LINEAR ALGEBRA

$4x-3y+z=11$

$2x+y-4z=-1$

$x+2y-2z=1$

$x+2y+z=$

$x+y-2z=3$

$2x+y+z=2$

B.S SECOND SAMESTER

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SECTION "D"

① Vectors
② Vector - 2-space and 3-space
③ Operation of vectors
④ Magnitude of vectors

① Express P and Q

$\overrightarrow{PQ}$
P(0,0), Q(4,5)

③ Find Magnitude of vectors
③ scalar product definition
$\theta = 90^0$, $\theta = 0^0$, $\theta = 180^0$

④ Scalar product as a component form

"Scaler product or Dot product"

Find the angle between

$A = 2i + 2j - K$ $B = 6i - 3j + 2K$

Solution:

We know that

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A| \cdot |B|} \longrightarrow (A)$$

First Find $\vec{A} \cdot \vec{B}$

$\vec{A} \cdot \vec{B} = (2i + 2j - K) \cdot (6i - 3j + 2K)$

$A \cdot B = 2i \cdot 6i + 2j \cdot (-3j) - K \cdot 2K$

$A \cdot B = 12 - 6 - 2$

$\boxed{A \cdot B = 4}$ — (i)

$\because i \cdot i = 1$

$\because K \cdot K = 1$

$\because j \cdot j = 1$

Now Find $|A| \cdot |B|$

$|A| = \sqrt{x^2 + y^2 + z^2}$

$|A| = \sqrt{(2)^2 + (2)^2 + (-1)^2}$

$|A| = \sqrt{4 + 4 + 1}$

$|A| = \sqrt{9} \Rightarrow \boxed{A = 3}$ — (ii)

$|B| = \sqrt{x^2+y^2+z^2}$

putting values

$|B| = \sqrt{(6)^2+(-3)^2+(2)^2}$

$|B| = \sqrt{36+9+4} = \sqrt{49}$

$\boxed{|B| = 7}$ — (iii)

Put (i), (ii) & (iii) in eq (A)

$\cos\theta = \frac{A \cdot B}{|A| \cdot |B|} = \cos\theta = \frac{4}{3 \times 7}$

$\cos\theta = 4/21$

$\theta = \cos^{-1}\left(\frac{4}{21}\right)$ - Ans

$\boxed{\theta = 79^\circ}$ approximately

Ans

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Q2:- Show that the vectors.

$A = 3i - 2j + k$

$B = i - 3j + 5k$

$C = 2i + j - 4k$

Form a right triangle

$A \circ B = (3i - 2j + k) \circ (i - 3j + 5k)$

$A \circ B = 3 + 6 + 5$

$\boxed{A \circ B = 14}$

$A \circ C = (3i - 2j + k) \circ (2i + j - 4k)$

$A \circ C = 6 - 2 - 4$

$\boxed{A \circ C = 0}$

$B \circ C = (i - 3j + 5k) \circ (2i + j - 4k)$

$B \circ C = 2 - 3 - 20$

$\boxed{B \circ C = -21}$

It is right angle triangle because $\boxed{A \cdot C = 0}$

Evaluate :-

① $(2i - j + 3k) \cdot (3i + 2j - k)$

ⓘ $|3A + 2B|$

$= |3(2i - j + 3k) + 2(3i + 2j - k)|$

$= |6i - 3j + 9k + 6i + 4j - 2k|$

$= |6i + 6i - 3j + 4j + 9k - 2k|$

$= |\cancel{12}k \; 12i + j + 7k|$

Now using mod formula

$= \sqrt{x^2 + y^2 + z^2}$

putting values

$= \sqrt{(12)^2 + (1)^2 + (7)^2}$

$= \sqrt{144 + 1 + 49}$

$\boxed{|3A + 2B| = \sqrt{194}}$ Ans.

(ii) $A = 2i - j + 3k$, $B = 3i + 2j - k$

$(2A + B) \cdot (A - 2B)$

$= (2(2i - j + 3k) + (3i + 2j - k)) \cdot ((2i - j + 3k) - (2(3i + 2j - k)))$

$= (4i - 2j + 6k + 3i + 2j - k) \cdot (2i - j + 3k) - (6i + 4j - 2k)$

$= (4i + 3i - 2j + 2j + 6k - k) \cdot (2i - 6i - j - 4j + 3k + 2k)$

$= (7i + 0j + 5k) \cdot (-4i - 5j + 5k)$

$= (-28 + 0 + 25)$

$\boxed{(2A + B) \cdot (A - 2B) = -3}$ Ans.

$A = i + 3j - 2K$, $B = 4i - 2j + 4K$

(i) $|3A + 2B|$

$|3(i + 3j - 2K) + 2(4i - 2j + 4K)|$

$= |3i + 9j - 6K + 8i - 4j + 8K|$

$= |11i + 5j + 2K|$

Now using Farmula

$= \sqrt{x^2 + y^2 + z^2}$

putt values

$= \sqrt{(11)^2 + (5)^2 + (2)^2}$

$= \sqrt{121 + 25 + 4}$

$\boxed{|3A + 2B| = \sqrt{150}}$ Ans.

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(ii) $(2A+B) \cdot (A-2B)$

$A = i+3j-2k \;,\; B = (4i-2j+4k)$

$\big(2(i+3j-2k)+(4i-2j+4k)\big) \cdot \big((i+3j-2k) - 2(4i-2j+4k)\big)$

$= (2i+6j-4k)+(4i-2j+4k) \cdot (i+3j-2k - 8i+4j-8k)$

$= (2i+4i+6j-2j-4k+4k) \cdot (i-8i+3j+4j-2k-8k)$

$= (6i+4j-0k) \cdot (-7i+7j-10k)$

$= (-42+28-0)$

$\boxed{2(A+B) \cdot (A-2B) = (-14)}$ Ans

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(3) Find the angle between

$A = 3i + 2j - 6K$

$B = 4i - 3j + K$

Sol We know that

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A| \cdot |B|}$$

First find $\vec{A} \cdot \vec{B}$

$(\vec{A} \cdot \vec{B}) = (3i + 2j - 6K) \cdot (4i - 3j + K)$

$(\vec{A} \cdot \vec{B}) = (12 - 6 - 6)$

$|\vec{A} \cdot \vec{B}| = 0$ ~~no~~ further solution is not possible

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(4) For what value of "a" are

