

# Recap Lecture 12

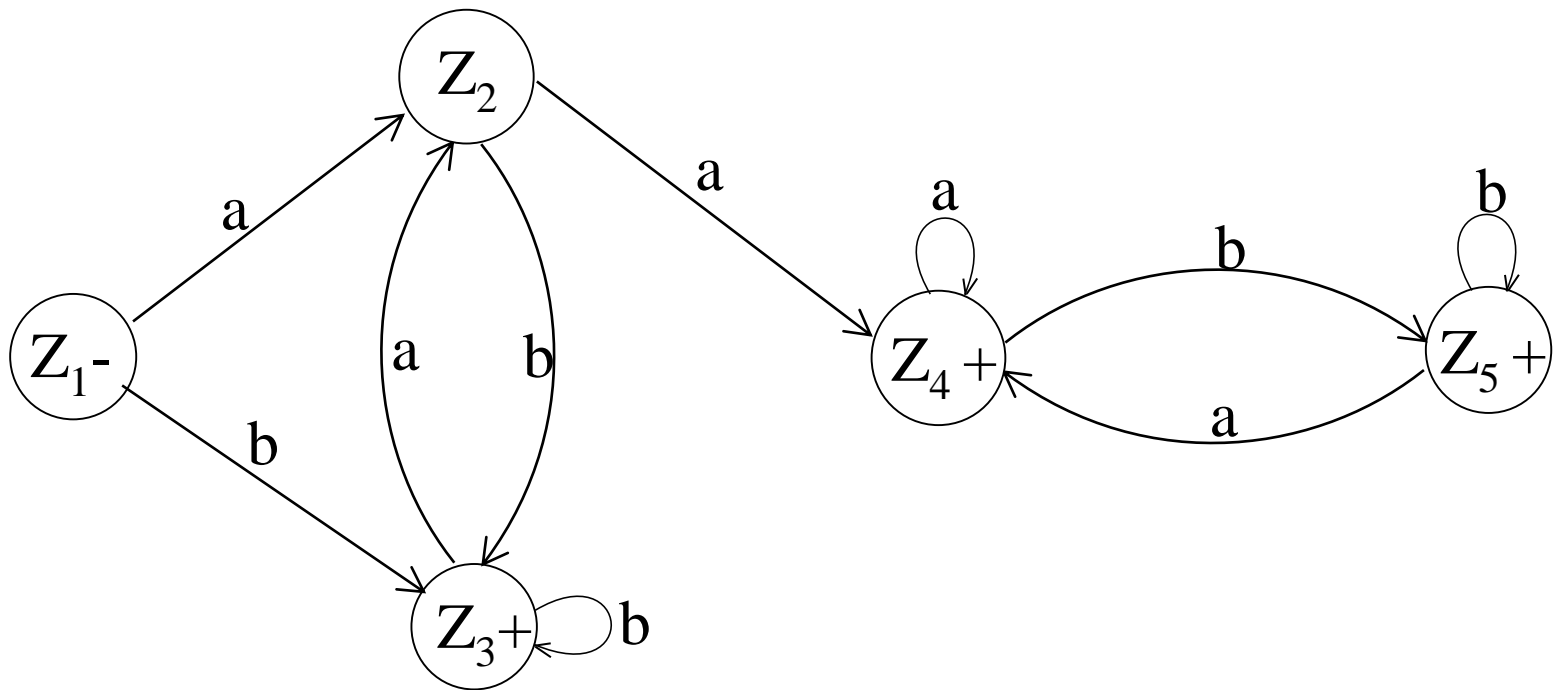


- ⌘ Examples of writing REs to the corresponding TGs, RE corresponding to TG accepting EVEN-EVEN language, Kleene's theorem part III (method 1: union of FAs), examples of FAs corresponding to simple REs, example of Kleene's theorem part III (method 1) continued

## Note

⌘ It may be noted that in the previous example  $FA_1$  contains two states while  $FA_2$  contains three states. Hence the total number of possible combinations of states of  $FA_1$  and  $FA_2$ , in sequence, will be six. For each combination the transitions for both a and b can be determined, but using the method in the example, number of states of  $FA_3$  was reduced to five.

