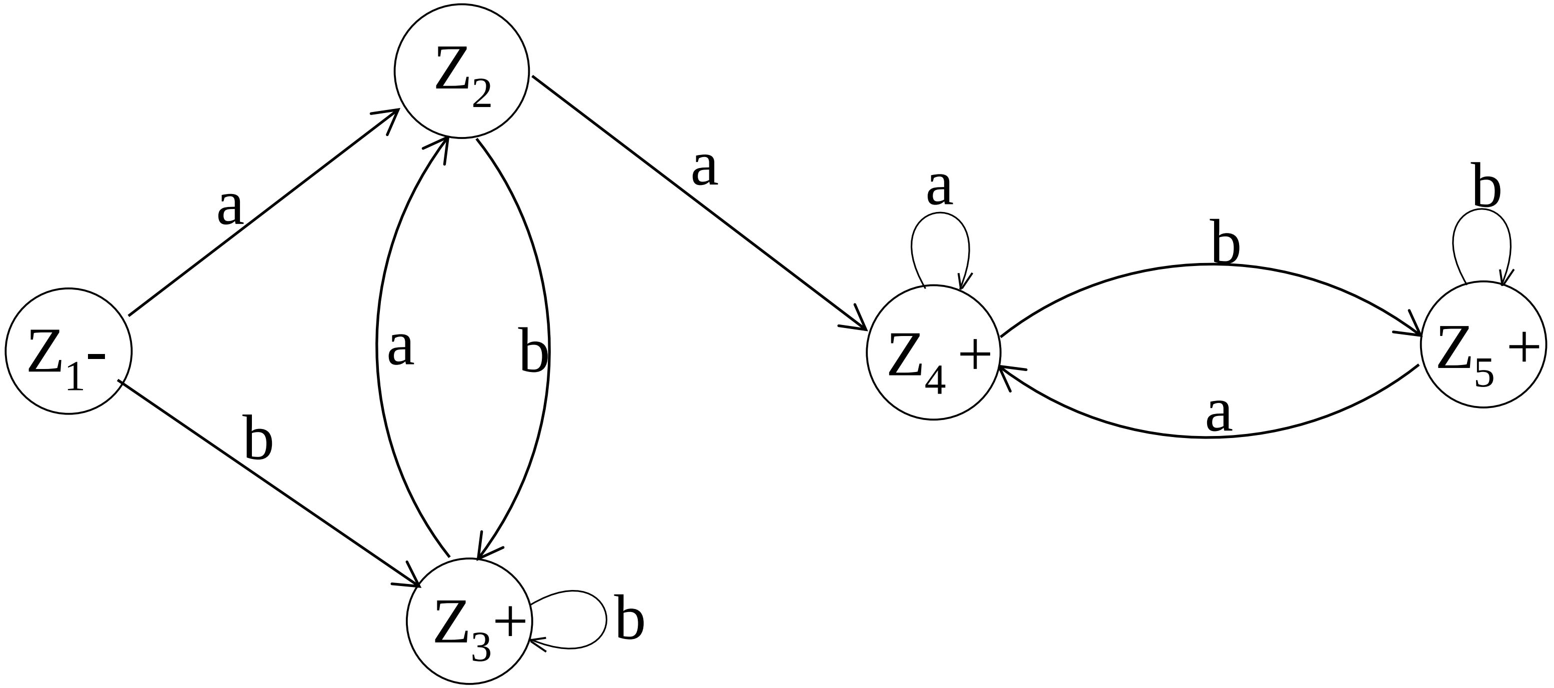
RECAP Lecture 13

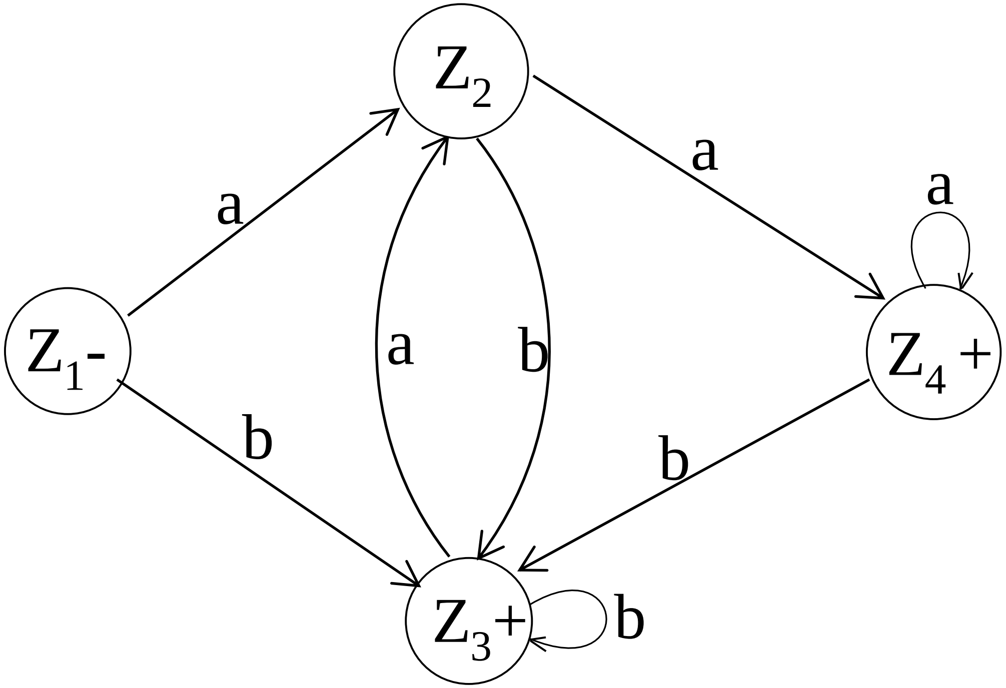
- Examples of Kleene's theorem part III (method 1) continued ,Kleene's theorem part III (method 2: Concatenation of FAs),
- Example of Kleene's theorem part III (method 2 : Concatenation of FAs)

