Recap Lecture 15

- Examples of Kleene’s theorem part III (method 3), NFA, examples, avoiding loop using NFA, example, converting FA to NFA, examples, applying an NFA on an example of maze

Application of an NFA

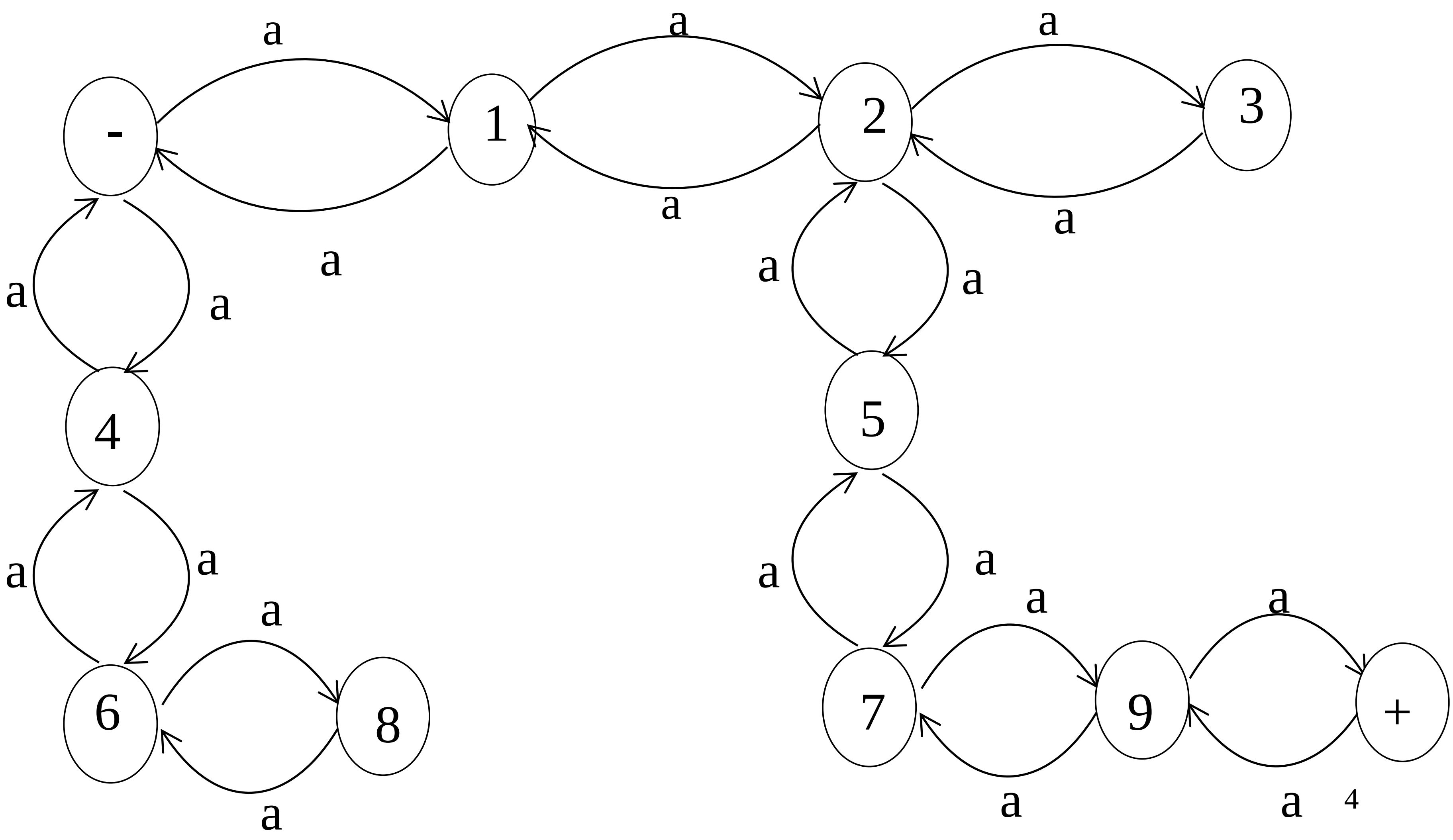
- There is an important application of an NFA in artificial intelligence, which is discussed in the following example of a maze

-	1	2	3
4	L	5	O
6	M	7	P
8	N	9	+

Example Continued ...

- and + indicate the initial and final states respectively. One can move only from a box labeled by other then L, M, N, O, P to such another box. To determine the number of ways in which one can start from the initial state and end in the final state, the following NFA using only single letter a, can help in this regard

Example continued ...



Example continued ...