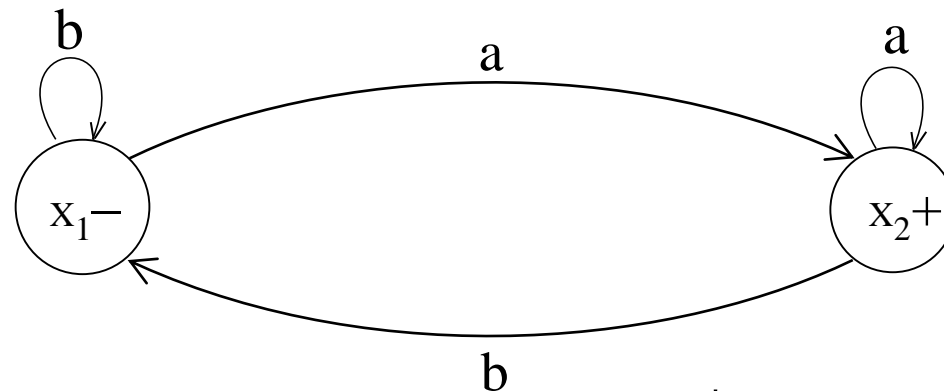
# Task



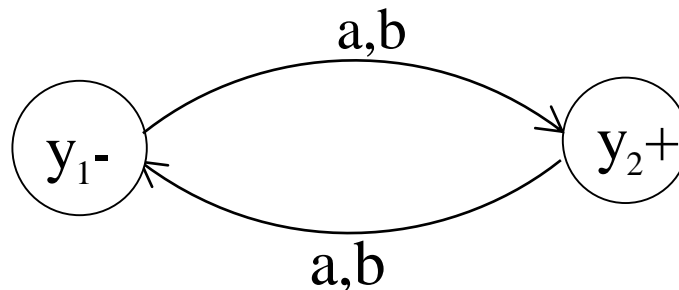
Build an FA equivalent to the previous FA

# Example

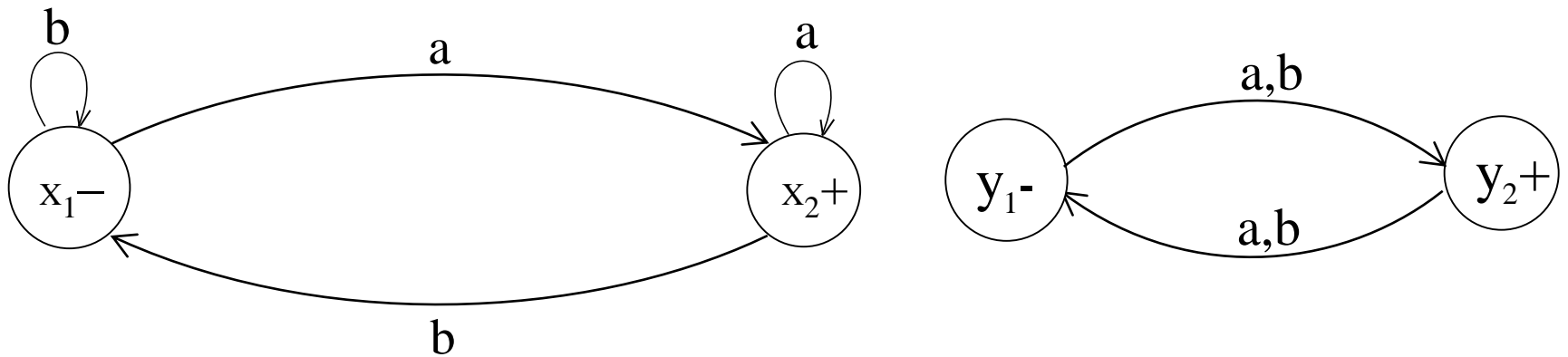
Let  $r_1 = (a+b)^*a$  and the corresponding  $FA_1$  be