Task

- Build an FA equivalent to the following FA



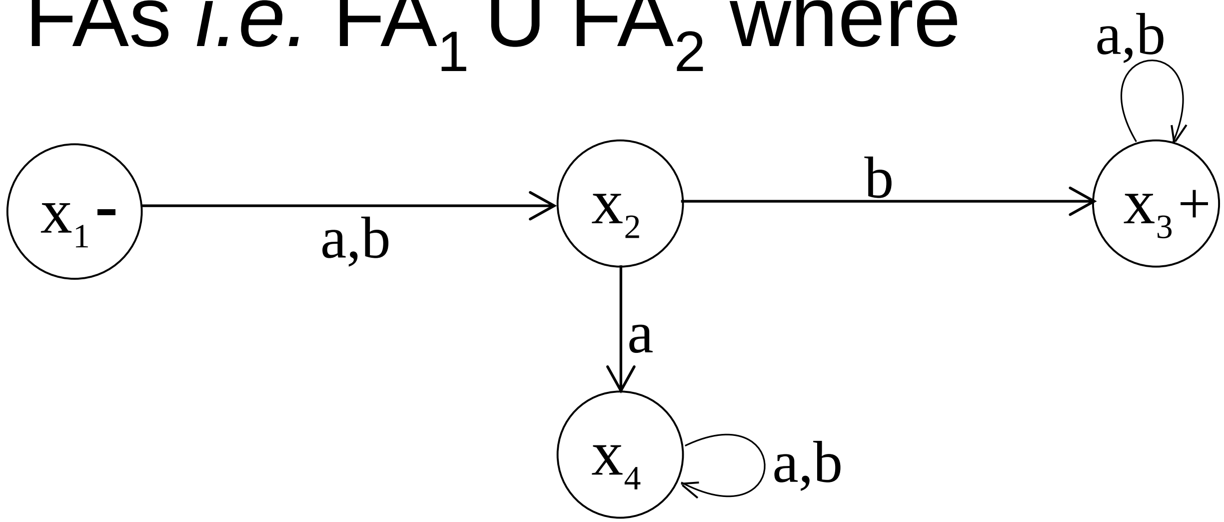
Solution of the Task



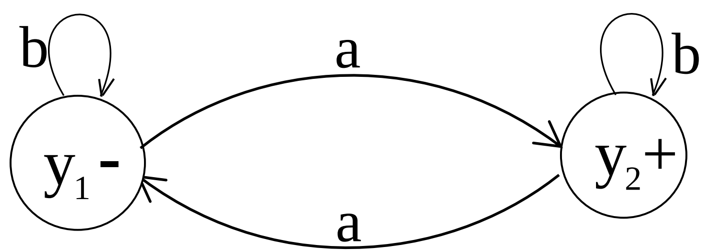
Task

Build an FA corresponding to the union of these two FAs *i.e.* FA_1 U FA_2 where

FA_1



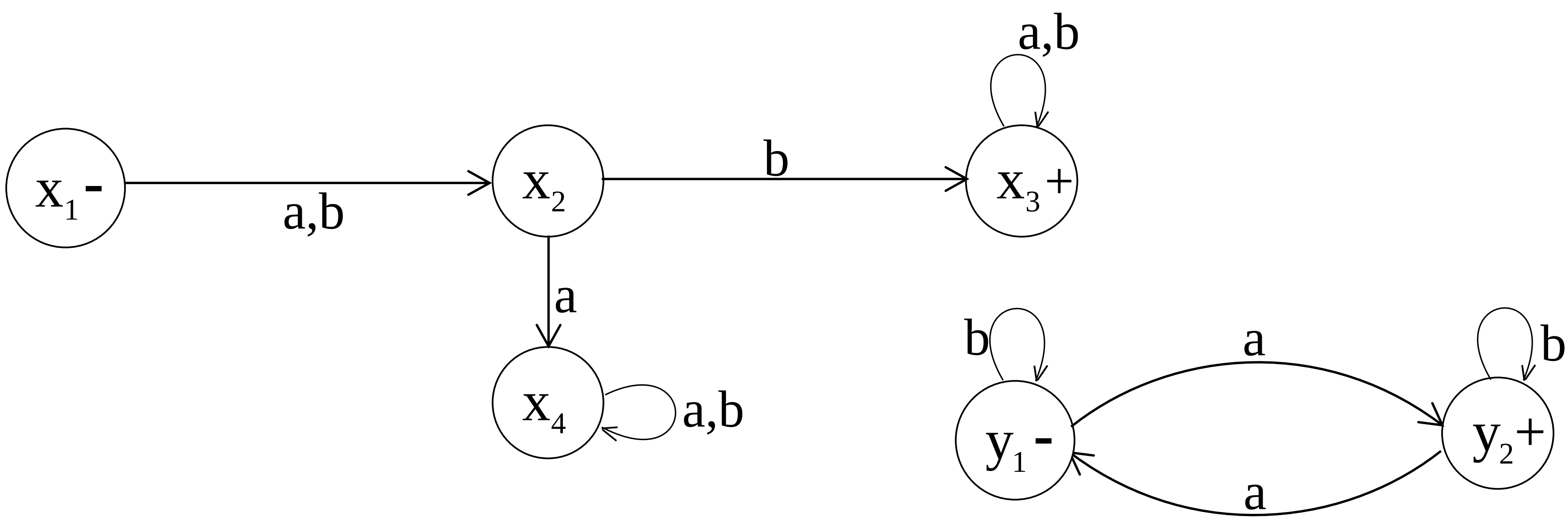
FA_2



Task solution

- RE corresponding to FA_1 may be $(a+b)b(a+b)^*$ which generates the language of strings, defined over $\Sigma=\{a,b\}$, **with b as second letter.**
- RE corresponding to FA_2 may be $b^*a(b+ab^*a)^*$ which generates the language of strings, defined over $\Sigma=\{a,b\}$, **with odd number of a's.**