- It can be observed that the shortest path which leads from the initial state and ends in the final state, consists of six steps *i.e.* the shortest string accepted by this machine is aaaaaa. The next larger accepted string aaaaaaaa.Thus if this NFA is considered to be a TG then the corresponding regular expression may be written as

  aaaaaa(aa)^{*}

  Which shows that there are infinite many required ways

Note

- It is to be noted that every FA can be considered to be an NFA as well , but the converse may not true.
- It may also be noted that every NFA can be considered to be a TG as well, but the converse may not true.

It may be observed that if the transition of null string is also allowed at any state of an NFA then what will be the behavior in the new structure. This structure is defined in the following

NFA with Null String

Definition: If in an NFA, λ is allowed to be a label of an edge then the NFA is called NFA with λ (NFA- λ). An NFA- λ is a collection of three things

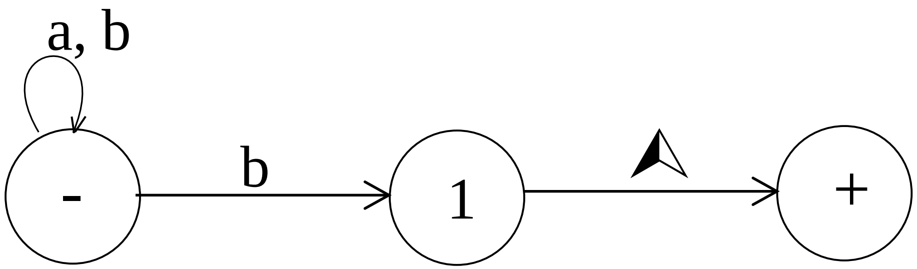
(1) Finite many states with one initial and some final states.

(2) Finite set of input letters, say, Σ ={a, b, c}.

(3) Finite set of transitions, showing where to move if a letter is input at certain state. There may be more than one transitions for certain letter and there may not be any transition for a certain letter. The transition of λ is also allowed at any state.

Example

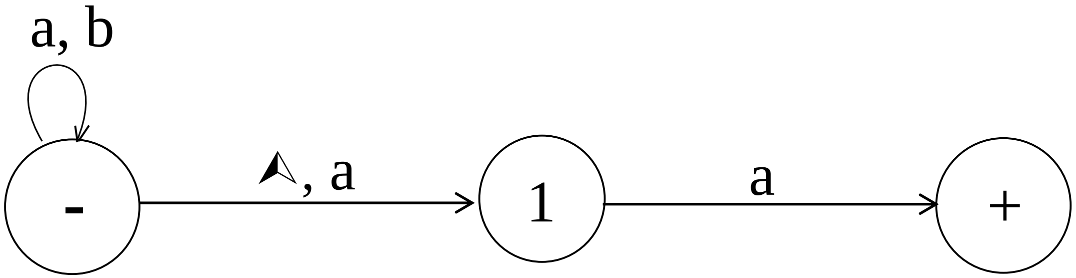
Consider the following NFA with Null string



The above NFA with Null string accepts the language of strings, defined over Σ = {a, b}, **ending in b**.

Example

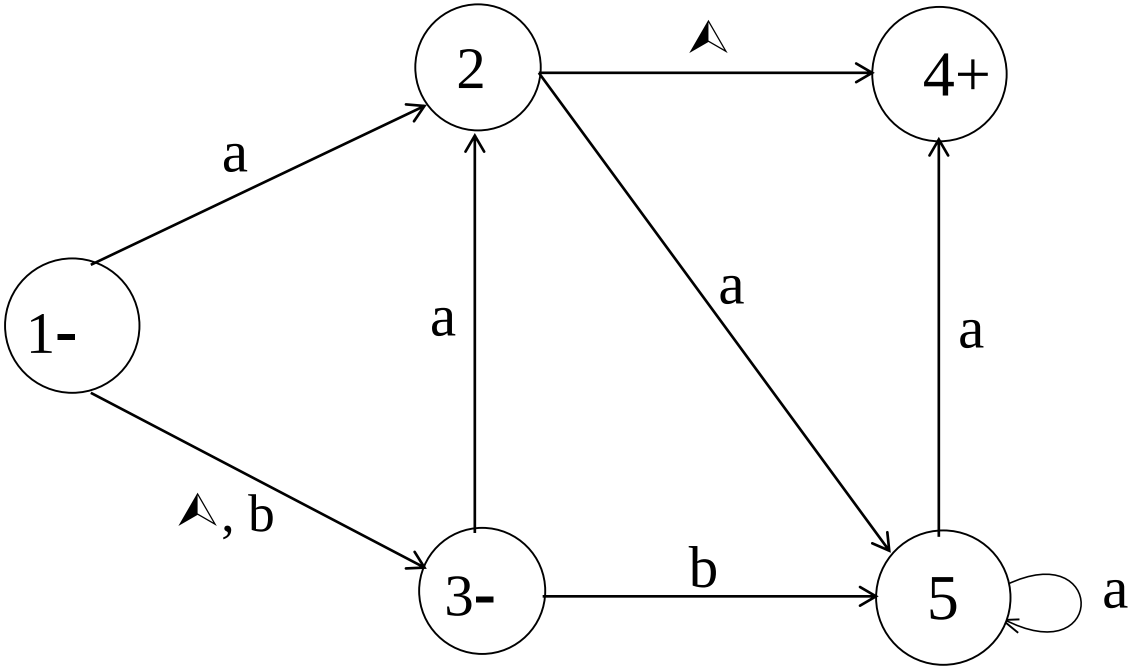
Consider the following NFA with Null string



