

Recap lecture 9



- ⌘ **TGs accepting the languages: containing aaa or bbb, beginning and ending in different letters, beginning and ending in same letters, EVEN-EVEN, a's occur in even clumps and ends in three or more b's, example showing different paths traced by one string, Definition of GTG**

Task Solution ...

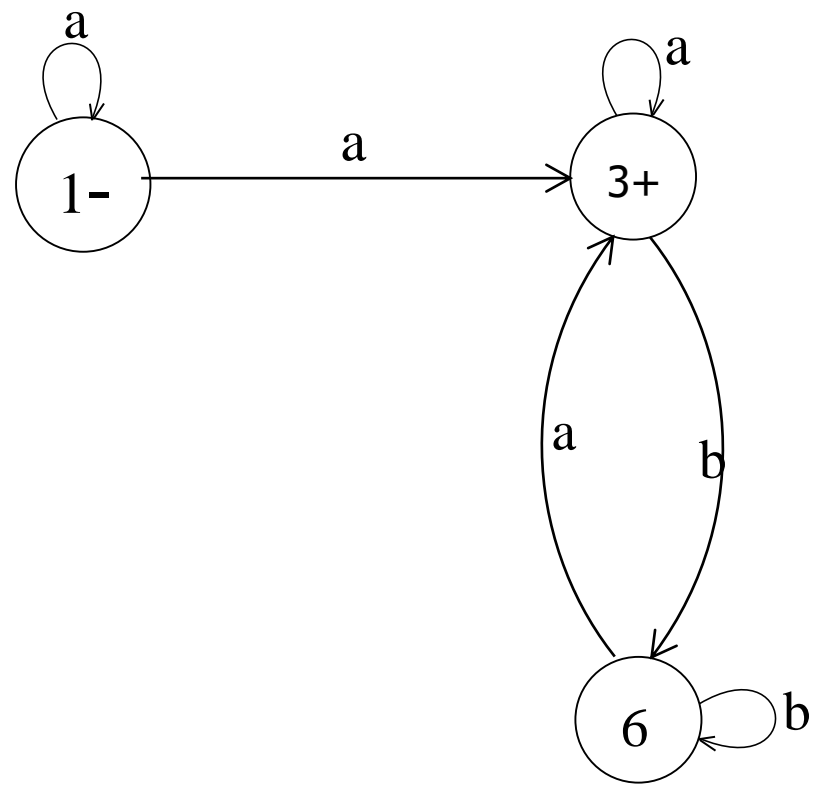
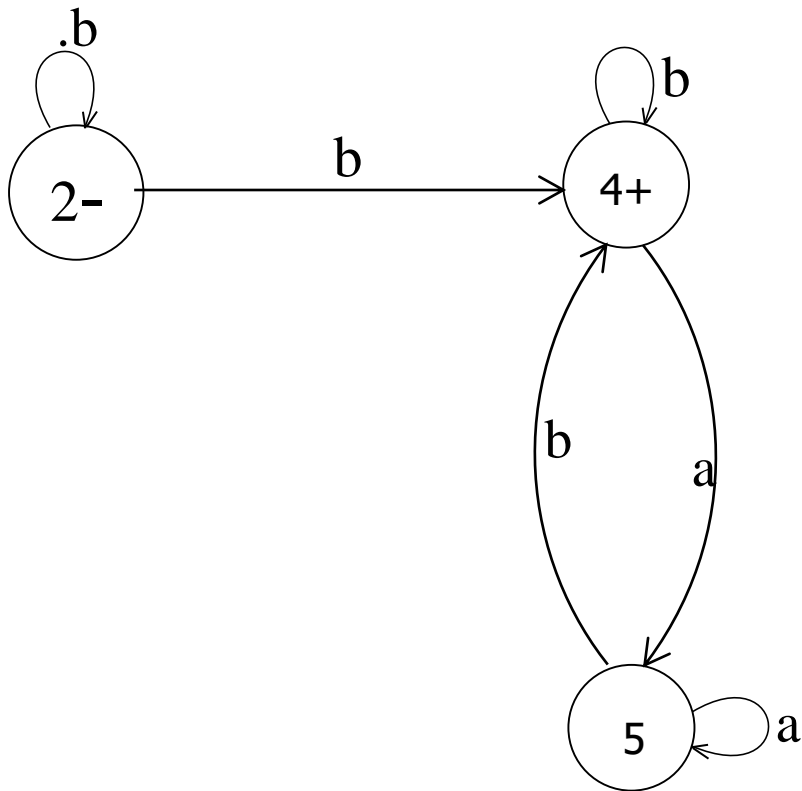


Build a TG accepting the language L of strings, defined over $\Sigma = \{a, b\}$, **beginning with and ending in the same letters**

Solution: The language L may be expressed by $a + b + a(a + b)^*a + b(a + b)^*b$

The language L may be accepted by the following TG

Task Solution



Generalized Transition Graphs

A generalized transition graph (GTG) is a collection of three things

- 1) Finite number of states, at least one of which is start state and some (maybe none) final states.
- 2) Finite set of input letters (Σ) from which input strings are formed.
- 3) Directed edges connecting some pair of states labeled with regular expression.