$A = a\hat{i} - 2\hat{j} + 1\hat{k}$

$B = 2a\hat{i} + a\hat{j} - 4\hat{k}$ perpendicular

<u>Solutions:-</u>

$\vec{A} \cdot \vec{B} = 0$ are given

$(a\hat{i} - 2\hat{j} + 1\hat{k}) \cdot (2a\hat{i} + a\hat{j} - 4\hat{k}) = 0$

$= 2a^2 - 2a - 4 = 0$

b.s devide by 2

$\frac{\cancel{2}a^2}{\cancel{2}} - \frac{\cancel{2}a}{\cancel{2}} = \frac{\cancel{4}^{2}}{\cancel{2}} = 0/2$

$a^2 - a - 2 = 0$

by factorization:

$a^2 - 2a + a - 2 = 0$

$a(a-2) + 1(a-2) = 0$

$(a-2) + (a+1) = 0$

$a - 2 = 0$ | $a + 1 = 0$

$\boxed{a = 2}$ | $\boxed{a = -1}$ Ans

$\vec{a} = \cancel{x+y}$

$\vec{a} = x_1 i + y_1 j + z_1 K$

$\vec{b} = x_2 i + y_2 j + z_2 K$

$\vec{a} \times b = (x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{K}) \times (x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{K})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & K \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

For Finding cross product

It is just an example.

$a \times b = |a| \cdot |b| \sin\theta \hat{n}$

$\hat{n} = \dfrac{a \times b}{|a \times b|}$ unit vector

Formula

$\vec{a} = |a| \cdot \hat{a}$

$\hat{a} = \dfrac{\vec{a}}{|a|}$

Vector product or cross product:

Q1: $A = 2i - 3j - K$

$B = i + 4j - 2K$

(a) $A \times B$

Solution:

$A \times B = (2i - 3j - K) \times (i + 4j - 2K)$

$$A \times B = \begin{bmatrix} \hat{i} & \hat{j} & \hat{K} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{bmatrix}$$

Expand by R_1

$$i \begin{bmatrix} -3 & -1 \\ 4 & -2 \end{bmatrix} - j \begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix} + K \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

$$i\left((-3 \times -2) - (-1) \times 4\right) - j\left(2 \times -2 - (-1) \times 1\right)$$

$$+ K\left(2 \times 4 - (-3) \times 1\right)$$

$$= i(6 + 4) - j(-4 + 1) + K(8 + 3)$$

$$= i(10) - j(-3) + K(11)$$

$$\boxed{A \times B = 10\hat{i} + 3\hat{j} + 11\hat{K}} \text{ Ans.}$$

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(b) $B \times A$

Solutions:-

$B \times A = (i + 4j - 2k) \times (2i - 3j - k)$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -2 \\ 2 & -3 & -1 \end{bmatrix}$$

$A \times B =$ Expand by R_1

$$i \begin{bmatrix} 4 & -2 \\ -3 & -1 \end{bmatrix} - j \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} + k \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$$

$= i(-4-6) - j[-1+4) + k(-3-8)$

$= i(-10) - j(3) + k(-11)$

$\boxed{(B \times A) = -10i - 3j - 11k}$ Ans.

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(c) $(A+B) \times (A-B)$

$((2i - 3j - k) + (i + 4j - 2k)) \times ((2i - 3j - k)$
$- (i + 4j - 2k))$

$$= (2i - 3j - 1k + i + 4j - 2k) \times (2i - 3j - k - i \mp 4j + 2k)$$

$$= (3i + j - 3k) \times (i - 7j + k)$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{bmatrix}$$

$$= i \begin{bmatrix} 1 & -3 \\ 7 & 1 \end{bmatrix} - j \begin{bmatrix} 3 & -3 \\ 1 & 1 \end{bmatrix} + k \begin{bmatrix} 3 & 1 \\ 1 & -7 \end{bmatrix}$$

$$\cancel{= i(7+3) - j(3+3) + k(-21-1)}$$

$\cancel{(A+B) \times (A-B)} =$

$$= i(1-21) - j(3+3) + k(-21-1)$$

$$\boxed{(A+B) \times (A-B) = -20\,i - 6j - 22k}$$ Ans.

(d) Find unit vector

$$\hat{n} = \frac{A \times B}{|A \times B|} \quad \text{—— (A)}$$

$|A \times B|$ First find $A \times B$

where $A = 2i - 3j - k$

$B = i + 4j - 2k$

$$A \times B = (2i - 3j - k) \times (i + 4j - 2k)$$

$$A \times B = \begin{bmatrix} \hat{i} & \hat{j} & k \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{bmatrix}$$

Expand by R_1

$$\hat{i} \begin{bmatrix} -3 & -1 \\ 4 & -2 \end{bmatrix} - j \begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix} + k \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

$$i(6+4) - j(-4+1) + k(8+3)$$

$$i(10) - j(-3) + k(11)$$

$$\boxed{A \times B = 10i + 3j + 11k}$$

Now find $|A \times B|$

$$|A \times B| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{(10)^2 + (3)^2 + (11)^2}$$

$$= \sqrt{100 + 9 + 121} = \boxed{\sqrt{230} = |A \times B|}$$

putting values in eq (A)

$$\boxed{n^{\wedge} = \frac{10i + 3j + 11k}{\sqrt{230}}}$$ Ans.

Q2: $A = 3i - j + 2K$

$B = 2i + j - K$

$C = i - 2j + 2K$

(a) $(A \times B) \times C$

Solutions:-

First find $(A \times B)$

$(A \times B) = (3i - j + 2K) \times (2i + j - K)$

$$= \begin{bmatrix} i & j & K \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$i \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} - j \begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix} + K \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$i(1-2) - j(-3-4) + K(3+2)$

$i(-1) - j(-7) + K(5)$

$(A \times B) = -i + 7j + 5K$

Now Find $(A \times B) \times C$

$(A \times B) \times C = (-i + 7j + 5K) \times (i - 2j + 2K)$

$$= \begin{bmatrix} i & j & k \\ -1 & 7 & 5 \\ 1 & -2 & 2 \end{bmatrix}$$

$$= i\begin{bmatrix} 7 & 5 \\ -2 & 2 \end{bmatrix} - j\begin{bmatrix} -1 & 5 \\ 1 & 2 \end{bmatrix} + k\begin{bmatrix} -1 & 7 \\ -2 & 2 \end{bmatrix}$$

$$i(14+10) - j(-2-5) + k(14$$

$$i\begin{bmatrix} 7 & 5 \\ -2 & 2 \end{bmatrix} - j\begin{bmatrix} -1 & 5 \\ 1 & 2 \end{bmatrix} + k\begin{bmatrix} -1 & 7 \\ 1 & -2 \end{bmatrix}$$