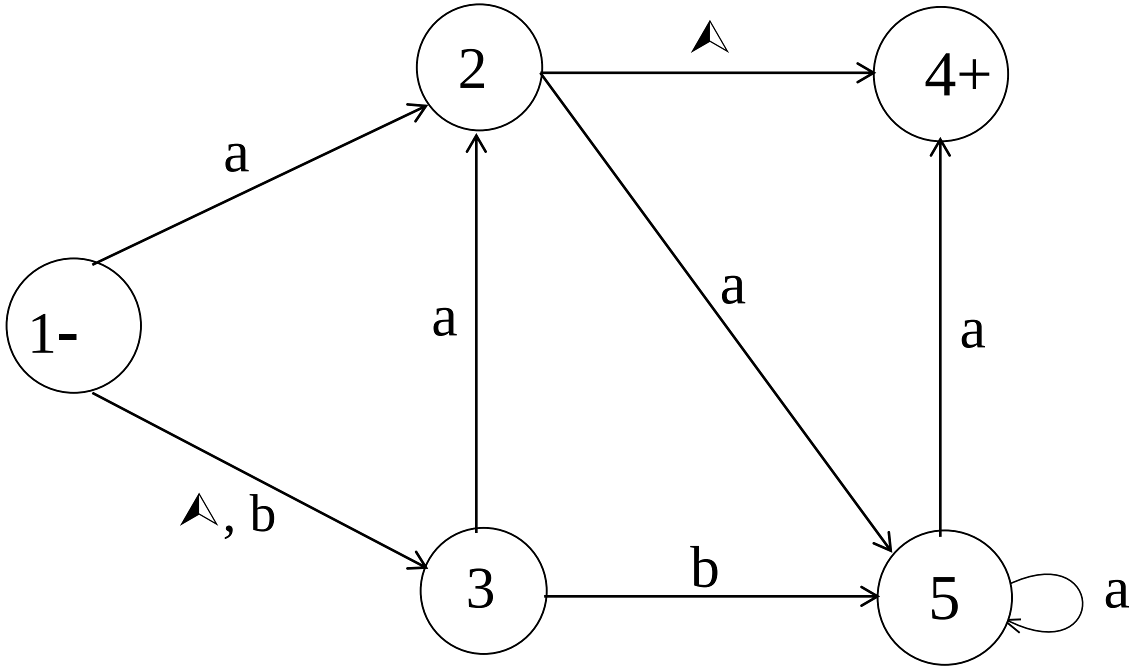
The above NFA with Null string accepts the language of strings, defined over Σ = {a, b}, **ending in a**.