Solution continued …

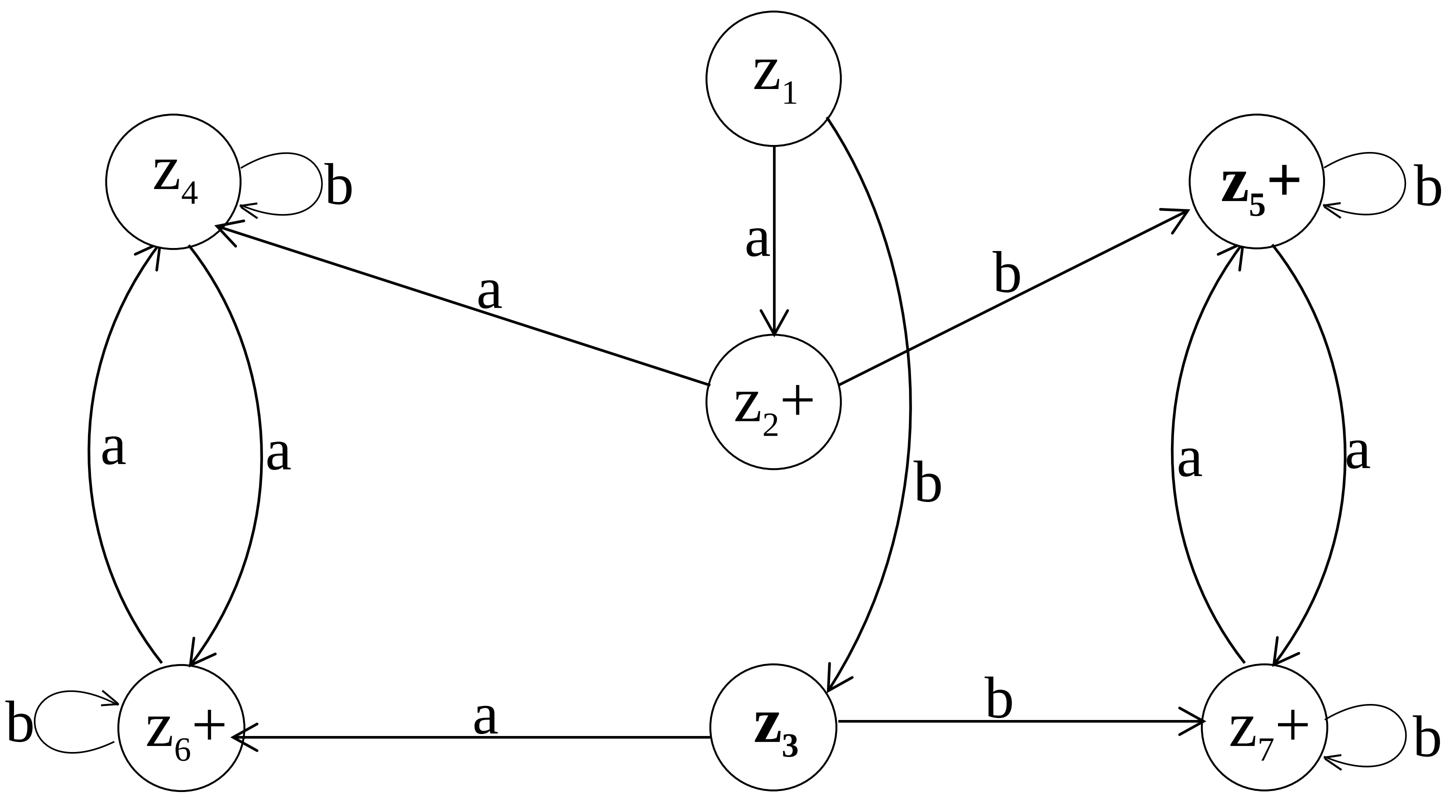


Old States	New States after reading	
	a	b
z_1-🕓(x_1,y_1)	**(x_2,y_2) 🕓z_2**	**(x_2,y_1) 🕓 z_3**

Solution continued …

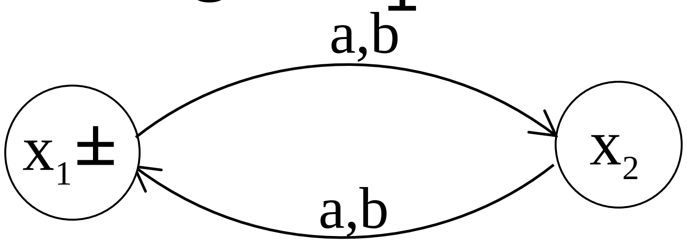
Old States	New States after reading	
	a	b
z_2+🕓(x_2 , y_2)	(x_4,y_1)🕓z_4	(x_3,y_2)🕓 z_5
z_3 🕓(x_2,y_1)	(x_4,y_2)🕓z_6	(x_3,y_1)🕓 z_7
z_4🕓(x_4,y_1)	(x_4,y_2)🕓 z_6	(x_4,y_1)🕓 z_4
z_5+🕓(x_3,y_2)	(x_3,y_1)🕓 z_7	(x_3,y_2)🕓 z_5
z_6+🕓(x_4,y_2)	(x_4,y_1)🕓z_4	(x_4,y_2)🕓 z_6
z_7+🕓(x_3,y_1)	(x_3,y_2)🕓z_5	(x_3,y_1)🕓 z_7

Solution continued …

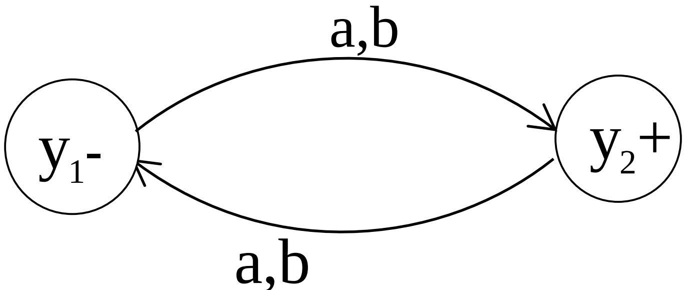


Example

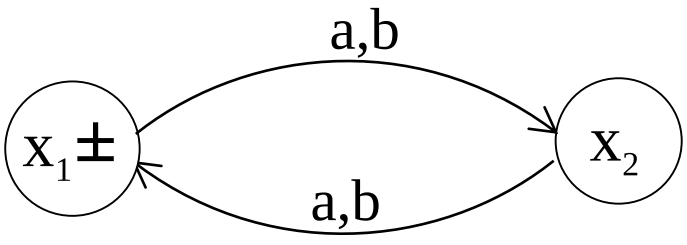
Let $r_1=((a+b)(a+b))^*$ and the corresponding FA_1 be