also  $r_2 = (a+b)((a+b)(a+b))^*$  or  $((a+b)(a+b))^*(a+b)$  and  $FA_2$  be



# Example continued ...

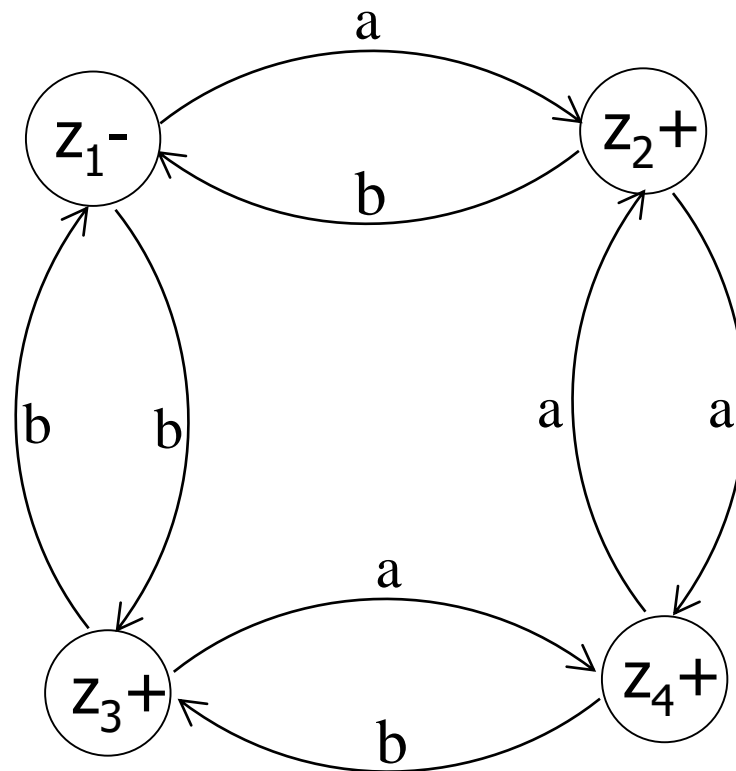


Old States	New States after reading	
	a	b
$z_1^- \cong (x_1, y_1)$	$(x_2, y_2) \cong z_2$	$(x_1, y_2) \cong z_3$

# Example continued ...

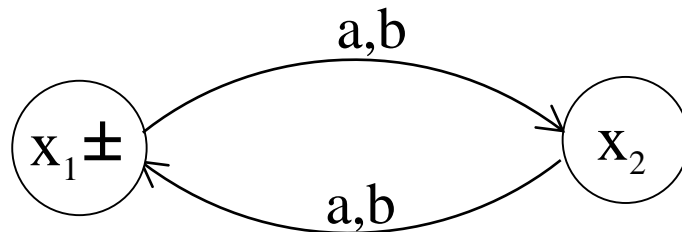
Old States	New States after reading	
	a	b
$z_2^+ \cong (x_2, y_2)$	$(x_2, y_1) \cong z_4$	$(x_1, y_1) \cong z_1$
$z_3^+ \cong (x_1, y_2)$	$(x_2, y_1) \cong z_4$	$(x_1, y_1) \cong z_1$
$z_4^+ \cong (x_2, y_1)$	$(x_2, y_2) \cong z_2$	$(x_1, y_2) \cong z_3$

