$$

$$= 1 + v - v + v^2$$

$$\frac{x\,dv}{dx} = 1 + v^2$$

$$\int\left(\frac{1}{1+v^2}\right)dv = \int \frac{1}{x}\,dx$$

$$\boxed{\tan^{-1} v = \ln x + c}$$

Question :-

$$y^2 - x(y+x)\frac{dy}{dx} = 0$$

$$y^2 = x(y+x)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y^2}{x(y+x)}$$

$$= \frac{y^2}{xy + x^2} \longrightarrow (i)$$

put $y = vx$ in LHS $\rightarrow$ (ii)

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \longrightarrow (iii)$$

Put (ii) & (iii) in (i)

$$v + \frac{x\,dv}{dx} = \frac{(vx)^2}{x(vx) + x^2}$$

$$= \frac{v^2x^2}{vx^2 + x^2}$$

$$v + \frac{x\,dv}{dx} = \frac{\cancel{x^2}(v^2)}{\cancel{x^2}(v+1)}$$

$$v + \frac{x\,dv}{dx} = \frac{v^2}{v+1}$$

$$\frac{x\,dv}{dx} = \frac{v^2}{v+1} - v$$

$$= \frac{v^2 - (v^2+v)}{v+1}$$

$$= \frac{\cancel{v^2} - \cancel{v^2} + v}{v+1}$$

$$\frac{x\,dv}{dx} = -\frac{v}{v+1}$$

$$\frac{v+1}{v}\,dv = -\frac{1}{x}\,dx$$

$$\frac{v}{v}\,dv + \frac{1}{v}\,dv = -\frac{1}{x}\,dx$$

$$\int 1\,dv + \int\frac{1}{v}\,dv = -\int\frac{1}{x}\,dx$$

$$v + \ln v = -\ln x + c$$

where $v = y/x$

Question: $(xy^2 + 2x^3)\,dy = y^3\,dx$

$$\frac{dy}{dx} = \frac{y^3}{xy^2+2x^3} \longrightarrow (i)$$

put $y = vx$ in LHS $\longrightarrow$ (ii)

$$\frac{d(y)}{dx} = \frac{d(vx)}{dx}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (iii)$$

put (ii) & (iii) in eq (i)

$$v + x\frac{dv}{dx} = \frac{(vx)^3}{x(vx)^2+2x^3}$$

$$= \frac{v^3x^3}{v^2x^3+2x^3}$$

$$= \frac{\cancel{x^3}(v^3)}{\cancel{x^3}(v^2+2)}$$

$$v + x\frac{dv}{dx} = \frac{v^3}{v^2+2}$$

$$x\frac{dv}{dx} = \frac{v^3}{v^2+2} - v$$

Taking LCM.

$$x\frac{dv}{dx} = \frac{\cancel{v^3} - \cancel{v^3} - 2v}{v^2+2}$$

$$x\frac{dv}{dx} = \frac{-2v}{v^2+2}$$

$$\frac{v^2+2}{2v}\,dv = -\frac{1}{x}\,dx$$

$$\frac{v^{\cancel{2}}}{2\cancel{v}} + \frac{\cancel{2}}{\cancel{2}v} = -\frac{1}{x}\,dx$$

$$\frac{v}{2} + \frac{1}{v} = -\frac{1}{x}\,dx$$

Taking integration

$$\int \frac{v}{2}\,dv + \int \frac{1}{v}\,dv = -\int \frac{1}{x}\,dx$$

$$\boxed{\frac{v^2}{4} + \ln v = -\ln x + c}$$

where $v = y/x$

Question:-

$$xy^2\,dy = y^3 - 2x^3\,dx$$

$$\frac{dy}{dx} = \frac{y^3 - 2x^3}{xy^2} \longrightarrow (i)$$

put $y = vx$ in LHS $\rightarrow$ (ii)

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \rightarrow \text{(iii)}$$

put (ii) & (iii) in (i)

$$v + \frac{x\,dv}{dx} = \frac{(vx)^3 - 2x^3}{x(vx)^2}$$

$$= \frac{v^3x^3 - 2x^3}{v^2x^3}$$

$$= \frac{\cancel{x^3}(v^3-2)}{\cancel{x^3}(v^2)}$$

$$v + \frac{x\,dv}{dx} = \frac{v^3-2}{v^2}$$

$$\frac{x\,dv}{dx} = \frac{v^3-2}{v^2} - v$$

Taking LCM

$$\frac{x\,dv}{dx} = \frac{\cancel{v^3} - 2 - \cancel{v^3}}{v^2}$$

$$\frac{x\,dv}{dx} = \frac{-2}{v^2}$$

$$\frac{v^2}{2}\,dv = -\,{}^1\!/_x\,dx$$

Taking integration.