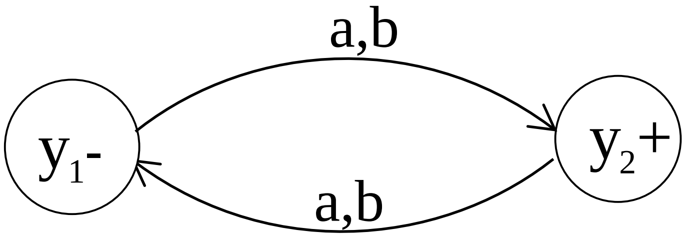


also $r_2 = (a+b)((a+b)(a+b))^*$ or $((a+b)(a+b))^*(a+b)$ and FA_2 be



Example continued …



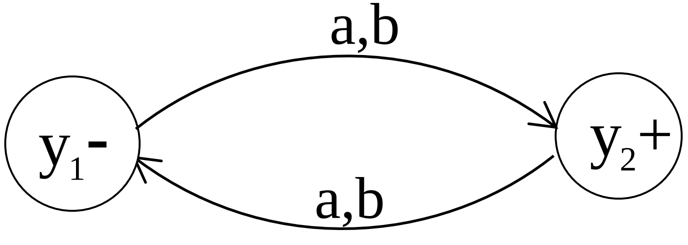


Old States	New States after reading	
	a	b
z_1-🕓(x_1,y_1)	(x_2,y_2) 🕓z_2	(x_2,y_2) 🕓 z_2

Example continued …

Old States	New States after reading	
	a	b
z_2+🕓(x_2,y_2)	(x_1,y_1) 🕓z_1	(x_1,y_1) 🕓 z_1

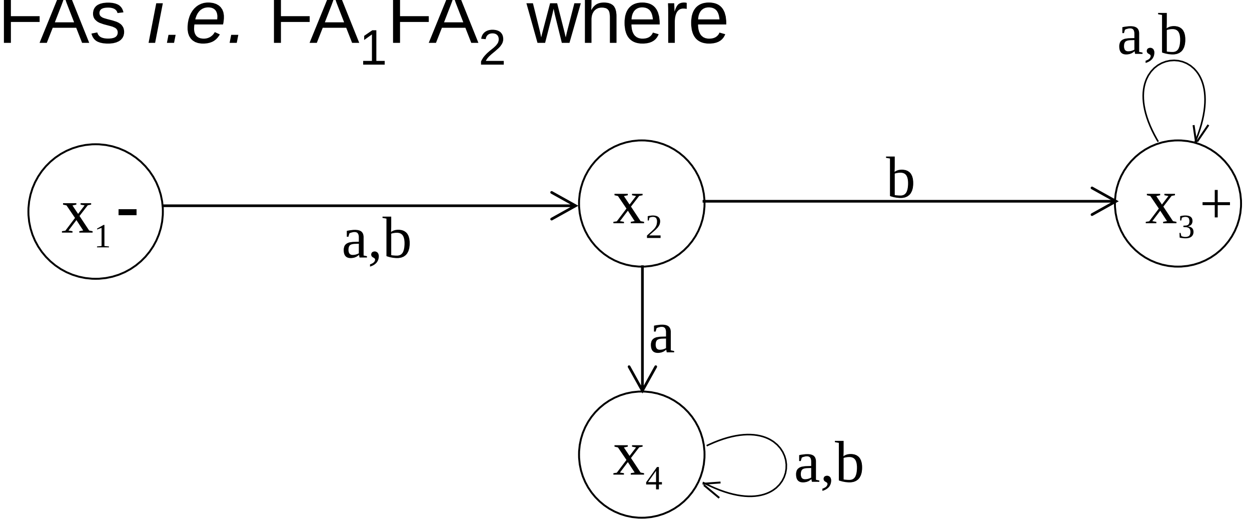
Example continued …



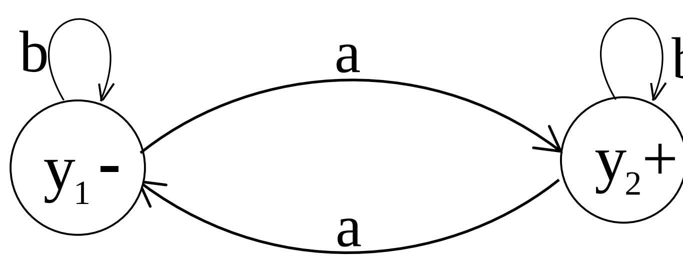
Task

Build FA corresponding to the concatenation of these two FAs *i.e.* FA_1FA_2 where

FA_1



FA_2



Kleene's Theorem Part III Continued …

- **Method3: (Closure of an FA)**

  Building an FA corresponding to r^*, using the FA corresponding to r.