# Example continued ...

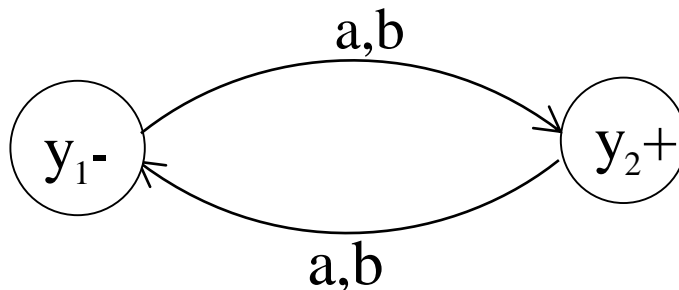


# Example

Let  $r_1 = ((a+b)(a+b))^*$  and the corresponding  $FA_1$  be

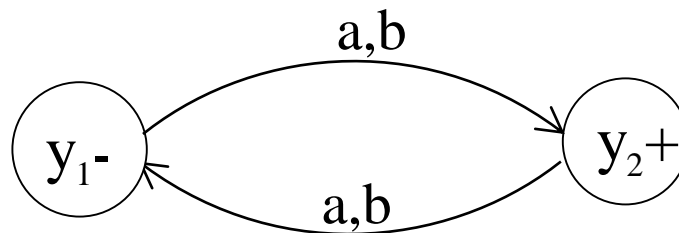
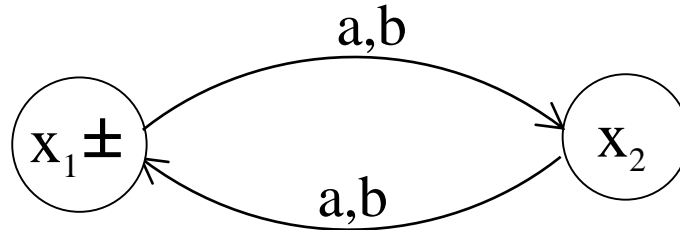


also  $r_2 = (a+b)((a+b)(a+b))^*$  or  $((a+b)(a+b))^*(a+b)$  and  $FA_2$  be





# Example continued ...

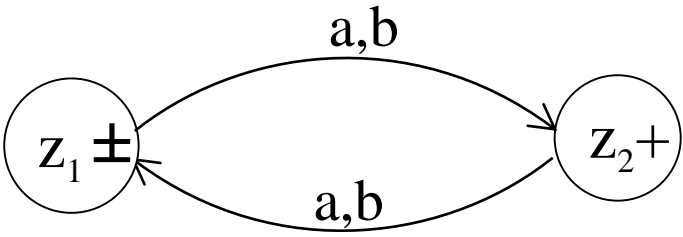


Old States	New States after reading	
	a	b
$z_1^\pm \cong (x_1, y_1)$	$(x_2, y_2) \cong z_2$	$(x_2, y_2) \cong z_2$

# Example continued ...

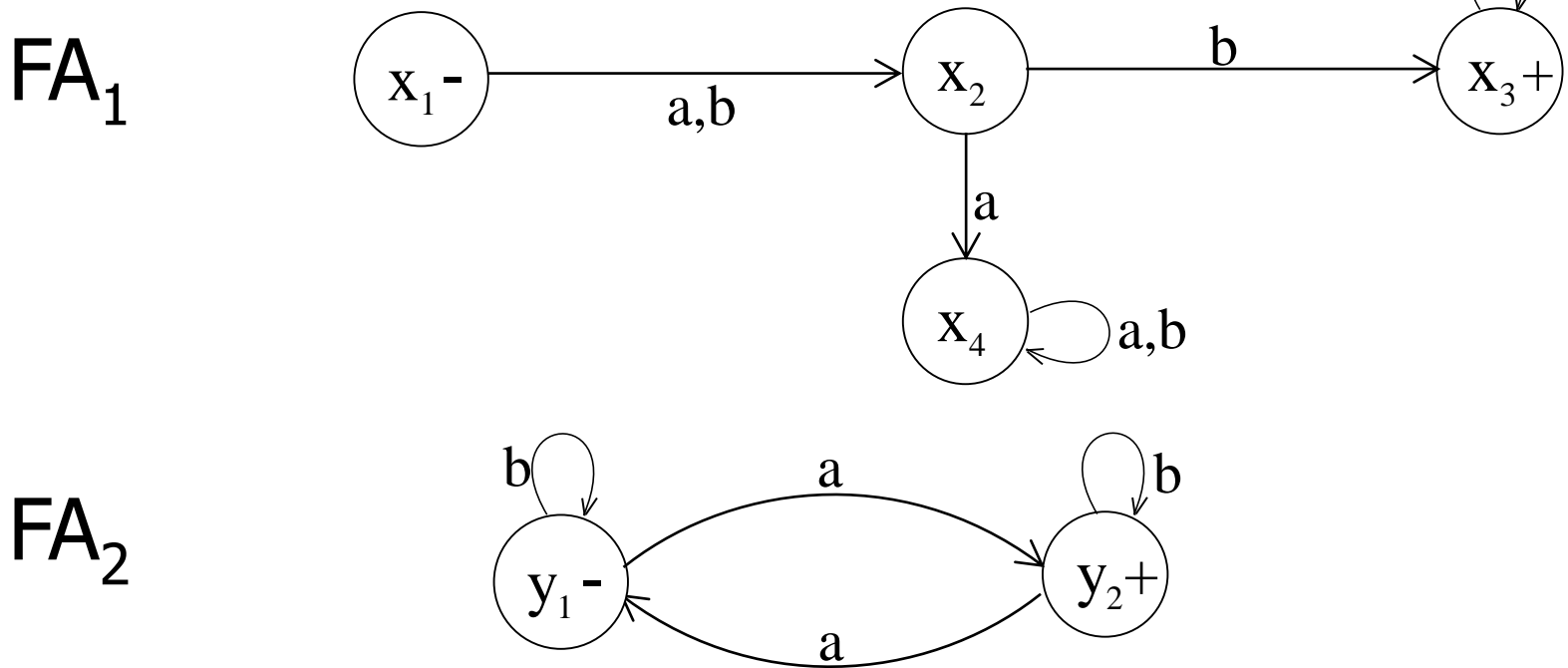
Old States	New States after reading	
	a	b
$z_2^+ \cong (x_2, y_2)$	$(x_1, y_1) \cong z_1$	$(x_1, y_1) \cong z_1$