Task

- Determine the regular expression of the following NFA-Λ

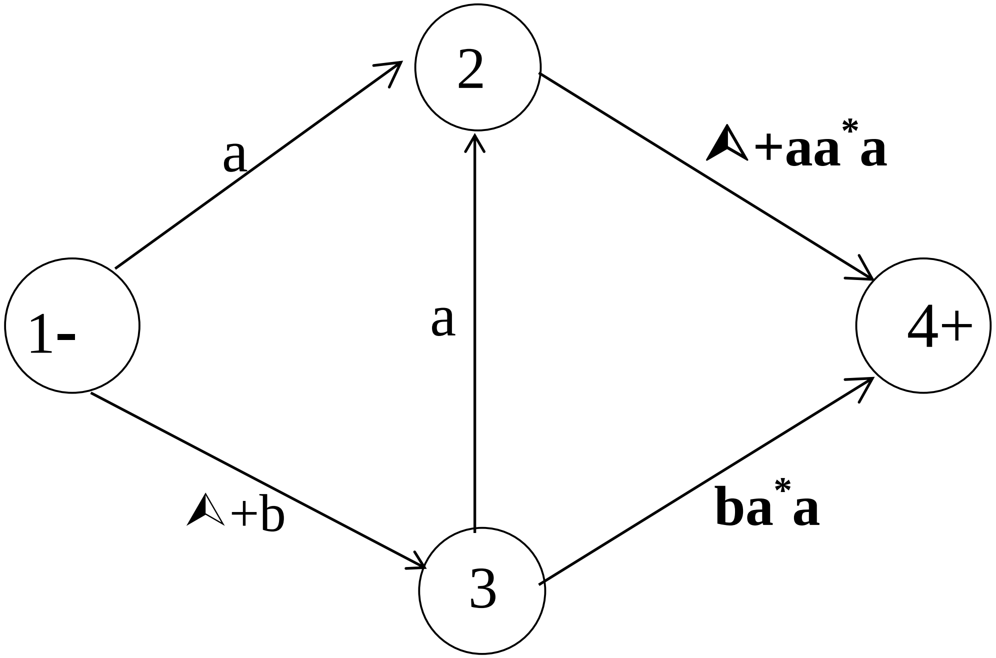


Solution of the Task



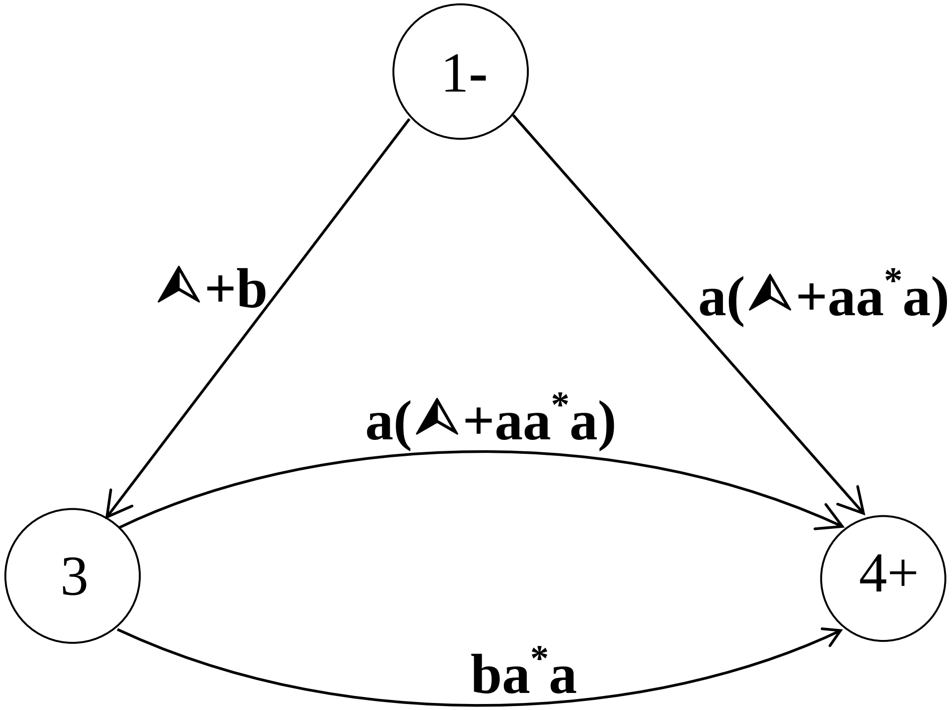
To eliminate state 5 the above NFA-Λ may be reduced to the following

Solution continued …



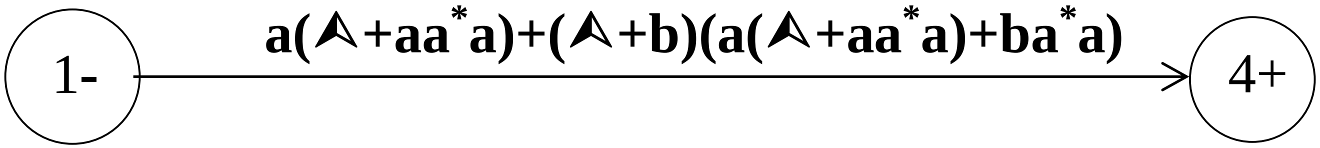
To eliminate state 2 the above NFA-Λ may be reduced to the following

Solution continued …



To eliminate state 3 the above NFA-Λ may be reduced to the following

Solution continued …



$a(\Lambda+aa^*a)+(\Lambda+b)(a(\Lambda+aa^*a)+ba^*a)$

1-

4+

Hence the RE is

$$\mathbf{a(\Lambda+aa^*a)+(\Lambda+b)(a(\Lambda+aa^*a)+ba^*a)}$$

Which may be reduced to

$\mathbf{a+aaa^*a+\underline{ba^*a+ba}+baaa^*a+bba^*a}$ OR

$\mathbf{a+aaa^*a+ba^*a+baaa^*a+bba^*a}$

Note

- It is to be noted that every FA may be considered to be an NFA-Λ as well, but the converse may not true.