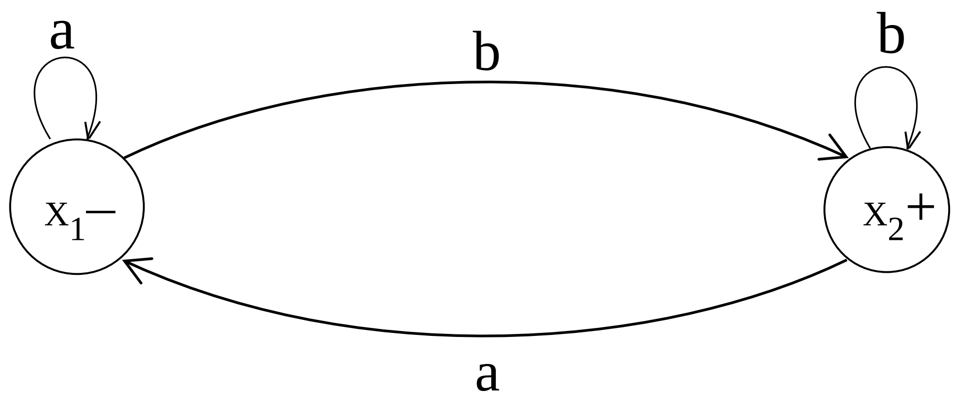
  It is to be noted that if the given FA already accepts the language expressed by the closure of certain RE, then the given FA is the required FA. However the method, in other cases, can be developed considering the following examples

Closure of FA Continued …

Closure of an FA, is same as concatenation of an FA with itself, except that the initial state of the required FA is a final state as well. Here the initial state of given FA, corresponds to the initial state of required FA and a non final state of the required FA as well.

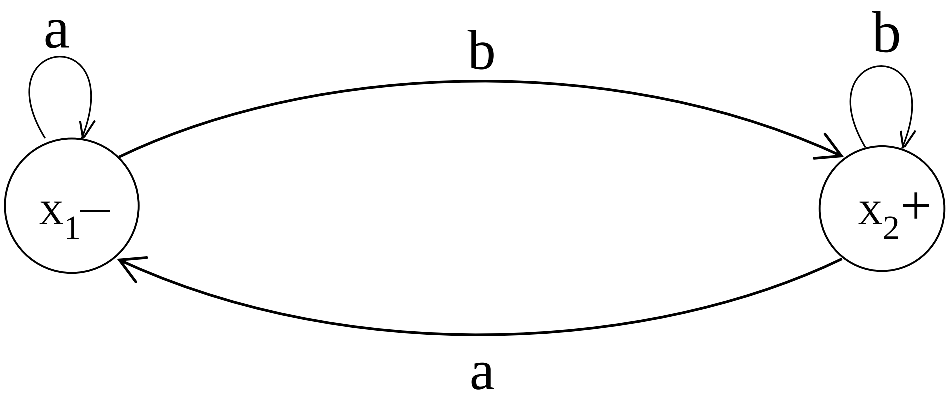
Example

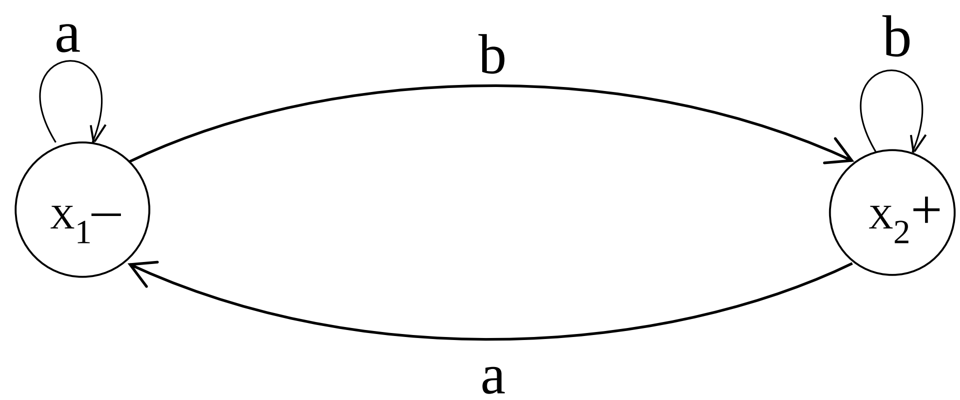
Let $r=(a+b)^*b$ and the corresponding FA be



then the FA corresponding to r^* may be determined as under

Example continued …





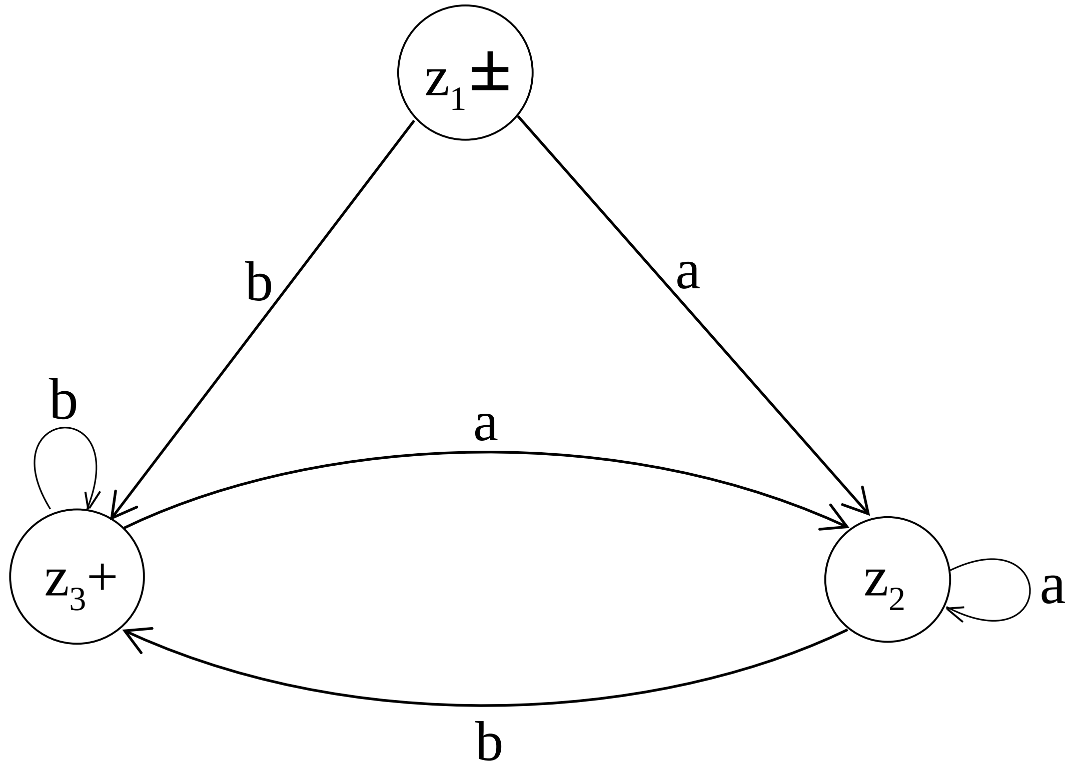
Old States	New States after reading	
	a	b
Final z_1 ± 🕓 x_1	x_1 🕓 z_2	(x_2,x_1) 🕓 z_3