# Example continued ...



# Task

Build an FA corresponding to the union of these two FAs *i.e.*  $FA_1 \cup FA_2$  where



# Kleene's Theorem Part III

## Continued ...

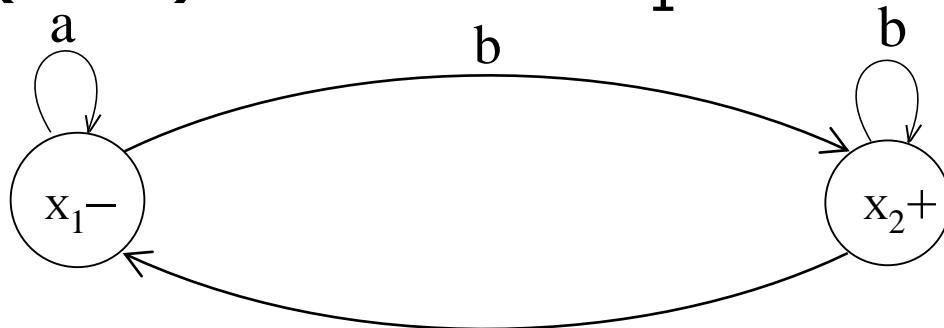


### ⌘ Method2 (Concatenation of two FAs):

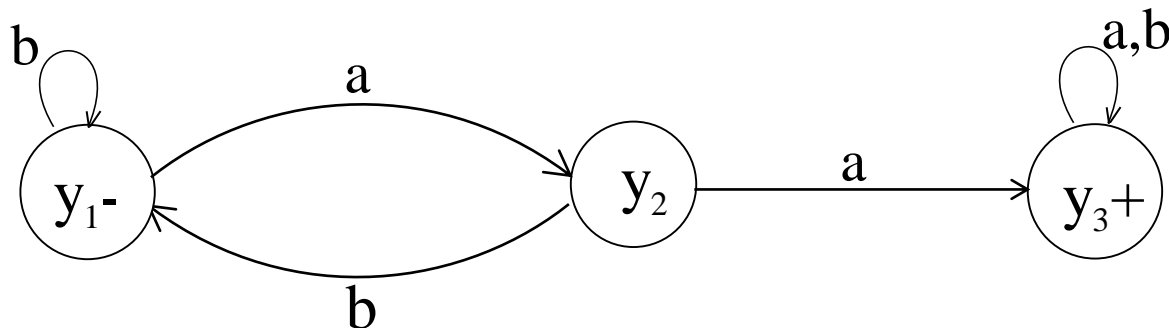
Using the FAs corresponding to  $r_1$  and  $r_2$ , an FA can be built, corresponding to  $r_1r_2$ . This method can be developed considering the following examples