$$i(14+10) - j(-2-5) + k(2-7)$$

$$i(24) - j(-7) + k(-5)$$

$$= 24i + 7j - 5k$$

$$\boxed{(A\times B)\times C = 24i + 7j - 5k}$$

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$$A \times (B \times C)$$

$$(B\times C) = (2i + j - k) \times (i - 2j + 2k)$$

$$= \begin{bmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$i\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix} - j\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix} + K\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}$$

$$i(2-2) - j(4+1) + K(-4-1)$$

$$i(0) - j(5) + K(-5)$$

$$0 - 5j - 5K$$

$$\boxed{(B\times C) = -5j - 5K}$$

Now $(B\times C)$ multiply with A

$$A\times(B\times C)\times A = (3i - j + 2K)\times(-5j - 5K)$$

$$= \begin{bmatrix} i & j & K\\ 3 & -1 & 2\\ 0 & -5 & -5\end{bmatrix}$$

$$= i\begin{bmatrix}-1 & 2\\ -5 & -5\end{bmatrix} - j\begin{bmatrix}3 & 2\\ 0 & -5\end{bmatrix} + K\begin{bmatrix}3 & -1\\ 0 & -5\end{bmatrix}$$

$$= i(5+10) - j(-15-0) + K(-15+0)$$

$$= i(15) - j(-15) + K(-15)$$

$$15i + 15j - 15K$$

$$A\times(B\times C) = \boxed{15i + 15j - 15K} \text{ Ans.}$$

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(c) $(A \times B) \times C = A \times (B \times C)$

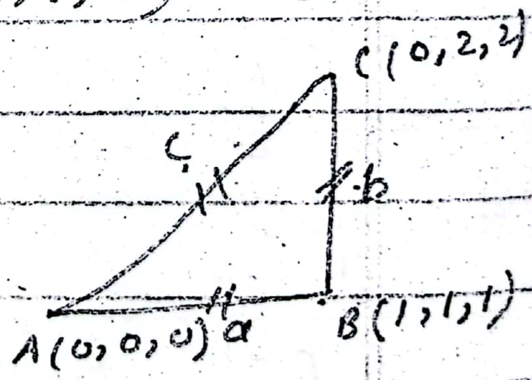
we have defined it but it is not equal to each other.

$$24i + 7j - 5k \neq 15i + 15j - 15k$$

Area of traingle & paralalogram:

$$\Delta = \frac{1}{2} |A \times B|$$

(i) Find the area of traingle ABC whose vertices are $A(0,0,0)$, $B(1,1,1)$ and $C(0,2,2)$



$$\vec{AB} = OB - OA$$

$$(1i + 1j + 1k) - (0i + 0j + 0k)$$

$$\boxed{\vec{AB} = 1\hat{i} + 1\hat{j} + 1k}$$

$AC = \vec{OC} - \vec{OA}$

$(0i + 2j + 2k) - (0i + 0j + 0k)$

$\vec{AB} = 0i + 2j + 2k$

$\boxed{AC = 0i + 2j + 2k}$ Ans.

$A = \frac{1}{2} |AB \times AC|$ —— A

$$AB \times AC = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{vmatrix}$$

$$= i\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} - j\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + k\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$i(2-2) - j(2-0) + k(2-0)$

$|AB \times AC| = 0i - 2j + 2k$

$= \sqrt{x^2 + y^2 + z^2}$

$\sqrt{(0)^2 + (-2)^2 + (2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$|AB \times AC| = \sqrt{8}$

Now put all these values in eq (A)

$\Delta = \frac{1}{2} \sqrt{|AB \times AC|}$ $\quad \because |AB \times AC| = \sqrt{8}$

$\Delta = \frac{1}{2} \sqrt{8}$

$\Delta = \frac{1}{2} \sqrt{2 \times 2 \times 2}$

$\Delta = \frac{1}{2} 2\sqrt{2}$

$\Delta = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$\boxed{\Delta = \sqrt{2}}$ Ans

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$\vec{A} = 2i + 4j$, $B = -3i + 2j$

$AB = OB - OA$

$\vec{AB} = (-3i + 2j) - (2i + 4j)$

$\vec{AB} = -5i - 2j$

$\boxed{AB = -5i - 2j}$ Ans.

$\vec{a} = 2i - 3j - k$ $\quad \vec{b} = i + 4j - 2k$

$a \times b = -\vec{b} \times \vec{a}$

$(a+b) \times (a-b) = 2(\vec{a} \times \vec{b})$

Scaler tripple product:-

$\vec{a}, \vec{b}$ and $\vec{c}$

$(a \times b) \times c \longrightarrow$ Vector

$a \times (b \times c) \longrightarrow$ Vector

$a \cdot (b \times c) \longrightarrow$ Scaler

$a \cdot (b \cdot c) \longrightarrow$ Does'nt exist

but answer will come in vector.

$\vec{a} = x_1 i + y_1 j + z_1 k$

$\vec{b} = x_2 i + y_2 j + z_2 k$

$\vec{c} = x_3 i + y_3 j + z_3 k$

$$a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b)$$

First find

$$b \times c = \begin{vmatrix} i & j & k \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

It just example.

Now i am going to solve the question on next page

Find scaler tripple product:-

$\vec{a} = 3i + 2k$
$\vec{b} = i + 2j + k$
$\vec{c} = -j + 4k$

we can also write it like

$\vec{a} = 3i + 0 \cdot j + 2k$
$\vec{b} = i + 2j + k$
$\vec{c} = 0i - j + 4k$

Find $a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b)$

Solutions:-

$a \cdot (b \times c)$

$$a \cdot (b \times c) = \begin{vmatrix} 3 & 0 & 2 \\ 1 & 2 & 1 \\ 0 & -1 & 4 \end{vmatrix}$$

Expand by R_1

$$3\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} - 0\begin{bmatrix} 1 & 1 \\ 0 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$3(8+1) - 0(4-0) + 2(-1-0)$

$3(9) - 0(4) + 2(-1)$

$= 27 - 2$

$a \cdot (b \times a) = 25$

$\boxed{a \cdot (b \times c) = 25}$

Now find $b \cdot (c \times a)$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

Expand by R_1

$$1\begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ 3 & 2 \end{bmatrix} + 1\begin{bmatrix} 0 & -1 \\ 3 & 0 \end{bmatrix}$$

$1(-2-0) - 2(0-12) + 1(0+3)$

$1(-2) - 2(-12) + 1(3)$

$-2 + 24 + 3$

$= -2 + 27$

$= 25$

$\boxed{b \cdot (c \times a) = 25}$

Find $c\cdot(a\times b)$

$$c\cdot(a\times b) = \begin{bmatrix} 0 & -1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Expand by R_1

$$= 0\begin{bmatrix} 0 & 2 \\ 2 & 1 \end{bmatrix} - (-1)\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \underset{\text{four}}{4}\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= 0[0-2) + 1(3-2) + 4(6-0)$$

$$= 0 + 1(1) + 4(6)$$

$$= 0 + 1 + 24$$

$$= 1 + 24$$

$$= 25$$

$$\boxed{c\cdot(a\times b) = 25}$$

Hence prove That

$$a\cdot(b\times c) = b\cdot(c\times a) = c\cdot(a\times b)$$
$$25 = 25 = 25$$

Ans.