Example continued …

Old States	New States after reading	
	a	b
Non-final z_2🕓x_1	x_1🕓z_2	(x_2,x_1)🕓z_3
z_3+🕓(x_2,x_1)	x_1🕓z_2	(x_2,x_1)🕓z_3

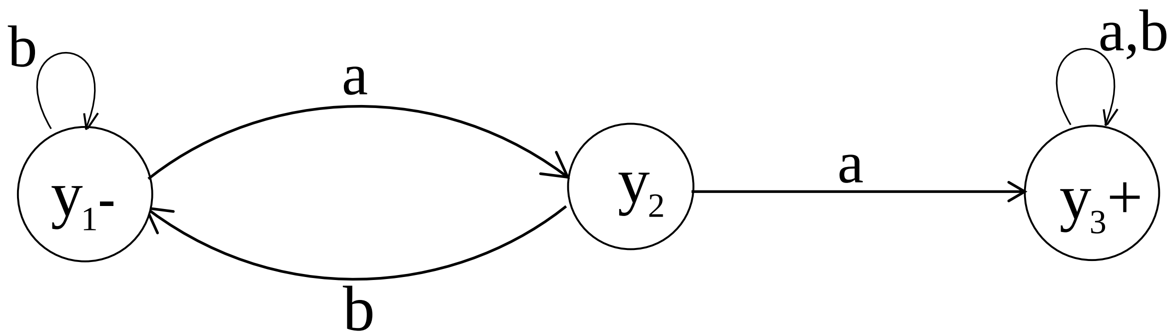
The corresponding transition diagram may be as under