- Similarly every NFA-Λ may be considered to be a TG as well, but the converse may not true.

NFA to FA

Two methods are discussed in this regard.

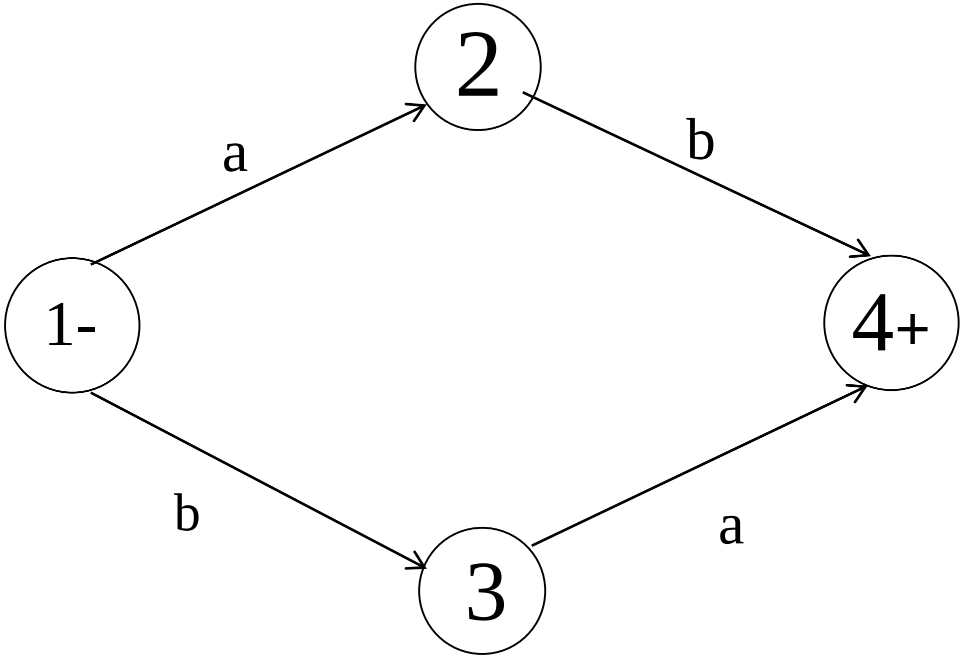
Method 1: Since an NFA can be considered to be a TG as well, so a RE corresponding to the given NFA can be determined (using Kleene’s theorem). Again using the methods discussed in the proof of Kleene’s theorem, an FA can be built corresponding to that RE. Hence for a given NFA, an FA can be built equivalent to the NFA. Examples have, indirectly, been discussed earlier.

NFA to FA continued …

Method 2: Since in an NFA, there more than one transition for a certain letter and there may not be any transition for certain letter, so starting from the initial state corresponding to the initial state of given NFA, the transition diagram of the corresponding FA, can be built introducing an empty state for a letter having no transition at certain state and a state corresponding to the combination of states, for a letter having more than one transitions. Following are the examples