Show that Scalar triple product is zero.

$\vec{a} = \hat{i} - \hat{j} + 0 \cdot \hat{k}$

$\vec{b} = 0\hat{i} + \hat{j} - 1\hat{k}$

$\vec{c} = -\hat{i} + 0\hat{j} + 1\hat{k}$

(i) $a \cdot (b \times c)$

Solution:-

$$a \cdot (b \times c) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

Expand by R_1

$$1 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} + 1 \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix} + 0 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$1(1+0) + 1(0-1) + 0(0+1)$

$1(1) + 1(-1) + 0(1)$

$= 1 - 1 + 0 = 1 - 1$

$$\boxed{\vec{a} \cdot (b \times c) = 0}$$

$b \circ (c \times a)$

$$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Expand by R_1

$$= 0 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - 1 \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} - 1 \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$= 0(0+1) - 1(0-1) - 1(1-0)$$

$$= 0(1) - 1(-1) - 1(1)$$

$$= 0 + 1 - 1$$

$$= 1 - 1 = 0$$

$$\boxed{b \circ (c \times a) = 0}$$

Now find $c \circ (a \times b)$

$$\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

Expand by R_1

$$-1\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} - 0\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + 1\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= -1(1-0) - 0(-1-0) + 1(1+0)$$

$$= -1(1) - 0(-1) + 1(1)$$

$$= -1 - 0 + 1$$

$$= -1 + 1 = 0$$

$$\boxed{c \cdot (a \times b) = 0}$$

Hence prove that scaler triple product zero

$a \cdot (b \times c) = 0$, $b \cdot (c \times a) = 0$

& $c \cdot (a \times b) = 0$

Q5: $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$

$\vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$

$\vec{c} = \hat{i} + \hat{j} + \hat{k}$

Find

① $\vec{a} \times \vec{b}$

$(\hat{a} \times b) = (2\hat{i} - 3\hat{j} - \hat{k}) \times (\hat{i} + 4\hat{j} - 2\hat{k})$

$$a \times b = \begin{bmatrix} i & j & k \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{bmatrix}$$

Expand by R_1

$$= i\begin{bmatrix} -3 & -1 \\ 4 & -2 \end{bmatrix} - j\begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix} + k\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

$$= i\,(6+4) - j\,(-4+1) + k\,(8+3)$$

$$i(10) - j(-3) + k(11)$$

$$10i + 3j + 11k$$

$$\boxed{a \times b = 10i + 3j + 11k} \text{ Ans:}$$

$$a \times c = (2i - 3j - k) \times (i + j + k)$$

$$a \times c = \begin{bmatrix} i & j & k \\ 2 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

Expand by R_1

$$i\begin{bmatrix} -3 & -1 \\ 1 & 1 \end{bmatrix} - j\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} + k\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$$

$= i(-3+1) - j(2+1) + k(2+3)$

$i(-2) - j(3) + k(5)$

$= -2i - 3j + 5k$

$\boxed{a \times c = -2i - 3j + 5k}$

(iii)

$b \times c$

$b \times c = (i + 4j - 2k) \times (i + j + k)$

$$= \begin{bmatrix} i & j & k \\ 1 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

Expand by R_1

$= i \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} - j \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} + k \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$

$= i(4+2) - j(1+2) + k(1-4)$

$= i(6) - j(3) + k(-3)$

$= 6i - 3j - 3k$

$\boxed{b \times c = 6i - 3j - 3k}$ Ans.

(v) $(a+b)\times(a-c)$

$(a+b)\times(a-c) =$

$= \left((2i-3j-1k)+(i+4j-2k)\right)\times\left((2i-3j-1k)-(i+j+1k)\right)$

$= (2i-3j-1k+i+4j-2k)\times(2i-3j-1k-i-j-k)$

$= (3i+j-\underset{\text{Three}}{3}k)\times(i-4j-2k)$

$$= \begin{bmatrix} i & j & k \\ 3 & 1 & -3 \\ 1 & -4 & -2 \end{bmatrix}$$

Expand by R_1

$i\begin{bmatrix} 1 & -3 \\ -4 & -2 \end{bmatrix} - j\begin{bmatrix} 3 & -3 \\ 1 & -2 \end{bmatrix} + k\begin{bmatrix} 3 & 1 \\ 1 & -4 \end{bmatrix}$

$i(-2-12)-j(-6+3)+k(-12-1)$

$i(-14)-j(-3)+k(-13)$

$\boxed{(a+b)\times(a-c) = 14i+3j-13k}$

Ans:

(vi)

$a \times (b \times c)$

first find $(b \times c)$

$b \times c = (i + 4j - 2k) \times (i + j + k)$

$$= \begin{bmatrix} i & j & k \\ 1 & 4 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= i \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} - j \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} + k \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$$

$= i(4+2) - j(1+2) + k(1-4)$

$= i(6) - j(3) + k(-3)$

$$\boxed{\cancel{a\times}(b \times c) = 6i - 3j - 3k}$$

Now ~~for~~ $a \times (b \times c)$ find.