Example continued …

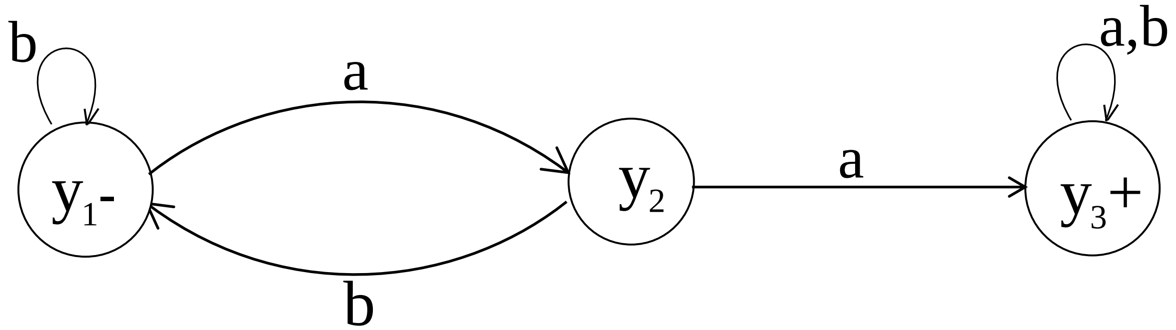


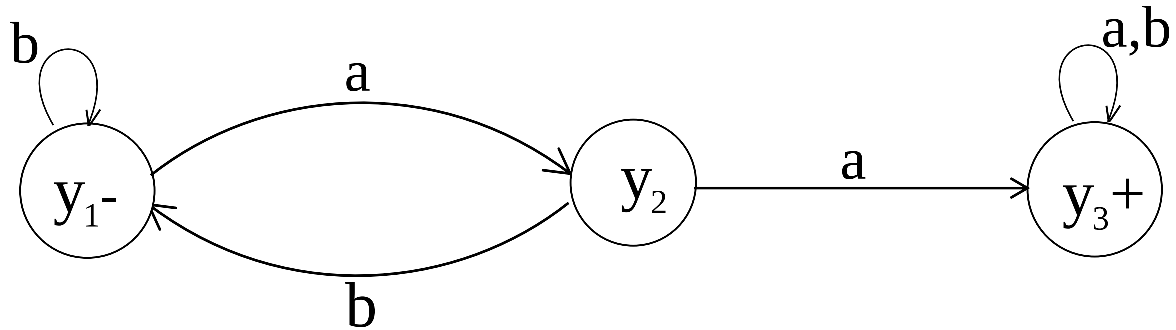
Example

Let r=$(a+b)^*aa(a+b)^*$ and the corresponding FA be



Example continued …



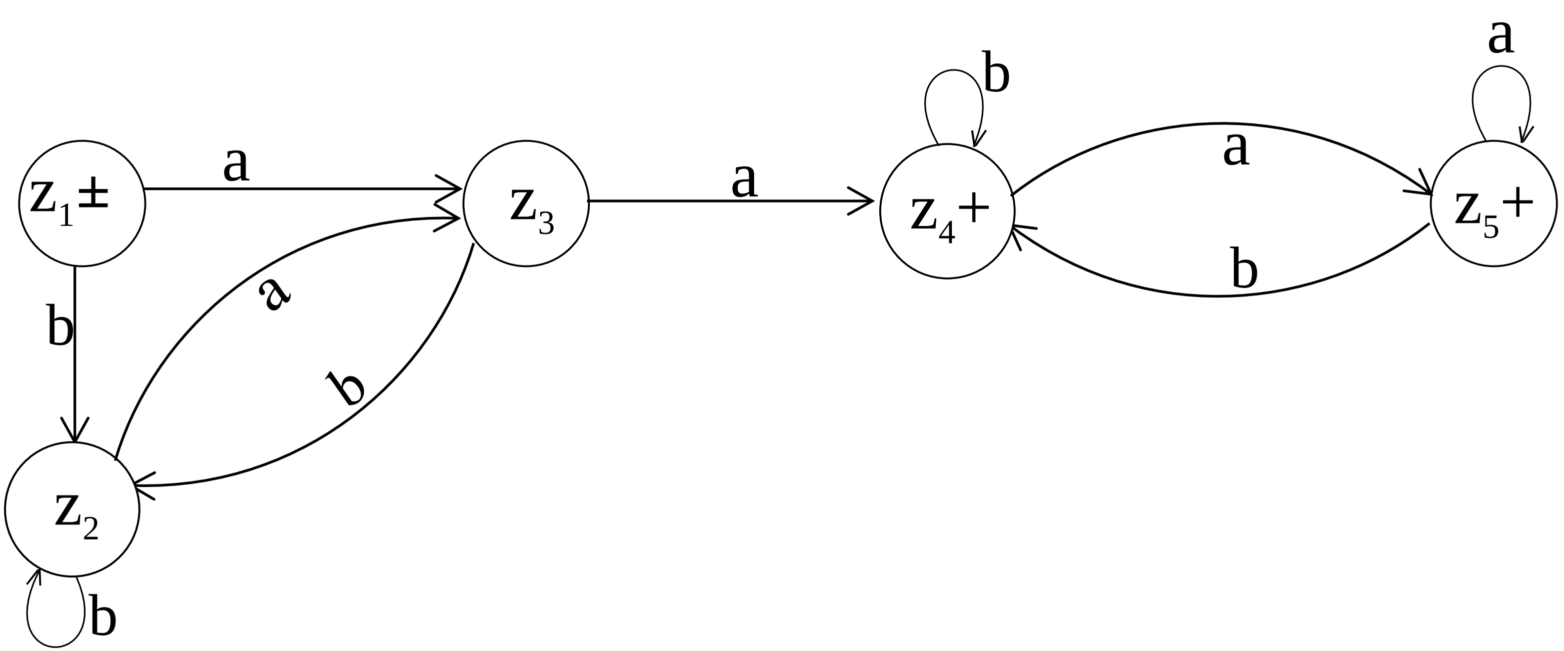


Old States	New States after reading	
	a	b
Final $z_1 \pm$ 🕓 y_1	y_2 🕓 z_3	y_1 🕓 z_2

Example continued …

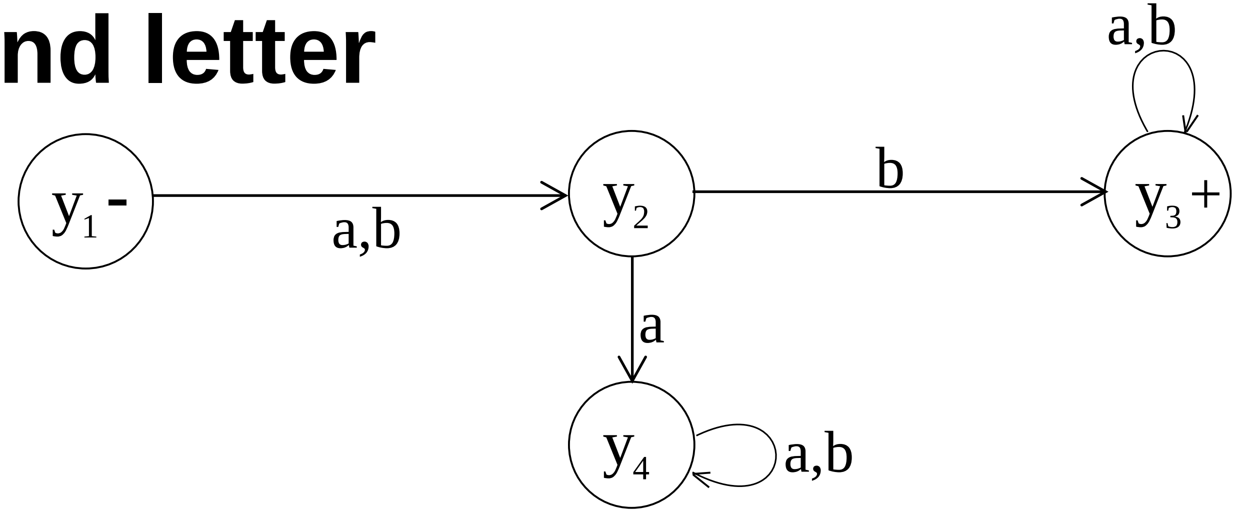
Old States	New States after reading	
	a	b
$z_2 \equiv y_1$	$y_2 \equiv z_3$	$y_1 \equiv z_2$
$z_3 \equiv y_2$	$(y_3, y_1) \equiv z_4$	$y_1 \equiv z_2$
$z_4^+ \equiv (y_3, y_1)$	$(y_3, y_1, y_2) \equiv z_5$	$(y_3, y_1) \equiv z_4$
$z_5^+ \equiv (y_3, y_1, y_2)$	$(y_3, y_1, y_2) \equiv z_5$	$(y_3, y_1) \equiv z_4$