# Example

Let  $r_1 = (a+b)^*b$  defines  $L_1$  and  $FA_1$  be



and  $r_2 = (a+b)^*aa(a+b)^*$  defines  $L_2$  and  $FA_2$  be



# Concatenation of two FAs

## Continued ...

Let  $FA_3$  be an FA corresponding to  $r_1r_2$ , then the initial state of  $FA_3$  must correspond to the initial state of  $FA_1$  and the final state of  $FA_3$  must correspond to the final state of  $FA_2$ . Since the language corresponding to  $r_1r_2$  is the concatenation of corresponding languages  $L_1$  and  $L_2$ , consists of the strings obtained, concatenating the strings of  $L_1$  to those of  $L_2$ , therefore ***the moment a final state of first FA is entered, the possibility of the initial state of second FA will be included as well.***

# Concatenation of two FAs

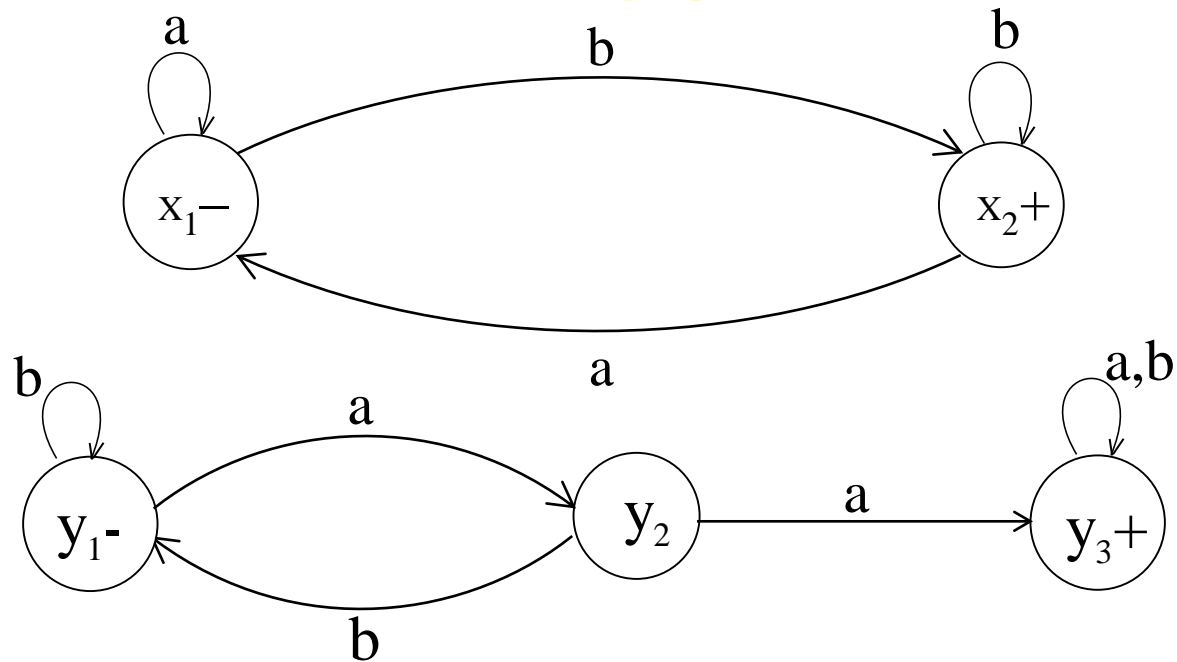
## Continued ...



Since, in general,  $FA_3$  will be different from both  $FA_1$  and  $FA_2$ , so the labels of the states of  $FA_3$  may be supposed to be  $z_1, z_2, z_3, \dots$ , where  $z_1$  stands for the initial state. Since  $z_1$  corresponds to the states  $x_1$ , so there will be two transitions separately for each letter read at  $z_1$ . It will give two possibilities of states which correspond to either  $z_1$  or different from  $z_1$ . This process may be expressed in the following transition table for all possible states of  $FA_3$