$a \times (b \times c) = (2i - 3j - k) \times (6i - 3j - 3k)$

$$= \begin{bmatrix} i & j & k \\ 2 & -3 & -1 \\ 6 & -3 & -3 \end{bmatrix}$$

Expand by R_1

$$i\begin{bmatrix}-3 & -1\\ -3 & -3\end{bmatrix} - j\begin{bmatrix}2 & -1\\ 6 & -3\end{bmatrix} + k\begin{bmatrix}2 & -3\\ 6 & -3\end{bmatrix}$$

$$i(9-3) - j(-6+6) + k(-6+18)$$

$$i(6) - j(0) + k(12)$$

$$\boxed{a\times(b\times c) = 6i - 0j + 12k}$$

=== 11 === 11 === 11 === 11

(vii) $(a\times b)\times c$

First find $(a\times b)$

$$(a\times b) = (2i - 3j - k)\times(i + 4j - 2k)$$

$$= \begin{bmatrix} i & j & k\\ 2 & -3 & -1\\ 1 & 4 & -2\end{bmatrix}$$

$$a\times b = i\begin{bmatrix}-3 & -1\\ 4 & -2\end{bmatrix} - j\begin{bmatrix}2 & -1\\ 1 & -2\end{bmatrix} + k\begin{bmatrix}2 & -3\\ 1 & 4\end{bmatrix}$$

$$i(6+4) - j(-4+1) + k(8+3)$$

$$i(10) - j(-3) + k(11)$$

$$10i + 3j + 11k$$

$$\boxed{a \times b = 10i + 3j + 11k}$$

Now find $(a \times b) \times c$

$$= (10i + 3j + 11k) \times (i + j + k)$$

$$= \begin{bmatrix} i & j & k \\ 10 & 3 & 11 \\ 1 & 1 & 1 \end{bmatrix}$$

Expand by R_1

$$= i\begin{bmatrix} 3 & 11 \\ 1 & 1 \end{bmatrix} - j\begin{bmatrix} 10 & 11 \\ 1 & 1 \end{bmatrix} + k\begin{bmatrix} 10 & 3 \\ 1 & 1 \end{bmatrix}$$

$$i(3-11) - j(10-11) + k(10-3)$$

$$= i(-8) - j(-1) + k(7)$$

$\cancel{a \times b \times}$ $\boxed{(a \times b) \times c = -8i + j + 7k}$

Ans

(viii) Find $a \times (b \times c) \neq (a \times b) \times c$

So we have solved it in the previous (vi) & (vii) questions which is not equal to each other.

= = = = = = = = = = = = = = =

(ix) Find $(a+b) \times (a-b)$

$((2i-3j-k)+(i+4j-2k))$

$\times ((2i-3j-k)-(i+4j-2k))$

$= (2i-3j-k+i+4j-2k) \times (2i-3j$
$-k-i-4j+2k)$

$= (3i-j-3k) \times (i-7j+k)$

$$= \begin{bmatrix} i & j & k \\ 3 & -1 & -3 \\ 1 & -7 & 1 \end{bmatrix}$$

Expand by R_1

$= i\begin{bmatrix} -1 & -3 \\ -7 & 1 \end{bmatrix} - j\begin{bmatrix} 3 & -3 \\ 1 & -1 \end{bmatrix} + k\begin{bmatrix} 3 & -1 \\ 1 & -7 \end{bmatrix}$

$i\,(-1-21) - j\,(3+3) + k\,(-21+1)$

$i\,(-22) - j\,(6) + k\,(-20)$

$= -22i - 6j - 20k$

$\boxed{(a+b)\times(a-b) = -22i - 6j - 20k}$

Ans

---

④ $\vec{a} = 2\hat{i} - 3\hat{j} + 0\hat{k}$

$\vec{b} = \hat{i} + \hat{j} - \hat{k}$

$c = 3\hat{i} + 0\hat{j} - k$

Show That

$a\cdot(b\times c) = b\cdot(c\times a)$

$a\cdot(b\times c) = \begin{bmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

Expand by R_1

$$= 2\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 3 & -1 \end{bmatrix} + 0\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= 2(-1+0) + 3(-1+3) + 0(0-3)$$

$$= 2(-1) + 3(+2) + 0(-3)$$

$$= 2(-1) + 3(2) + 0$$

$$= -2 + 6$$

$$\boxed{a \cdot (b \times c) = 4}$$

$$b \cdot (c \times a) = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 0 & -1 \\ 2 & -3 & 0 \end{bmatrix}$$

$$1\begin{bmatrix} 0 & -1 \\ -3 & 0 \end{bmatrix} - 1\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} - 1\begin{bmatrix} 3 & 0 \\ 2 & -3 \end{bmatrix}$$

$$1(0-3) - 1(0+2) - 1(-9-0)$$

$$1(-3) - 1(2) - (-9)$$

$$-3 - 2 + 9$$

$$-5 + 9 = b \cdot (c \times a) = 4$$

Hence prove That

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$4 = 4$$

Do The Points $(4,-2,1)$, $(5,1,6)$, $(2,2,-5)$ & $(3,5,0)$ lie in a plane.

Solutions:-

$A(4,-2,1) = 4i - 2j + k$

$B(5,1,6) = 5i + j + 6k$

$C(2,2,-5) = 2i + 2j - 5k$

$D(3,5,0) = 3i + 5j + 0k$

$\vec{a} = AB = B - A$

$= (5i + j + 6k) - (4i - 2j + k)$

$\vec{a} = 5i - 4i + j + 2j + 6k - k$

$\boxed{\vec{a} = i + 3j + 5k}$

$\vec{b} = \vec{AC} = \vec{C} - \vec{A}$

$(2i + 2j - 5k) - (4i - 2j + k)$

$\boxed{\vec{b} = -2i + 4j - 6k}$

$\vec{C} = AD = D - A$

$(3i + 5j + 0k) - (4i - 2j + k)$

$\vec{C} = 3i - 4i + 5j + 2j + 0k - k$

$\boxed{\vec{C} = -i + 7j - 1k}$

Now find $\vec{a} \cdot (\vec{b} \times \vec{c})$

$$= \begin{bmatrix} 1 & 3 & 5 \\ -2 & 4 & -6 \\ -1 & 7 & -1 \end{bmatrix}$$

Expand by R_1

$= 1(-4 + 42) - 3(2 - 6) + 5(-14 + 4)$

$= 1(38) - 3(-4) + 5(-10)$

$= 38 + 12 - 50$

$= 50 - 50$

$\boxed{\vec{a} \cdot (\vec{b} \times \vec{c}) = 0}$ Ans.

" LINEAR TRANSFORMATION":-

A Linear transformation L of R^n to R^m is a function Assigning a unique vector $L(U)$ in R^m to each U in R^n such that

(i) $L(U+V) = L(U) + L(V)$ for every U & V in R^n

(ii) $L(KU) = KL(U)$ for every U in R^n & every scaler constant

$$f(x, y) = x + y$$

$$U = (x_1, y_1)$$

$$V = (x_2, y_2)$$

$$U+V = (x_1, y_1) + (x_2, y_2)$$

$$U+V = (x_1 + x_2, y_1 + y_2)$$

$$L(U+V) = (x_1 + x_2) + (y_1 + y_2)$$

L.H.S

$L(U) = f(x_1, y_1) = x_1 + y_1$

$L(V) = f(x_2, y_2) = x_2 + y_2$

$L(U) + L(V) = (x_1 + y_1 + x_2 + y_2)$

$L(U) + L(V) = (x_1 + x_2) + (y_1 + y_2)$

R.H.S

Hence

R.H.S = L.H.S

Linear TRANSFERMATON

(ii) $U = (x_1, y_1)$ L.H.S

$KU = (Kx_1, Ky_1)$

$L(KU) = f(Kx_1, Ky_1)$

$L(KU) = Kx_1 + Ky_1$

$L(KU) = K(x_1 + y_1)$

R.H.S.