It may be noted that in GTG, the labels of transition edges are corresponding regular expressions

Example

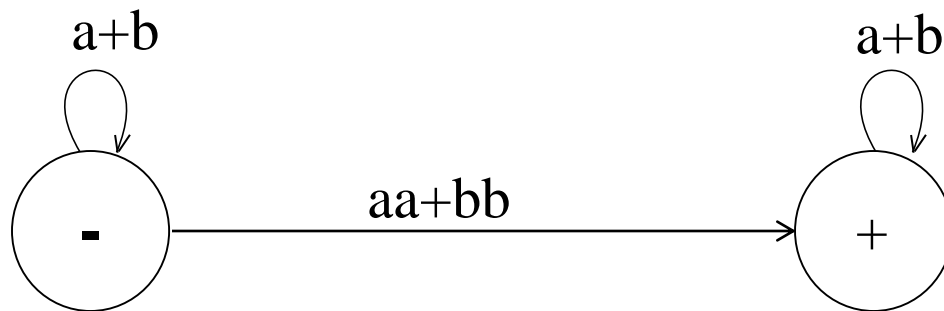
Consider the language L of strings, defined over $\Sigma=\{a,b\}$, containing **double a or double b**.

The language L can be expressed by the following regular expression

$$(a+b)^* (aa + bb) (a+b)^*$$

The language L may be accepted by the following GTG.

Example continued ...



Example

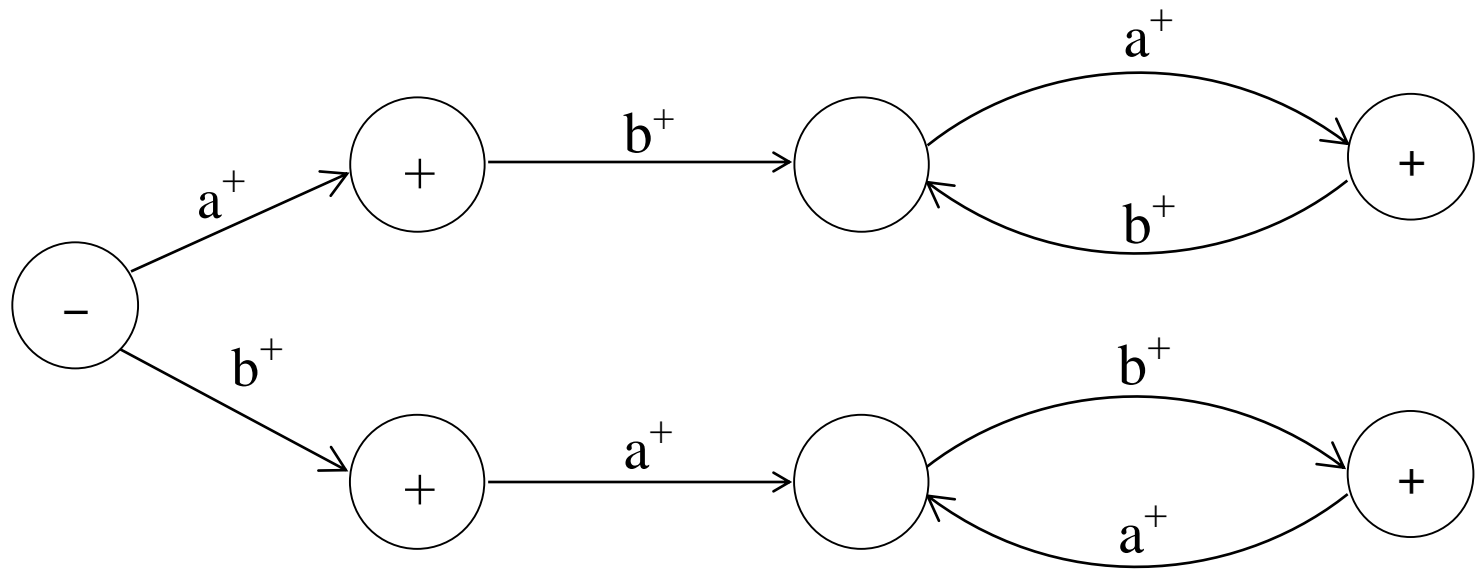
⌘ Consider the Language L of strings, defined over $\Sigma = \{a, b\}$, **beginning with and ending in same letters.**