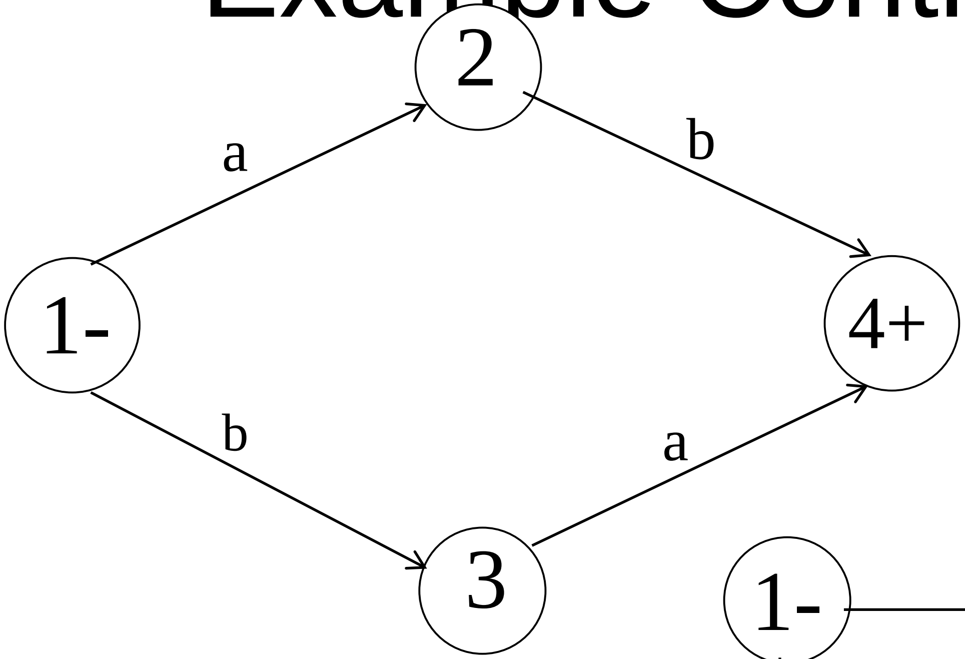
Example

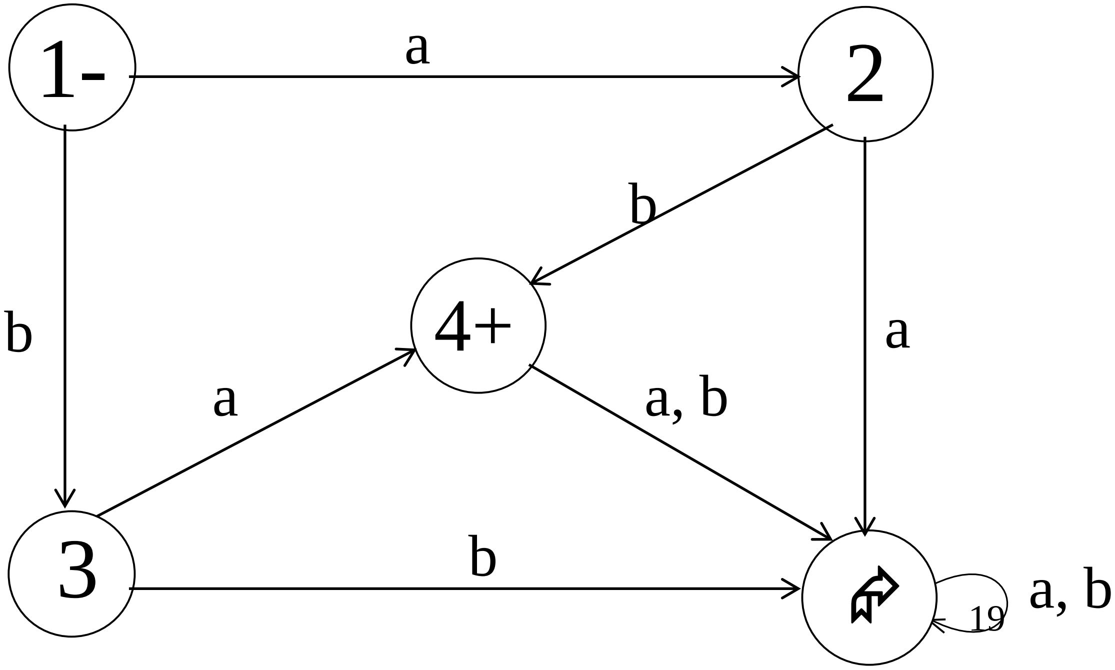
Consider the following NFA



Using the method discussed earlier, the above NFA may be equivalent to the following FA

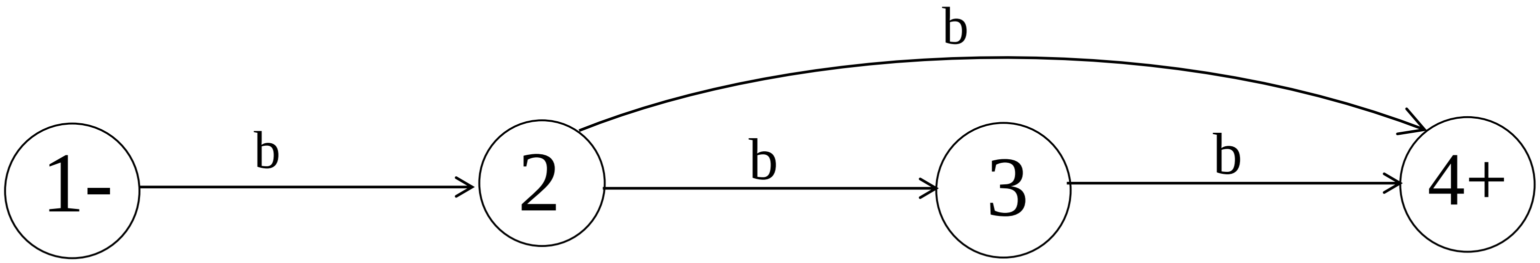
Example Continued ...