$$\int \frac{v^2}{2}\,dv = -\int \frac{1}{x}\,dx$$

$$\boxed{\frac{v^3}{3} = -\ln x + c}$$

where $v = y/x$

Question:-

$$(x^2 - 3y^2)\,dx + 2xy\,dy = 0$$

$$2xy\,dy = -(x^2-3y^2)\,dx$$

$$\frac{dy}{dx} = -\frac{(x^2-3y^2)}{2xy} \rightarrow \text{(i)}$$

put $y = vx$ in LHS $\rightarrow$ (ii)

$$\frac{dy}{dx} = \frac{d}{dx}(v.x)$$

$$= v + \frac{x\,dv}{dx} \rightarrow \text{(iii)}$$

put (ii) & (iii) in (i)

$$v + \frac{x\,dv}{dx} = \frac{-(x^2 - 3(vx)^2)}{2x(vx)}$$

$$= \frac{-(x^2 - 3v^2x^2)}{2x^2v}$$

$$= \frac{-\cancel{x^2}(1-3v^2)}{\cancel{x^2}(2v)}$$

$$v + \frac{x\,dv}{dx} = \frac{-1 + 3v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{-1+3v^2}{2v} - v$$

Taking LCM

$$\frac{x\,dv}{dx} = \frac{-1+3v^2-2v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{-1+v^2}{2v}$$

$$\frac{2v}{v^2-1}\,dv = \frac{1}{x}\,dx$$

now taking integration

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{1}{x}\,dx$$

$$\boxed{\ln(v^2-1) = \ln x + C}$$

where $v = y/x$

Question:-

$$\frac{dy}{dx} = \frac{x^2+y^2}{xy} \rightarrow (I)$$

put $y = vx$ in LHS $\rightarrow$ (II)

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$= v + \frac{x\,dv}{dx} \rightarrow (III)$$

put (II) & (III) in (I)

$$v + \frac{x\,dv}{dx} = \frac{x^2+(vx)^2}{x(vx)}$$

$$= \frac{x^2+v^2x^2}{vx^2}$$

$$= \frac{\cancel{x^2}(1+v^2)}{\cancel{x^2}(v)}$$

(2.6)

$$\frac{x\,dv}{dx} = \frac{1+v^2}{v} - v$$

taking LCM

$$\frac{x\,dv}{dx} = \frac{1+v^2-v^2}{v}$$

$$\frac{x\,dv}{dx} = \frac{1}{v}$$

$$v\,dv = \frac{1}{x}\,dx$$

taking integration

$$\int v\,dv = \int \frac{1}{x}\,dx$$

$$\boxed{\frac{v^2}{2} = \ln x + C}$$

where $v = y/x$

Questions :-

$$(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$$

$$(x^2 + 3xy + y^2)\,dx = x^2\,dy$$

we can write at as

(2.7)

$$x^2\,dy = (x^2 + 3xy + y^2)\,dx$$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} \rightarrow (i)$$

put $y = vx \rightarrow (ii)$

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \rightarrow (iii)$$

put (ii) & (iii) in eq (i)

$$v + \frac{x\,dv}{dx} = \frac{x^2 + 3x(vx) + (vx)^2}{x^2}$$

$$= \frac{x^2 + 3x^2 v + x^2 v^2}{x^2}$$

$$= \frac{\cancel{x^2}(1 + 3v + v^2)}{\cancel{x^2}}$$

$$v + \frac{x\,dv}{dx} = 1 + 3v + v^2$$

$$\frac{x\,dv}{dx} = 1 + 3v + v^2 - v$$

$$= 1 + 2v + v^2$$

$$\frac{1}{1+2v+v^2}\,dv = \frac{1}{x}\,dx$$

$$\because (a+b)^2 = a^2 + b^2 + 2ab$$

$$\frac{1}{(1+v)^2}\,dv = \frac{1}{x}\,dx$$

$$(1+v)^{-2}\,dv = 1/x\,dx$$

now taking integration

$$\int (1+v)^{-2}\,dv = \int 1/x\,dx$$

$$\frac{(1+v)^{-2+1}}{-2+1} = \ln x + c$$

$$\frac{(1+v)^{-1}}{-1} = \ln x + c$$

$$\boxed{\frac{-1}{(1+v)} = \ln x + c}$$

where $v = y/x$

Non Homogeneous Differential Equation:

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

case 1:-

$$\frac{a_1}{b_1} \neq \frac{a_2}{b_2}$$

case 2:-

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

Question:- cas 1 scenario.