$U = (x_1, y_1)$

$KU = (Kx_1, Ky_1)$

✓ $L(U) = f(x_1, y_1)$

$L(KU) = f(Kx_1, Ky_1)$

(H) (i) $f(x,y) = x \cdot y \rightarrow$ (1)

(ii) $f(x,y) = x \cdot y^2$ (iii) $f(x+y) = 2(x+y)$

$KL(U) = K(x_1 + y_1)$

∴ Hence prove That

$L(KU) = KL(U)$

$R\cdot H\cdot S = R\cdot H\cdot S$

(12). which of The following are linear transformation

(a) $L(x, y) = (x+1, y, x+y)$

Solution:-

Let $U = (x_1, y_1)$ & $V = (x_2, y_2)$ be The two vector in R^2

$L(U+V) = L(U) + L(V)$ Find

L·H·S

$L(U+V)$

$U+V = (x_1, y_1) + (x_2, y_2)$

$= (x_1 + x_2, y_1 + y_2)$

$L(U+V) = L(x_1 + x_2, y_1 + y_2)$

$L(U+V) = (x_1+x_2+1, y_1+y_2,$
$x_1+x^2+y_1+y_2)$

we have, put value in eq

$x+1, \; y, \; x+y$

$(x_1+x_2+1), (y_1+y_2), x_1+x_2+y_1+y_2$

So we get

$L(U+V) = (x_1+x_2+1, y_1+y_2, x_1+x_2$
$+y_1+y_2)$

R.H.S

$L(U)+L(V)$

$L(U)+L(V)$

$L(U) = L(x_1, y_1) = (x_1+1, y_1, x_1+y_1)$

$L(V) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$

$L(U)+L(V) = (x_1+1, y_1, x_1+y_1)$
$+(x_2+1, y_2, x_2+y_2)$

$L(U)+L(V) = (x_1+x_2+2, y_1+y_2+$
$x_1+x_2+y_1+y_2)$

It is not linear transformation because L.H.S $\neq$ R.H.S

(3) $L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y \\ x-z \end{bmatrix}$ $L: R^3 \to R^3$

Suppose:

$U = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$

$L(U) = L\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix}$ --- (i)

$V = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$L(V) = L\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2+y_2 \\ y_2 \\ x_2-z_2 \end{bmatrix}$ --- (ii)

$L(U+V) = L(U) + L(V)$

R.H.S =

$L(U) + L(V)$

$= \begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix} + \begin{bmatrix} x_2+y_2 \\ y_2 \\ x_2-z_2 \end{bmatrix}$

$$= \begin{bmatrix} x_1 + y_1 + x_2 + y_2 \\ y_1 + y_2 \\ x_1 - z_1 + x_2 - z_2 \end{bmatrix}$$

$L.H.S = \{ \quad U + V = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$$= \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix}$$

$$L(U+V) = L\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 + x_2 + y_1 + y_2 \\ y_1 + y_2 \\ x_1 + x_2 - z_1 - z_2 \end{bmatrix}$$

$$L(U+V) = \begin{bmatrix} x_1 + y_1 + x_2 + y_2 \\ y_1 + y_2 \\ x_1 - z_1 + x_2 - z_2 \end{bmatrix}$$

$L.H.S = R.H.S$ It is linear transformation

$L(x, y) = $ // ——— //

③ $L(x, y) = (x^2+x,\ y-y^2)$

$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} w \\ w^2+x \\ y-z \end{bmatrix}$ (No Linear transformation)

$L(x, y) = (x^2+x,\ y-y^2)$

Solution: R.H.S

$U = (x_1+y_1)$

$L(U) = L(x_1+y_1) = x_1$

$= (x_1^2+x_1,\ y_1-y_2^2)$

$V = (x_2, y_2)$

$L(V) = L(x_2, y_2) = (x_2^2+x_2, y_2-y_2^2)$

$L(U)+L(V) = (x_1^2+x_2^2+x_1+x^2$
$+ y_1+y_2-y_1^2-y_2^2)$

L.H.S

$U+V = (x_1+y_1)+(x_2+y_2)$

$U+V = (x_1+y_1+x_2+y_2)$

$= (x_1+x_2+y_1+y_2)$

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$$L(U+V) = \left((x_1+x_2)^2 + (x_1+x_2) + (y_1+y_2) - (y_1+y_2)^2\right)$$

$$= \left(x_1^2 + x_2^2 + 2x_1x_2\right) + (x_1+x_2), (y_1+y_2) + \left(y_1^2 - y_2^2 - 2y_1y_2\right)$$

R.H.S ≠ L.H.S

Not Linear transformation

=====//=====//=====//=====

REAL VECTOR Space:

Difinitions:- A real vector space is a set of element of "V" together with two operation ⊕ and ⊙ satisfying the following properties:

(A) If U & V are any element of "V" then U⊕V is an in U.

(i.e) "V" is closed under the operation ⊕

(a) $U+V = V \oplus U$ for 'U' an 'V' in "V"

~~(b) $U \oplus V = V \oplus U$~~

(b) $U \oplus (V \oplus w) = (U+V) \oplus w$ for 'U', 'V' and 'w' in "V".

(e) There is an element is in "V" Such That $U \oplus 0 = 0 \oplus U = U$ For all in V.

(f) For each U in 'V' there is an element $-U$ in V Such that

$U \oplus -U = 0$

(B) if U is any element of 'V' and 'c' is any real Number Then $c \odot U$ is in 'V'

(e) $c \odot (U \oplus V) = c \odot U \oplus c \odot V$

(f) $(c \oplus d) \odot U = c \cdot U \oplus d \odot U$

(g) $c \odot (d \odot U) = (c \odot d) \odot U$

(h) $1 \odot U = U$ For all U in "V".

For example..