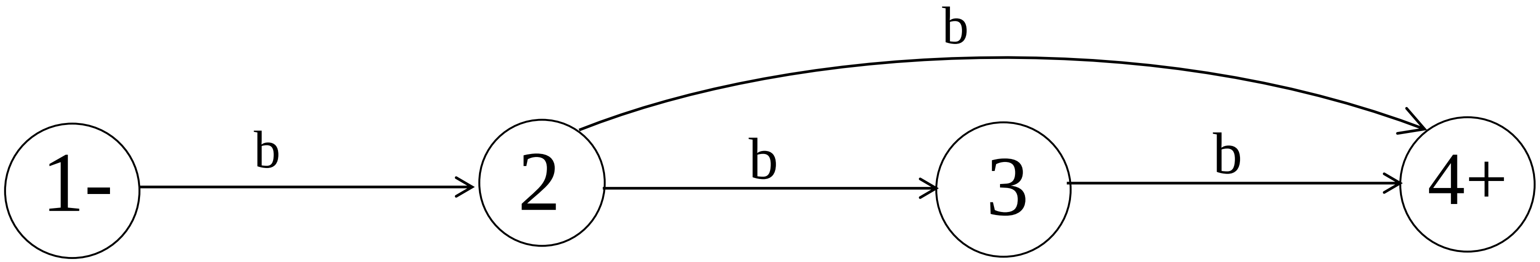
Example

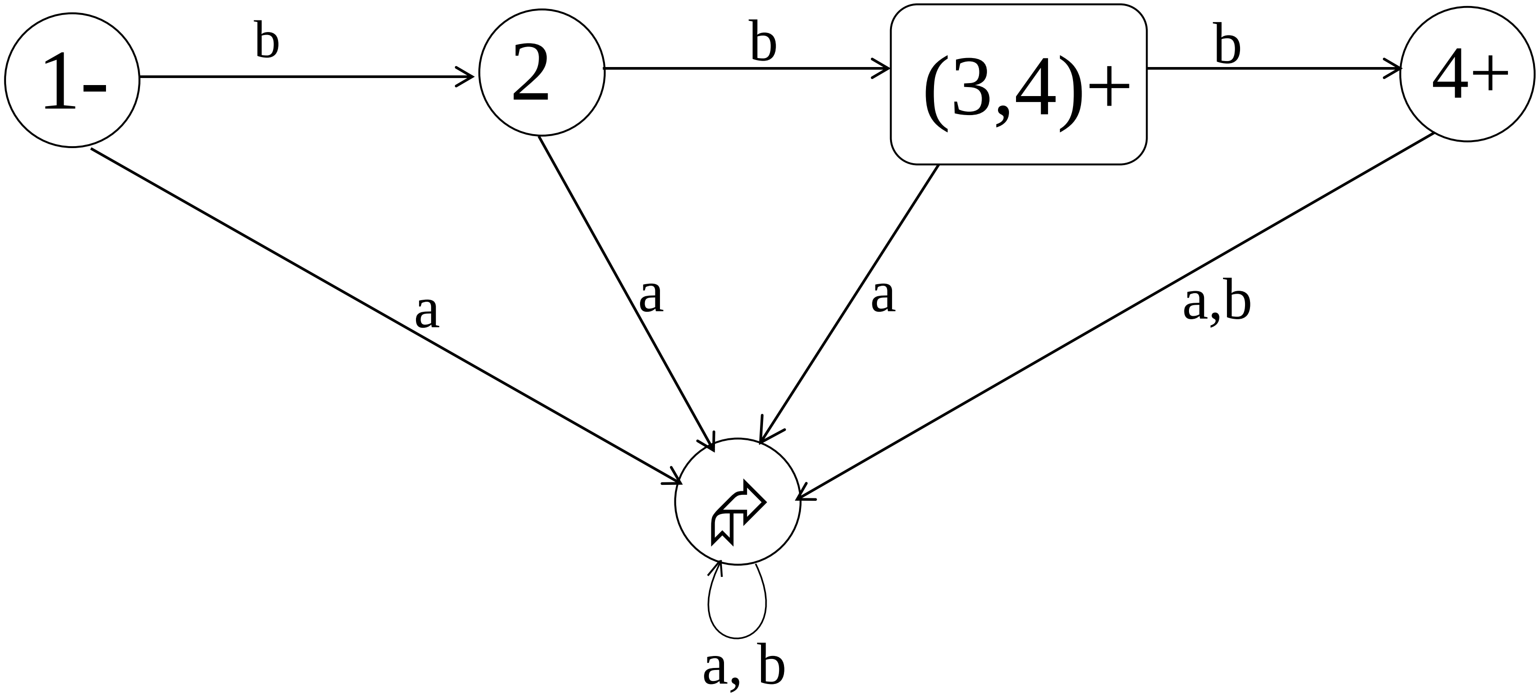
- A simple NFA that accepts the language of strings defined over Σ ={a,b}, **consists of bb and bbb**