$$\frac{dy}{dx} = \frac{x + 3y - 5}{x - y - 1}$$

Let

$$x = X + h$$
$$y = Y + k$$

$$\frac{dy}{dx} = \frac{(X+2) + 3(Y+1) - 5}{(X+2) - (Y+1) - 1}$$

$$= \frac{X + \cancel{5} + 3Y - \cancel{5}}{X + \cancel{2} - Y - \cancel{2}}$$

$$\frac{dy}{dx} = \frac{X + 3Y}{X - Y} \longrightarrow (1)$$

Rough. W

$$\frac{a_1}{b_1} = \frac{1}{3}$$

$$\frac{a_2}{b_2} = \frac{1}{-1} = -1$$

— x — x —

$$h + 3k - 5 = c$$
$$\underline{\pm h \mp k \mp 1 = c}$$
$$4k - 4 = 0$$
$$4k = 4$$
$$k = 4/4 = 1$$
$$h + 3(1) - 5 = c$$
$$h - 2 = c$$
$$h = 2$$

Let

$y = vx \longrightarrow$ (ii)

$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \longrightarrow$ (iii)

put (ii) & (iii) in (i)

$$v + \frac{x\,dv}{dv} = \frac{X + 3(vX)}{X - (vX)}$$

$$= \frac{X + 3vX}{X - vX}$$

$$v + \frac{x\,dv}{dx} = \frac{\cancel{X}(1+3v)}{\cancel{X}(1-v)}$$

$$v + \frac{x\,dv}{dx} = \frac{1+3v}{1-v}$$

$$\frac{x\,dv}{dx} = \frac{1+3v}{1-v} - v$$

Taking LCM

$$= \frac{1+3v - v(1-v)}{1-v}$$

$$= \frac{1+3v - v + v^2}{1-v}$$

$$\frac{x\,dv}{dx} = \frac{1+2v+v^2}{1-v}$$

$$\frac{x\,dv}{dx} = \frac{(1+v)^2}{(1-v)}$$

$$\frac{1-v}{(1+v)^2}\,dv = \frac{1}{x}\,dx$$

$$\frac{1}{(v+1)^2}dv - \frac{v+1-1}{(v+1)^2}\,dv = \frac{1}{x}\,dx$$

$$\int\frac{1}{(v+1)^2}\,dv - \int\frac{(v+1)}{(v+1)^2}dv + \int\frac{1}{(v+1)^2}dv = \int\frac{1}{x}\,dx$$

$$\int(v+1)^{-2}\,dv - \int\frac{1}{(v+1)}\,dv + \int(v+1)^{-2}\,dv = \int\frac{1}{x}\,dv$$

$$\frac{(v+1)^{-2+1}}{-2+1} - \ln(v+1) + \frac{(v+1)^{-2+1}}{-2+1} = \ln x + C$$

$$\frac{(v+1)^{-1}}{-1} - \ln(v+1) + \frac{(v+1)^{-1}}{-1} = \ln x + C$$

$$\frac{1}{-1(v+1)} - \ln(v+1) + \frac{1}{-1(v+1)} = \ln x + C$$

$$\frac{1}{-v-1} + \ln(v+1) + \frac{1}{-v-1} = \ln x + C$$

where $v = Y/X$

also $X = x-2$

$Y = y-1$

rough

$x = X+2$

$y = Y+1$

Question:- case 1

$$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$$

Rough

$\frac{a_1}{b_1} = \frac{1}{2}$

$\frac{a_2}{b_2} = \frac{2}{1} = 2$

Let

$x = X + h$

$y = Y + K$

$$\frac{dy}{dx} = \frac{(X+1)+2(Y+1)-3}{2(X+1)+(Y+1)-3}$$

$2(h + 2K - 3) = 0$
$2h + K - 3 = 0$

$\cancel{2h} + 4K - 6 = 0$
$\pm \cancel{2h} \pm K \mp 3 = 0$

$3K - 3 = 0$
$3K = 3$
$K = 3/3 = 1$

$h + 2(1) - 3 = 0$
$h + 2 - 3 = 0$
$h - 1 = 0$
$h = 1$

$$\frac{dy}{dx} = \frac{X+\cancel{1}+2Y+\cancel{2}-\cancel{3}}{2X+\cancel{2}+Y+\cancel{1}-\cancel{3}}$$

$$\frac{dY}{dX} = \frac{X+2Y}{2X+Y} \longrightarrow (i)$$

Let

$$Y = VX \longrightarrow (ii)$$

$$\frac{dY}{dX} = \frac{d}{dX}(VX)$$

$$\frac{dY}{dX} = V + X\frac{dV}{dX} \longrightarrow (iii)$$

put (ii) & (iii) in eq (i)

$$V + X\frac{dV}{dX} = \frac{X+2(VX)}{2X+(VX)}$$

$$= \frac{X+2VX}{2X+VX}$$

$$= \frac{\cancel{X}(1+2V)}{\cancel{X}(2+V)}$$

$$V + X\frac{dV}{dX} = \frac{1+2V}{2+V}$$

$$X\frac{dV}{dX} = \frac{1+2V}{2+V} - V$$

now taking LCM

$$X\frac{dV}{dX} = \frac{1+2V-V(2+V)}{2+V}$$

$$X\frac{dV}{dX} = \frac{1+\cancel{2V}-\cancel{2V}-V^2}{2+V}$$

$$X\frac{dV}{dX} = \frac{1-V^2}{2+V}$$

$$\frac{2+v}{1-v^2}\,dv = \frac{1}{x}\,dx$$

$$\left(\frac{2}{1-v^2} + \frac{v}{1-v^2}\right)dv = \frac{1}{x}\,dx$$

$$\int\frac{2}{1-v^2}\,dv - \frac{1}{2}\int\frac{-2v}{1-v^2}\,dv = \int\frac{1}{x}\,dx$$