$V = \{ 0 \pm 1 \pm 2 \pm 3 \pm 4 \cdots \}$

Set of Integer:

$U = 2,\ V = 3$	$W = -1$
$U + V = V \oplus U$	$U \oplus (V \oplus W)$
$2 + 3 = 3 + 2$	$= (U+V) \oplus W$
$5 = 5$	$2 \oplus (3-1) = (2+3) - 1$
	$2 + (2) = (5) - 1$
	$4 = 4$

=====॥=====॥=====॥=====

Example #1

Consider the set of V of all ordered tripple of real number of the form (x, y, c) and define the operation $\oplus$ and $\odot$ by

$(x, y, 0) \oplus (x', y', 0) = (x + x', y + y', 0)$

$c \odot (x, y, 0) = (cx, cy, 0)$

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(c) $c \odot (x, y, 0) = (cx, cy, 0)$

(d) $U + O = O + U = U$

$O = (0, 0, 0)$

Q(2)

Real vector space

V is the set of all ordered pair of real number (x, y) where $x > 0$, $y > 0$

$(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$

and $c \odot (x, y) = (cx, cy)$

Sol :- $U = (x, y)$, $V = (x_1, y_1)$

and $w = (x_2, y_2)$

Now Apply condition

$U \oplus V = V \oplus U$

P.T.O

$U \oplus V = (x, y) \oplus (x_1, y_1)$

$= (x + x_1, y + y_1)$ L.H.S

R.H.S

$V + U = (x_1, y_1) + (x, y)$

$= (x_1 + x, y_1 + y) = (x + x_1, y + y_1)$

L.H.S = R.H.S

—— " —— " —— " ——

$U \oplus (V \oplus W) = (U \oplus V) \oplus W$

$U = (x, y)$ $V = (x_1, y_1)$

$W = (x_2, y_2)$

Now Apply Condition

L.H.S

$U \oplus (V \oplus W)$

$(V \oplus W) = (x_1, y_1) \oplus (x_2, y_2)$

$= (x_1 + x_2, y_1 + y_2)$

$$U \oplus (V \oplus W) = (x, y) \oplus (x_1 + x_2, y_1 + y_2)$$

$$= (x + x_1 + x_2, y + y_1 + y_2)$$

$(U \oplus V) \oplus W =$ R.H.S

$$U \oplus V) = (x, y) \oplus (x_1, y_1)$$

$$= (x + x_1, y + y_1)$$

$$(U \oplus V) \oplus W = (x + x_1, y + y_1) \oplus (x_2 + y_2)$$

$$= (x + x_1 + x_2, y + y_1 + y_2)$$

$$(U + V) + W = U \oplus (V \oplus W)$$

$$U + 0 = U \qquad U = (x, y)$$

$$U + 0 = U = (x, y) + (0, 0)$$

$$= (x + 0, y + 0)$$

$$\boxed{U + 0 = (x, y)}$$

$u + (-u) = 0$

$u + (-u) = (x, y) + (-x, -y)$

$= (x - x, y - y)$

$u + (-u) = 0, 0$

$\boxed{u + (-u) = 0}$ Ans.

= // = // = // =

$c \odot (u \oplus v) = c \odot u \oplus c \odot v$

$\Rightarrow c \odot ((x, y) + (x_1, y_1))$ L.H.S

$\Rightarrow c \odot (x + x_1, y + y_1)$

$\Rightarrow c \odot (x + x_1, y + y_1)$

$(cx + cx_1, cy + cy_1)$ = L.H.S

Now taking R.H.S

$c \odot u \oplus c \odot v$

$c(x, y) + c(x_1, y_1)$

$= (cx, cy) + (cx_1, cy_1)$

$= (cx + cx_1, cy + cy_1)$

Hence $L.H.S = R.H.S$

(6) $(c \oplus d) \odot U = c \odot U \oplus d \odot U$

$= (c \oplus d) \odot U$ $\quad \underline{R.H.S}$

$= (c+d) \odot (x, y)$

$= (c+d) \cdot x + (c+d) \cdot y$

$= cx + dx + cy + dy$

$= \boxed{cx + cy + dx + dy} = (c \oplus d) \odot U$

$\underline{L.H.S}$

$c \odot U \oplus d \odot U$

$c \odot (x, y) + d \odot (x, y)$

$c \odot (x+y) + d \odot (x+y)$

$c \odot (x+y) + d \odot (x+y)$

$cx + cy + dx + dy$

$\boxed{cx + cy + dx + dy} = c \odot U \oplus d \odot U$

Hence $R.H.S = L.H.S$

(7) $1 \odot U = U$

$(1.1) \cdot (x, y) = (x \cdot 1, y \cdot 1)$

$= (x, y)$

$\boxed{1 \cdot U = U}$ prove

= = = = = = = = = = = = = = =

" "SUBSPACE" "

Let V be a vector space then w will be Subspace of V if

(a) $U \oplus V$ is in W

(b) $K \in R$, $K \odot U$ is in W

(c) which of the following Subsets of R^3 are Subspace of R^3, The set of all vectors of the form.

(a) $(a, b, 2)$

$U = (a, b, 2)$

$V = (a_1, b_1, 2)$ are two vectors in W then

$U+V = (a, b, 2) + (a_1, b_1, 2)$

$(a+a_1, b+b_1, 2+2)$

$= (a+a_1, b+b_1, 4) \notin W$

It is not a subset of W.

(b) Let $U = (a, b, c)$ & $V = (a_1, b_1, c_1)$ are two vectors in W.

$a = c = 0, \ a_1 = c_1 = 0$

$U \oplus V = (a, b, c) \oplus (a_1, b_1, c_1)$

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$= (a + a_1, b + b_1, c + c_1)$

$= (0 + 0, b + b_1, 0 + 0)$

$a = 0$
$a_1 = 0$
$c = 0$
$c_1 = 0$

$= (0, b + b_1, 0) \in W$

$U + V$ is a subset of W

$K \odot U = K \odot (a, b, c)$

$= (Ka, Kb, Kc)$

where

$a = 0, c = 0$

$(K(0), Kb, K(0))$

$= (0, Kb, 0)$

This is subspace

Eigenvalues and Eigenvectors

Definition :- If (A) matrix is an $(n \times n)$ matrix, then a nonzero vectors (x) vector in R^n is called an eigenvector of A. If Ax a scaler multiple of x, that is $Ax = (\lambda) x$ scaler constant for some scaler λ. The scaler λ is called an eigenvalues of A and x is said to be eigenvector corresponding to λ.

Eigenvector of 2×2 matrix

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$ Taking 3 common

$$= 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \because x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= 3x$$

$$\boxed{= \lambda x}$$ → igon vector

igon value

Computing Eigenvalues & Eigenvector (x):

To obtain a general positive For

Finding eigenvalues & eigenvectors of an $(n \times n)$ matrix A.