Example continued …

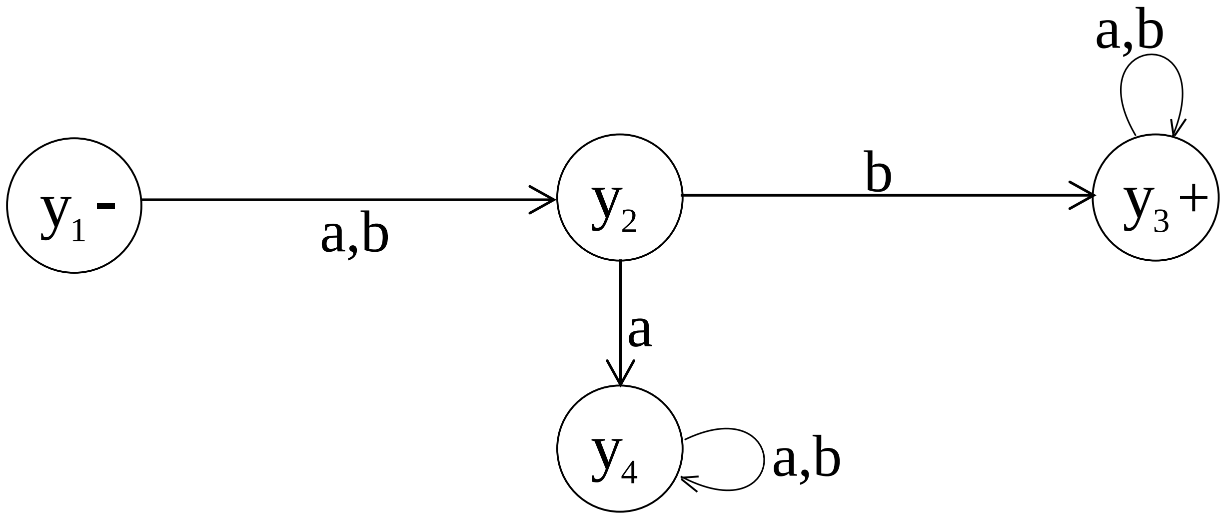


Example

Consider the following FA, accepting the language of strings with **b as second letter**



Example continued …

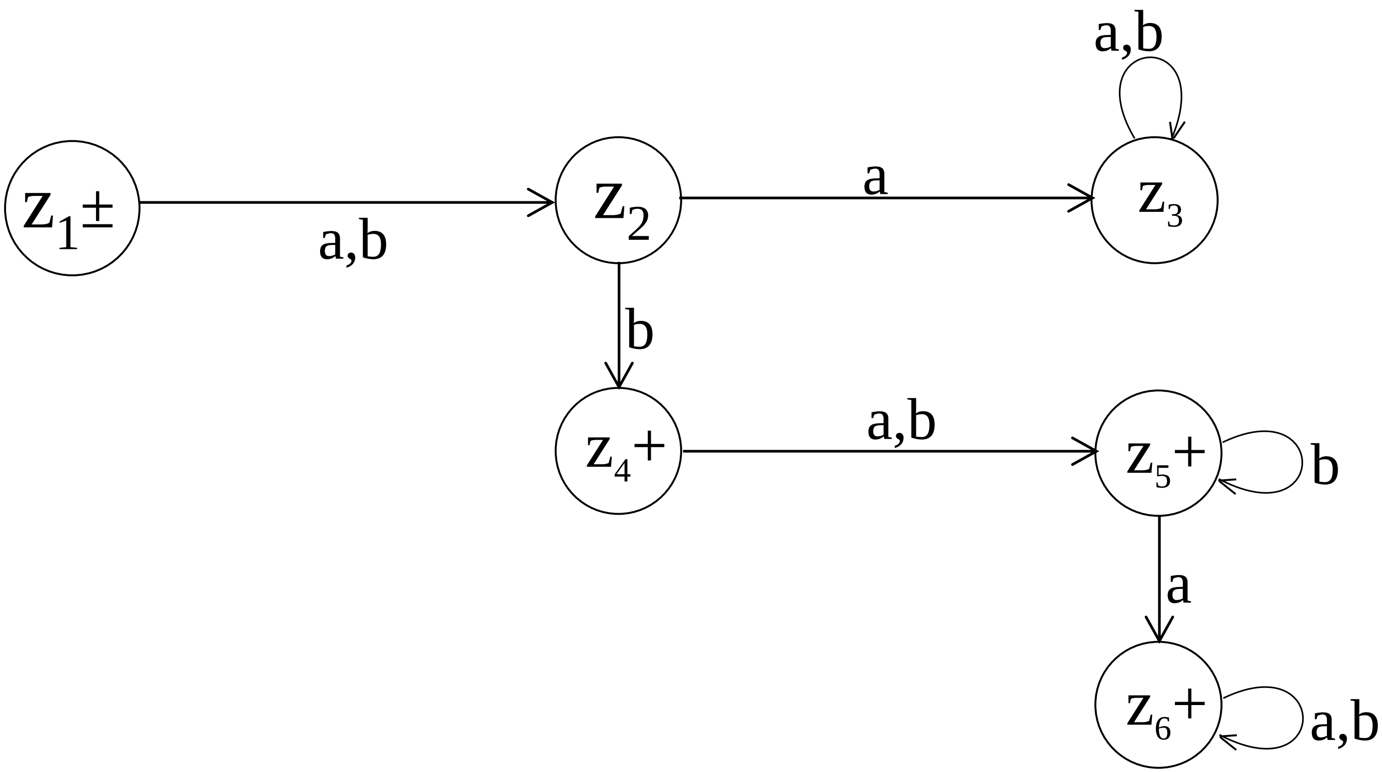


Old States	New States after reading	
	a	b
z_1± 🕓 y_1	y_2 🕓 z_2	y_2 🕓 z_2

Example continued ...

Old States	New States after reading	
	a	b
z_2🕓y_2	y_4🕓z_3	(y_3,y_1)🕓 z_4
z_3🕓y_4	y_4🕓z_3	y_4🕓z_3
z_4+🕓(y_3,y_1)	$(y_3\ ,y_1\ ,y_2)$🕓 z_5	$(y_3\ ,y_1\ ,y_2)$🕓 z_5
z_5+🕓(y_3,y_1,y_2)	$(y_3,y_1\ ,y_2\ ,y_4)$🕓 z_6	(y_3,y_1,y_2)🕓 z_5
z_6🕓$(y_1,y_1\ ,y_2\ ,y_4)$	$(y_1,y_1\ ,y_2\ ,y_4)$🕓z_6	$(y_1,y_1\ ,y_2\ ,y_4)$🕓z_6

Example continued ...



Summing Up

- Examples of Kleene's theorem part III (method 1) continued ,Kleene's theorem part III (method 2: Concatenation of FAs), Examples of Kleene's theorem part III(method 2:concatenation FAs) continued, Kleene's theorem part III (method 3:closure of an FA), examples of Kleene's theorem part III(method 3:Closure of an FA) continued