Consider $\int \frac{2}{1-v^2}\,dv$

$$\frac{2}{1-v^2} = \frac{A}{1+v} + \frac{B}{(1-v)} \rightarrow (i)$$

xing & divide by $(1+v)(1-v)$

$$2 = A(1-v) + B(1+v) \rightarrow (ii)$$

put $v+1 = 0$

$v = -1$

$2 = 2A + 0$

$A = 2/2 = 1$

put $v-1 = 0$

$v = 1$

$2 = B(2)$

$B = 1$

put values in — (i)

$$\frac{2}{(1+v)(1-v)} = \frac{1}{(1+v)} + \frac{1}{(1-v)}$$

$$\int\frac{1}{1+v}\,dv - \int\frac{-1}{1-v}\,dv - \frac{1}{2}\int\frac{-2v}{1-v^2}\,dv$$

$$= \frac{1}{x}\,dx$$

$$\ln(1+v) - \ln(1-v) - \frac{1}{2}\ln(1-v^2) = \ln x + C$$

where $v = Y/X$

and also

$X = x-1$

$Y = y-1$

$x = X+1$

$X = x-1$

$y = Y+1$

$Y = y-1$

—— x —— x —— x ——

Question:-

$$(2x-3y+4)\,dx + (3x-2y+1)\,dy = 0$$

$$(3x-2y+1)\,dy = -(2x-3y+4)\,dx$$

$$\frac{dy}{dx} = \frac{-(2x-3y+4)}{3x-2y+1}$$

$$\frac{dy}{dx} = \frac{-2x+3y-4}{3x-2y+1}$$

Let

$x = X+h$

$y = Y+k$

$$\frac{dY}{dX} = \frac{-2(X+1)+3(Y+2)-4}{3(X+1)-2(Y+2)+1}$$

$$= \frac{-2X+\cancel{2}+3Y+\cancel{6}-\cancel{4}}{3X+\cancel{3}-2Y-\cancel{4}+\cancel{1}}$$

$\frac{a_1}{b_1} = \frac{-2}{3}$

$\frac{a_2}{b_2} = \frac{3}{-2}$

$3(-2h+3k-4)=0$

$2(3h-2k+1)=0$

$-6h+9k-12=0$

$6h-4k+2=0$

$5k-10=0$

$5k = 10$

$k = 10/5$

$k = 2$

$$\frac{dy}{dx} = \frac{-2x+3y}{3x-2y} \rightarrow (i)$$

$-2h+3(2)-4=$

$-2h+6-4=$

$h=\frac{-2}{-2}$

$h=1$

put $y = vx \rightarrow (ii)$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (iii)$$

put (ii) & (iii) in eq (i)

$$v + x\frac{dv}{dx} = \frac{-2x+3(vx)}{3x-2(vx)}$$

$$= \frac{-2x+3vx}{3x-2vx}$$

$$= \frac{\cancel{x}(-2+3v)}{\cancel{x}(3-2v)}$$

$$v + x\frac{dv}{dx} = \frac{-2+3v}{3-2v}$$

$$x\frac{dv}{dx} = \frac{-2+3v}{3-2v} - v$$

Taking LCM.

problem

$$x\frac{dv}{dx} = \frac{-2+3v-v(3-2v)}{3-2v}$$

$$= \frac{-2+\cancel{3v}-\cancel{3v}+2v^2}{3-2v}$$

$$x\frac{dv}{dx} = \frac{-2+2v^2}{3-2v}$$

$$\frac{3-2v}{-2+2v^2}dv = \frac{1}{x}dx$$

$$\frac{3}{-2+2v^2}dv - \frac{2v}{-2+2v^2}dv = \frac{1}{x}dx$$

$$\frac{3}{2}\int\frac{1}{v^2-1}dv - \frac{1}{2}\int\frac{(2v)}{(v^2-1)}dv = \int\frac{1}{x}dx$$

consider

$$\int\frac{1}{v^2-1}dv$$

Take partial fraction

$$\frac{1}{v^2-1} = \frac{A}{(1+v)} + \frac{B}{(1-v)} \rightarrow (1)$$

xing & dividing by $(1+v)(1-v)$

$$1 = A(1-V) + B(1+V) \rightarrow (ii)$$

put $V+1 = 0$

$V = -1$

$1 = 2A$

$$\boxed{A = 1/2}$$

put $V-1 = 0$

$V = 1$

$B(1+1)$

$1 = B(2)$

$$\boxed{B = 1/2}$$

put in (i)