The language L may be expressed by the following regular expression

$$(a+b)^+ a(a + b)^* a + b(a + b)^* b$$

This language may be accepted by the following GTG

Example



Example

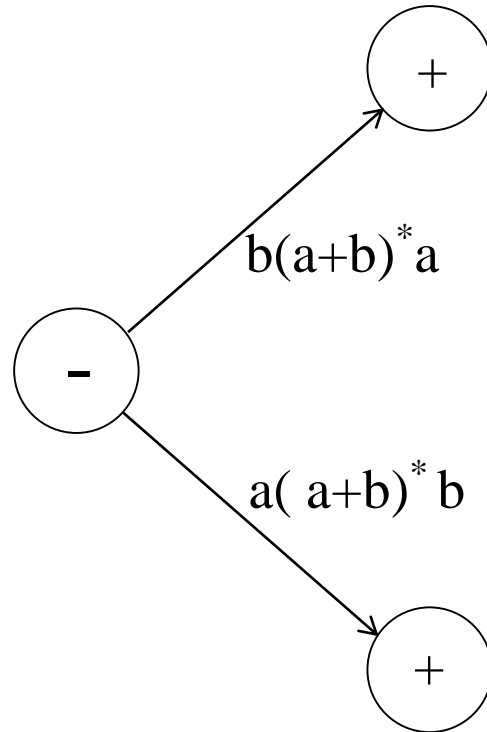
Consider the language L of strings of, defined over $\Sigma = \{a, b\}$, **beginning and ending in different letters.**

The language L may be expressed by RE

$$a(a + b)^*b + b(a + b)^*a$$

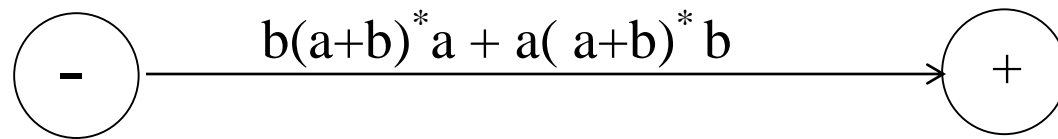
The language L may be accepted by the following GTG

Example Continued ...



The language L may be accepted by the following GTG as well

Example Continued ...



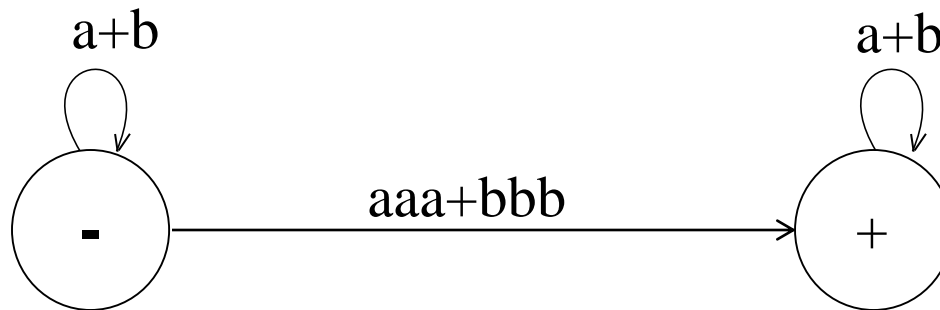
Example

Consider the language L of strings, defined over $\Sigma = \{a, b\}$, **having triple a or triple b.** The language L may be expressed by RE

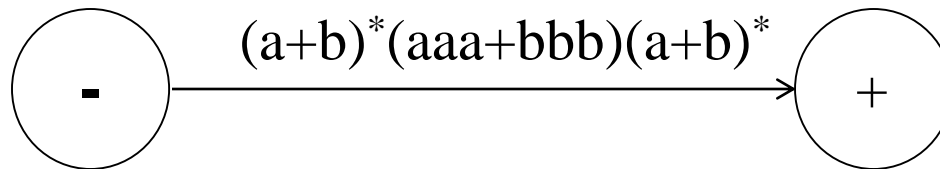
$$(a+b)^* (aaa + bbb) (a+b)^*$$

This language may be accepted by the following GTG

Example Continued ...



OR



Nondeterminism



⌘ TGs and GTGs provide certain relaxations *i.e.* there may exist more than one path for a certain string or there may not be any path for a certain string, this property creates **nondeterminism** and it can also help in differentiating TGs or GTGs from FAs. Hence an FA is also called a Deterministic Finite Automaton (DFA).

Kleene's Theorem



⌘ **If** a language can be expressed by

1. FA or
2. TG or
3. RE

then it can also be expressed by other two as well.

It may be noted that the theorem is proved, proving the following three parts

Kleene's Theorem continued ...



Kleene's Theorem Part I

If a language can be accepted by an FA then it can be accepted by a TG as well.

Kleene's Theorem Part II

If a language can be accepted by a TG then it can be expressed by an RE as well.

Kleene's Theorem Part III

If a language can be expressed by a RE then it can be accepted by an FA as well.

Kleene's Theorem continued ...



Proof(Kleene's Theorem Part I)

Since every FA can be considered to be a TG as well, therefore there is nothing to prove.

Summing Up



- ⌘ Definition of GTG, examples of GTG accepting the languages of strings: containing aa or bb, beginning with and ending in same letters, beginning with and ending in different letters, containing aaa or bbb,
- ⌘ Nondeterminism, Kleene's theorem (part I, part II, part III), proof of Kleene's theorem part I