- The above NFA can be converted to the following FA

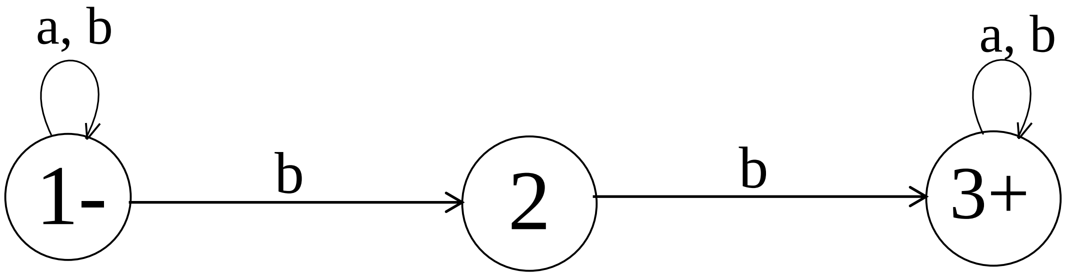
Example Continued ...



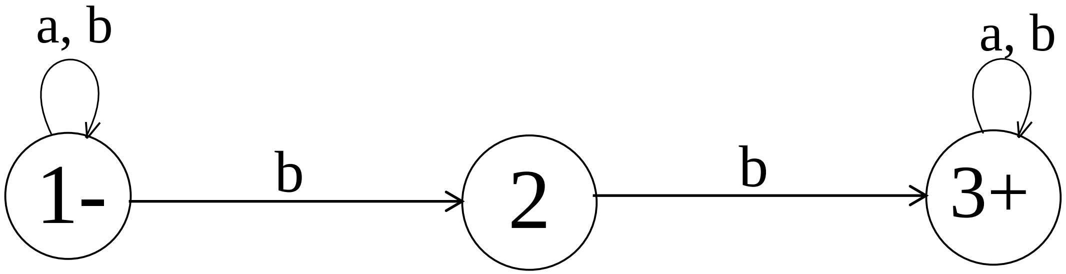


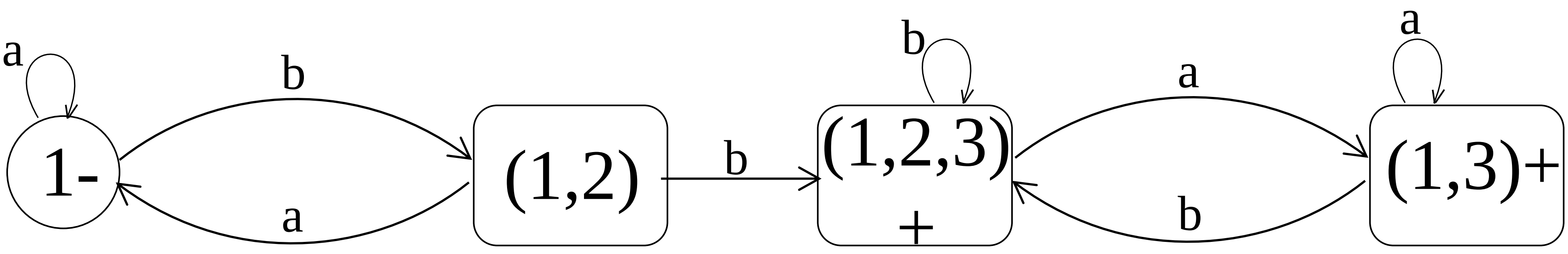
Task

- Build an FA corresponding to the following NFA which accepts the language of strings **containing bb**



Solution of the Task





It may be noted that the above method seems to be complicated, hence an easier method discussed by Martin, follows as

NFA to FA continued …

Method 3: As discussed earlier that in an NFA, there may be more than one transition for a certain letter and there may not be any transition for certain letter, so starting from the initial state corresponding to the initial state of given NFA, the transition table along with new labels of states, of the corresponding FA, can be built introducing an empty state for a letter having no transition at certain state and a state corresponding to the combination of states, for a letter having more than one transitions. Following are the examples

Summing Up

- Applying an NFA on an example of maze, NFA with null string, examples, RE corresponding to NFA with null string (task), converting NFA to FA (method 1,2,3) examples