$$\int \frac{1}{(1+V)(1-V)} = \frac{1}{2(1+V)} + \frac{1}{2(1-V)}$$

$$\frac{3}{2}\int \left(\frac{1}{2(1+V)} + \frac{1}{2(1-V)}\right) dv - \frac{1}{2}\int \frac{2V}{V^2-1} du =$$

$$\frac{3}{4}\int \frac{1}{1+V} dv + \frac{3}{4}\int \frac{1}{1-V} dv - \frac{1}{2}\int \frac{2V}{V^2-1} dv = \frac{1}{X}$$

$$\boxed{= \frac{3}{4} \ln(1+V) + \frac{3}{4} \ln(1-V) - \frac{1}{2} \ln(V^2-1) = \ln X}$$

where $V = Y/X$

$X = x-1$

$Y = y-2$

Question:-

case II :- $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5} \rightarrow (i)$$

Let $(y-x) = z \rightarrow (ii)$

$$\frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{dz}{dx} \rightarrow (iii)$$

put (ii) & (iii) in eq (i)

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

Taking LCM

$$\frac{dz}{dx} = \frac{\cancel{z}+1-\cancel{z}-5}{z+5}$$

$$= \frac{-4}{z+5}$$

$$\int z+5 \, dz = \int -4 \, dx$$

$$\frac{z^2}{2} + 5z = -4x + C$$

where $z = (y-x)$

Question

$(2x+3y-1)\,dx + (2x+3y+2)\,dy = 0$

$$\frac{dy}{dx} = \frac{-(2x+3y-1)}{2x+3y+2}$$

$$\frac{dy}{dx} = \frac{-(2x+3y)+1}{(2x+3y)+2} \longrightarrow (i)$$

Let $2x+3y = z$

$$\frac{dy}{dx} = z \longrightarrow (ii)$$

$$\frac{d}{dx}(2x+3y) = \frac{dz}{dx}$$

$$2 + \frac{3dy}{dx} = \frac{dz}{dx} \rightarrow (ii)$$

$$\frac{3dy}{dx} = \frac{dz}{dx} - 2$$

$$\frac{dy}{dx} = \frac{1}{3}\frac{dz}{dx} - \frac{2}{3} \rightarrow (iii)$$

put (ii) & (iii) in (i)

$$\frac{1}{3}\frac{dz}{dx} - \frac{3}{2} = \frac{-(z)+1}{z+2}$$

$$\frac{1}{3}\frac{dz}{dx} = \frac{-z+1}{z+2} + \frac{2}{3}$$

Taking Lcm.

$$\frac{1}{3}\frac{dz}{dx} = \frac{-3z+3+2z+4}{\cancel{3}(z+2)}$$

$$\frac{dz}{dx} = \frac{-z+7}{z+2}$$

$$\int \frac{z+2}{-z+7}\,dz = \int 1\,dx$$

$$= \int \frac{z}{z-7}\,dz + 2\int \frac{1}{z-7}\,dz = -\int dx$$

$$= \int \frac{z+7-7}{z-7}\,dz + 2\int \frac{1}{z-7}\,dz = -\int dx$$

$$= \int \frac{z-7}{z-7}\,dz + 7\int \frac{1}{z-7} + 2\int \frac{1}{z-7}\,dz = -\int dx$$

$$= \int 1\,dz + 7\int \frac{1}{z-7} + 2\int \frac{1}{z-7}\,dz = -\int dx$$

$$= z + 7\ln(z-7) + 2\ln(z-7) = -x + c$$

where

$z = 2x + 3y.$

Exact Differential equations:

An equation of the form $M(x,y)\,dx + N(x,y)\,dy = 0$ is said to be Exact differential equation of

$$\frac{dM}{dy} = \frac{dN}{\partial x}$$

$$f(x,y) = x^2 + 2xy + x\sin y$$

$$\frac{\partial f}{\partial x} = 2x + 2y + \sin y$$

$$\frac{\partial f}{\partial y} = 0 + 2x + x\cos y$$

$$\frac{\partial f}{\partial y} = 0 + 2 + \cos y$$

$$\frac{\partial^2 f}{\partial y} = 0 + 2 + \cos y$$

$$\frac{\partial^3 f}{\partial x} = 0 + 0 + 0$$

$$\frac{\partial^3 f}{\partial x^2} = 0$$

$$\frac{\partial^2 f}{\partial x} = 0 + 2 + \cos y.$$

$$\frac{\partial M}{\partial y} = ?$$

$$\frac{\partial M}{\partial x} = ?$$

$$\frac{\partial^2 M}{\partial x \partial y} = ?$$

$$\frac{\partial M}{\partial y} = 4\tan x \frac{\partial}{\partial y}(y) + x \frac{\partial}{\partial y}(\tan y)$$