We will begin with the problem of finding eigenvalues of A.

Note first that the equation

$Ax = \lambda x$ can be rewritten as

$Ax = \lambda I x$ or equivalently as

$$(\lambda I - A)x = 0$$

Theorm (1)

If (A) is an $(n \times n)$ matrix, Then λ is

an eigenvalues of A
if only if it
satisfies the equation

Let $(\lambda I - A) = 0$ → Characteristics equation of A

This

$Ax = \lambda x$

$Ax = \lambda I x$ — identity matrix

$(\lambda Ix - Ax) = 0$

First find λ

$(\lambda I - A)x = 0$

Let $(\lambda I - A) = 0$

Q 1:

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ Find

Aigenvalues and Eigenvectors

Note for Eigen values find only λ
& Eigen vector find only x

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ identity matrix

$$\lambda I = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{bmatrix}$$

$\lambda I - A =$

Let $(\lambda I - A) = \begin{bmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{bmatrix}$

find magnitude

$$(\lambda - 3)(\lambda + 1) - 0 = 0$$

$\lambda - 3 = 0$	$\lambda + 1 = 0$
$\lambda = 3$	$\lambda = -1$

$$(\lambda I - A)x = 0$$

$$\begin{bmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For $\lambda = 3$

$$= \begin{bmatrix} 0 & 0 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 0x_1 \quad 0x_2 = 0$

$\Rightarrow -8x_1 + 4x_2 = 0$

$\Rightarrow -8x_1 + 4x_2 = 0$

$\cancel{-}\cancel{8}^{2}x_1 = \cancel{-}\cancel{4}x^{2}$

$2x_1 = x_2$

put $x_2 = r$

$2x_1 = r$ b.s divide by 2

$x_1 = \dfrac{r}{2}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$? $x = r/2$

$x_2 = r$

$$\boxed{x = \begin{bmatrix} r/2 \\ r \end{bmatrix}}$$ Ans.

=== // === // === // ===

Example (2)

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$\lambda = ?$

$x = ?$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 3 & +1 \\ -2 & \lambda - 2 \end{bmatrix}$$

let $\lambda I - A = \begin{bmatrix} \lambda - 3 & 1 \\ -2 & \lambda - 2 \end{bmatrix}$

$$(\lambda - 3)(\lambda - 2) - 1 \times -2 = 0$$

$$(\lambda - 3)(\lambda - 2) + 2 = 0$$

$$(\lambda - 3)(\lambda - 2) + 2 = 0$$

$(\lambda - 3) + 2 = 0$	$(\lambda - 2) + 2 = 0$
$\lambda - 3 = -2$	$\lambda - 2 \neq -2$
$\lambda =$	

$\lambda - 3 = +2$ | $\lambda - 2 = -2$

$\lambda = -2+3$ $\quad$ $\lambda = -2+2$

$\lambda = 1$ $\quad$ $\lambda = 0$

$\lambda = \{1, 0\}$

for $\lambda = 1$

$(\lambda I - A)x = 0$

$$\begin{bmatrix} \lambda - 3 & 1 \\ -2 & \lambda - 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For $\lambda = 1$

$$= \begin{bmatrix} 1-3 & 1 \\ -2 & 1-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x_1 + 1x_2 = 0$

$-2x_1 - x_2 = 0$

$\cancel{+}2x_1 = \cancel{+}x_2$ — (i)

$-2x_1 = x_2$ — (ii) or $2x_1 = -x_2$

~~Let~~ put $x_2 = r$ in eq (i)

$$\frac{\cancel{2}x_1}{\cancel{2}} = \frac{r}{2}$$

$$\boxed{x_1 = \frac{r}{2}}$$

put $x_2 = r$ in eq (ii)

$$\frac{\cancel{2}x_1}{\cancel{2}} = \frac{-r}{2}$$

$$\boxed{x_1 = -r/2}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r/2 \\ r \end{bmatrix} \quad \text{Ans.}$$

$$X = \begin{bmatrix} -r/2 \\ r \end{bmatrix} \quad \text{Ans.}$$

— // — // — //

Eigenvalues of 3×3 Matrix

(1) Find the eigenvalues of

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

The characteristics form

Let $(\lambda I - A) = 0$

Let $\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix} \right) = 0$

$= \text{Let} \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -4 & +17 & \lambda - 8 \end{bmatrix} = 0$

$= \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -4 & 17 & \lambda - 8 \end{bmatrix} = 0$

Expand by R_1

$$= \lambda \begin{bmatrix} \lambda & -1 \\ 17 & \lambda-8 \end{bmatrix} + 1 \begin{vmatrix} 0 & -1 \\ -4 & \lambda-8 \end{bmatrix} + 0 \begin{bmatrix} - & - \\ - & - \end{bmatrix} = 0$$

$$= \lambda \begin{vmatrix} \lambda & -1 \\ 17 & \lambda-8 \end{vmatrix} + 1 \begin{bmatrix} 0 & -1 \\ -4 & \lambda-8 \end{vmatrix} + 0 \begin{vmatrix} - & - \\ - & - \end{vmatrix} = 0$$

$$\lambda(\lambda(\lambda-8)+(17)) + 1(0-4) = 0$$

$$\lambda(\lambda^2 - 8\lambda + 17) - 4 = 0$$

$$\lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$$

Using Synthitic Division

Method.

$$\begin{array}{c|cccc} & 1 & -8 & 17 & -4 \\ 1 & & +1 & -7 & 10 \\ \hline & 1 & -7 & 10 & 6 \end{array}$$

Check Again

$$\begin{array}{c|cccc} & 1 & -8 & 17 & -4 \\ 4 & & 4 & -16 & 4 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$\lambda = 4$

one power decrease.

$\lambda^2 - 4\lambda + 1 = 0$

using Quardatic farmula

$a = 1$
$b = -4$
$c = 1$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$\lambda = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{2\times 2\times 3}}{2}$$

$$\lambda = 2 \pm \sqrt{3}$$

$$\lambda = \frac{\cancel{4} \pm \cancel{2}\sqrt{3}}{\cancel{2}}$$

$\boxed{\lambda = 2 \pm \sqrt{3}}$ Ans.

$$\boxed{\begin{aligned} \lambda_1 &= 4 \\ \lambda_2 &= 2 + \sqrt{3} \\ \lambda_3 &= 2 - \sqrt{3} \end{aligned}}$$

CamScanner

$\Sigma \overline{100}$

(3) $A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$

(4) $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -1 & 3 & 2 \end{bmatrix}$

Homework

Linear Algebra
Full course