$$\frac{\partial M}{\partial y} = 4\tan x + x\sec^2 y$$

$$\frac{\partial M}{\partial x} = ?$$

$$\frac{\partial M}{\partial x} = 4y\sec^2 x + \tan y$$

$$\frac{\partial^2 M}{\partial x y} = 4\sec^2 x + \sec^2 y.$$

$$\frac{\partial^3 M}{\partial x y} = 4\sec^2 x + \sec^2 y$$

Question:

$$2xy\,dx + (x^2-1)\,dy = 0$$

$M = 2xy \qquad N = (x^2-1)$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$$

$$\frac{\partial (M)}{\partial y} = \frac{\partial (2xy)}{\partial y}$$

$$\boxed{\frac{\partial M}{\partial y} = 2x}$$

$$\frac{\partial N}{\partial x} = \frac{\partial (x^2-1)}{\partial x}$$

$$\frac{\partial N}{\partial x} = 2x - 0$$

$$\boxed{\frac{\partial N}{\partial x} = 2x}$$

$$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial y}$$

Exact

$\int M\,dx + \int$ (terms of N free from x) $dy = C$

As

$$\int 2xy\,dx + \int (-1)\,dy = C$$

$$2y\int x\,dx - \int dy = C$$

$$\cancel{2}y\frac{x^2}{\cancel{2}} - y = C$$

$\boxed{yx^2 - y = C}$ Answer.

Question:

$$(y+2xy^2)\,dx + (x+2x^2y)\,dy = 0$$

$M = y + 2xy^2 \qquad N = x + 2x^2y$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y+2xy^2)$$

$$\boxed{\frac{\partial M}{\partial y} = 1 + 4xy}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x+2x^2y)$$

$$\boxed{\frac{\partial N}{\partial x} = 1 + 4xy}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 1+4xy$$

Exact

$\int M dx + \int (\text{term of } N \text{ free from } x)\, dy =$

$$\int (y + 2xy^2)\, dx + \int 0 \; dy = C$$

$$\int y\, dx + 2y^2 \int x\, dx = C$$

$$xy + \frac{2y^2x^2}{2} = C$$

$$\boxed{xy + x^2y^2 = C}$$ Answer.

Question:

$$(2x-1)\,dx + (3y+7)\,dy = 0$$

$M = 2x-1 \qquad N = 3y+7$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2x-1)$$

$$\frac{\partial M}{\partial y} = 0$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(3y+7)$$

$$\frac{\partial N}{\partial x} = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 0$$

Exact

$$\int (2x-1)\,dx + \int 3y+7 \;\; dy = C$$

$$\int 2x\, dx - \int 1\, dx + 3\int y\, dy + 7\int dy = C$$

$$2\int x\, dx - \int 1\, dx + 3\int y\, dy + 7\int dy = C$$

$$\frac{2x^2}{2} - x + \frac{3y^2}{2} + 7y = C$$

$$\boxed{x^2 - x + \frac{3y^2}{2} + 7y = C}$$ Answer.

Question:

$$(2x+y)\,dx + (x-6y)\,dy = 0$$

$M = 2x+y \qquad N = x-6y$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2x+y)$$

$$= 0+1$$

$$\boxed{\frac{\partial M}{\partial y} = 1}$$

$$\frac{\partial N}{\partial x} = \frac{\partial(x-6y)}{\partial x}$$

$$= 1+0$$

$$\boxed{\frac{\partial N}{\partial x} = 1}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 1$$

Exact

$$\int(2x+y)\,dx - 6\int y\,dy = C$$

$$2\int x\,dx + y\int dx - 6\int y\,dy = C$$

$$\frac{2x^2}{2} + xy - \frac{\cancel{6}^{3}y^2}{\cancel{2}} = C$$

$$\boxed{x^2 + xy - 3y^2 = C}$$ Answer

Question:

$$(5x+4y)\,dx + (4x-8y^3)\,dy = 0$$

$M = 5x+4y \qquad N = 4x-8y^3$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(5x+4y)$$

$$\frac{\partial M}{\partial y} = 0+4$$

$$\boxed{\frac{\partial M}{\partial y} = 4}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(4x-8y^3)$$

$$= 4-0$$

$$\boxed{\frac{\partial N}{\partial x} = 4}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 4$$

Exact

$$\int(5x+4y)\,x - 8\int y^3\,dy = C$$

$$\frac{5x^2}{2} + 4xy - \frac{\cancel{8}^{2}y^4}{\cancel{4}} = C$$

$$\boxed{\frac{5x^2}{2} + 4xy - 2y^4 = C}$$ Answer

Question: $(\sin y - y\sin x)dx + (\cos x + x\cos y)dy = 0$

$M = \sin y - y\sin x \quad N = \cos x + x\cos y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(\sin y - y\sin x)$

$\boxed{\frac{\partial M}{\partial y} = \cos y - \sin x}$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(\cos x + x\cos y)$

$\frac{\partial N}{\partial x} = -\sin x + \cos y$

$\boxed{\frac{\partial N}{\partial x} = \cos y - \sin x}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \cos y - \sin x$

Exact

$\int \sin y - y\sin x - \int y\, dy = c$

$\sin y \int dx - y\int \sin x\, dx - \int y\, dy = c$

$x\sin y - y(-\cos x) - \frac{y^2}{2} = c$

$\boxed{x\sin y + y\cos x - y^2/2 = c}$ Answer

Question :- $(2xy^2 - 3)dx + (2x^2y + 4)dy = 0$

Solution: $M = (2xy^2 - 3) \qquad N = 2x^2y + 4$

$\frac{\partial M}{\partial y} = \frac{\partial (2xy^2-3)}{\partial y} \qquad \frac{\partial N}{\partial x} = \frac{\partial(2x^2y+4)}{\partial x}$

$= 2x \cdot 2y - 0 \qquad = 2y \cdot 2x + 0$

$\boxed{\frac{\partial M}{\partial y} = 4xy} \qquad \boxed{\frac{\partial N}{\partial x} = 4xy}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 4xy$

Exact

putting values in formula

$\int M + \int N$ term include only $y = c$

$\int (2xy^2 - 3)dx + \int (4)dy = c$

$2y^2\int x\,dx - 3\int dx + 4\int dy = c$

$\frac{\cancel{2}y^2x^2}{\cancel{2}} - 3x + 4y = c$

$\boxed{x^2y^2 - 3x + 4y = c}$ Answer

Question: $(x^2-y^2)\,dx - 2xy\,dy = 0$

Solution: $M = x^2 - y^2$ $\quad N = -2xy$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial (x^2-y^2)}{\partial y} \qquad \frac{\partial N}{\partial x} = \frac{\partial(-2xy)}{\partial x}$$

$$\Rightarrow \boxed{\frac{\partial M}{\partial y} = -2y} \qquad \boxed{\frac{\partial N}{\partial x} = -2y}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = -2y$$

Exact

putting values in formula

$\int M + \int$ terms of N free from $x = c$

$$\Rightarrow \int (x^2-y^2)\,dx + 0 = c$$

$$\Rightarrow \frac{x^{2+1}}{2+1} - y^2x = c$$

$$\Rightarrow \boxed{\frac{x^3}{3} - xy^2 = c}$$ Answer.

Question: $(x^3+y^3)\,dx + 3xy^2\,dy = 0$

Solution $M = x^3+y^3$ $\quad N = 3xy^2$

$$\frac{\partial M}{\partial y} = \frac{\partial (x^3+y^3)}{\partial y} \qquad \frac{\partial N}{\partial x} = \frac{\partial (3xy^2)}{\partial x}$$

$$\boxed{\frac{\partial M}{\partial y} = 3y^2} \qquad \boxed{\frac{\partial N}{\partial x} = 3y^2}$$

Exact

$\Rightarrow \int M + \int$ N terms free from $x = C$

$$\Rightarrow \int (x^3+y^3)\,dx + 0 = C$$

$$\Rightarrow \frac{x^{3+1}}{3+1} + xy^3 = C$$

$$\Rightarrow \boxed{x^4/4 + xy^3 = C}$$ Answer.

Question:- $(3x^2y + e^y)\,dx + (x^3 + xe^y - 2y)\,dy = 0$

$M = 3x^2y + e^y$ $\quad N = x^3 + xe^y - 2y$

$$\frac{\partial M}{\partial y} = \frac{\partial (3x^2y + e^y)}{\partial y} \qquad \frac{\partial N}{\partial x} = \frac{\partial (x^3 + xe^y - 2y)}{\partial x}$$

$$\frac{\partial M}{\partial y} = 3x^2 + e^y \qquad \frac{\partial N}{\partial x} = 3x^2 + e^y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3x^2 + e^y$$

Exact

$\int M\,dx + \int$ N terms free from $x = c$

$$\int (3x^2y + e^y)\,dx + \int -2y\,dy = c$$

$$3y\int x^2\,dx + e^y\int dx - 2\int y\,dy = c$$

$$\frac{3x^3y}{3} + xe^y - \frac{2y^2}{2} = c$$

$$\boxed{x^3y + xe^y - y^2 = c}$$ Answer.

Question:

$(x+y)^2\,dx + (2xy + x^2 - 1)\,dy = 0$

$M = (x+y)^2$ $\qquad N = 2xy + x^2 - 1$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x+y)^2$ $\qquad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2xy + x^2 - 1)$

$= 2(x+y)\frac{\partial}{\partial y}(x+y)$ $\qquad \frac{\partial N}{\partial x} = 2y + 2x$

$\frac{\partial M}{\partial y} = 2(x+y)$ $\qquad \frac{\partial N}{\partial x} = 2(x+y)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 2(x+y)$

Exact

$\int M\,dx + \int N$ terms free from x $dy = c$

$\int (x+y)^2 + \int -1\,dy = c$

$\frac{(x+y)^{2+1}}{2+1} - y = c$

$\frac{(x+y)^3}{3} - y = c$

$y(1) = 1$

put $y = 1$ & $x = 1$

$\frac{(1+1)^3}{3} - 1 = c$

$\frac{(2)^3}{3} - 1 = c$

$\frac{8}{3} - 1 = c$

Taking LCM

$\frac{8-3}{3} = c$

$\boxed{5/3 = c}$ Answer.

Question:-

$(4y + 2t - 1)\,dt + (6y + 4t - 1)\,dy = 0$

$M = 4y + 2t - 1$ $\qquad N = 6y + 4t - 1$

$\frac{\partial M}{\partial y} = \frac{\partial(4y+2t-1)}{\partial y}$ $\qquad \frac{\partial N}{\partial t} = \frac{\partial}{\partial t}(6y + 4t - 1)$

$\frac{\partial M}{\partial y} = 4$ $\qquad \frac{\partial N}{\partial t} = 4$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t} = 4$

Exact

$\int M\,dt + \int N$ terms free from t $dy = c$

$\int (4y + 2t - 1)\,dt + \int (6y - 1)\,dy = c$

$4ty + \frac{2t^2}{2} - t$

$4ty + t^2 - t + \frac{6y^2}{2} - y = c$

$4ty + t^2 - t + 3y^2 - y = c$

$y_{(-1)} = 2$

put $y = 2$ & $t = -1$

$4(-1)(2) + (-1)^2 - (-1) + 3(2)^2 - 2 = c$

$-8 + 1 + 1 + 12 - 2 = c$

$14 - 10 = c$

$\boxed{4 = c}$ Answer.

$y_{(0)} = 1$

Question:-

$(e^x + y)\,dx + (2 + x + ye^y)\,dy = 0$

$M = e^x + y \qquad N = 2 + x + ye^y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(e^x + y) \qquad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2 + x + ye^y)$

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 1$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 1$

Exact

$\int M\,dx + \int N$ terms free from x $dy = c$

$\int (e^x + y)\,dx + \int (2 + ye^y)\,dy = c$

$e^x + xy + 2y + \int ye^y\,dy = c$

$e^x + xy + 2y + \left(y \int e^y - \int \frac{\partial}{\partial y}(y) . \int e^y\,dy\right) dy = c$

$e^x + xy + 2y + ye^y - \int 1 . e^y\,dy = c$

$e^x + xy + 2y + ye^y - e^y = c$

putting values

$y = 1$ & $x = 0$

$e^0 + (0)(1) + 2(1) + (1)e^{(1)} - e^{(1)} = c$

$1 + 0 + 2 + \cancel{e^1} - \cancel{e^1} = c \qquad \because e^0 = 1$

$1 + 2 = c$

$\boxed{3 = c}$ Answer.

Question:

$$\left(1-\frac{3}{y}+x\right)\frac{dy}{dx}+y=\frac{3}{x}-1$$

$$\left(1-\frac{3}{y}+x\right)\frac{\partial y}{\partial x}=\frac{3}{x}-1-y$$

$$\left(1-\frac{3}{y}+x\right)dy=\left(\frac{3}{x}-1-y\right)dx$$

$$-\left(\frac{3}{x}-1-y\right)dx+\left(1-\frac{3}{y}+x\right)dy=0$$

multiplying by $-$

$$\left(\frac{3}{x}-1-y\right)dx-\left(1-\frac{3}{y}+x\right)dy=0$$

$$M=\frac{3}{x}-1-y \qquad N=-\left(1-\frac{3}{y}+x\right)$$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}\left(\frac{3}{x}-1-y\right) \qquad \frac{\partial N}{\partial x}=\frac{\partial}{\partial x}-\left(1-\frac{3}{y}+x\right)$$

$$\frac{\partial M}{\partial y}=-1 \qquad \frac{\partial N}{\partial x}=-1$$

Exact

putting values in formula

$\int M + \int N$ terms free from $x = C$

$$\int\left(\frac{3}{x}-1-y\right)dx-\int\left(1-\frac{3}{y}\right)dy=C$$

$$3\int\frac{1}{x}dx-\int dx-y\int dx-\int dy+3\int\frac{1}{y}dy=C$$

$$\boxed{3\ln x-x-xy-y+3\ln y=C}$$ Answer