# Example continued ...

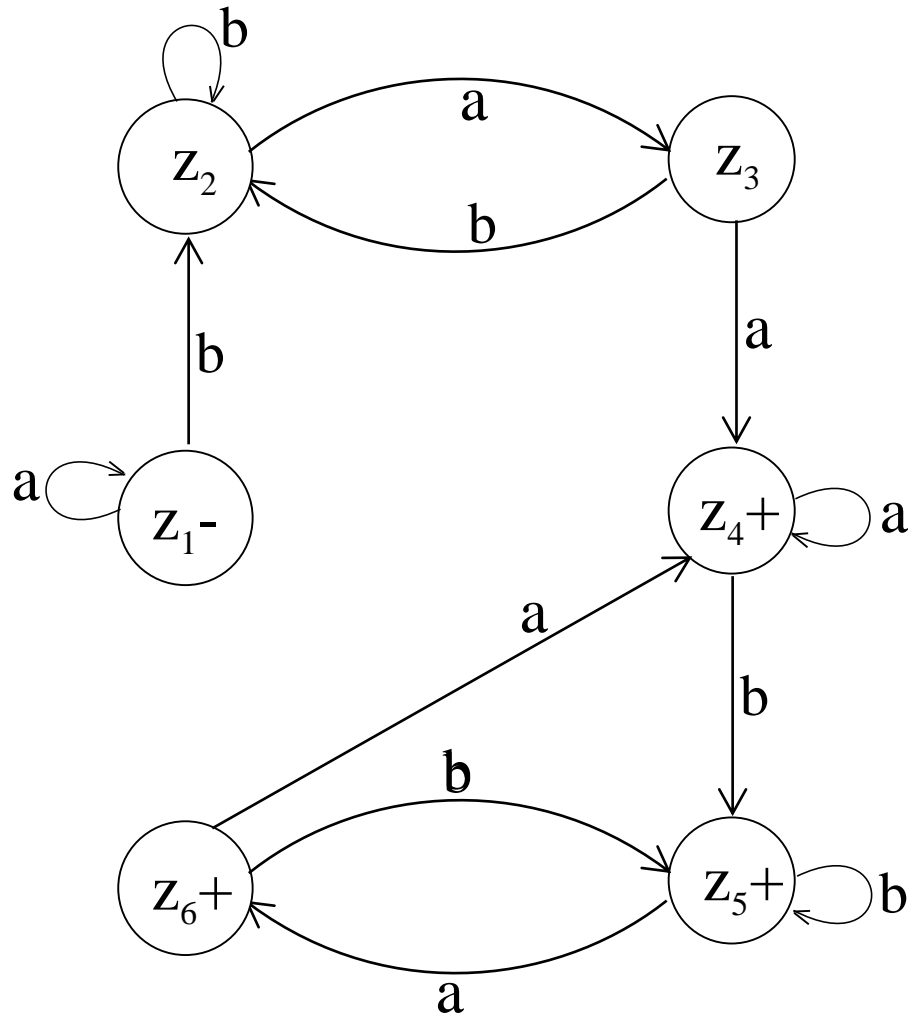


Old States	New States after reading	
	a	b
$z_1^- \cong x_1$	$x_1 \cong z_1$	$(x_2, y_1) \cong z_2$

## Example continued ...

Old States	New States after reading	
	a	b
$z_2 \cong (x_2, y_1)$	$(x_1, y_2) \cong z_3$	$(x_2, y_1) \cong z_2$
$z_3 \cong (x_1, y_2)$	$(x_1, y_3) \cong z_4$	$(x_2, y_1) \cong z_2$
$z_4^+ \cong (x_1, y_3)$	$(x_1, y_3) \cong z_4$	$(x_2, y_1, y_3) \cong z_5$
$z_5^+ \cong (x_2, y_1, y_3)$	$(x_1, y_2, y_3) \cong z_6$	$(x_2, y_1, y_3) \cong z_5$
$z_6^+ \cong (x_1, y_2, y_3)$	$(x_1, y_3) \cong z_4$	$(x_2, y_1, y_3) \cong z_5$

# Example continued ...



# Summing Up



- ⌘ Examples of Kleene's theorem part III  
(method 1) continued ,Kleene's theorem part III  
(method 2: Concatenation of FAs),
- ⌘ Example of Kleene's theorem part III  
(method 2 : Concatenation of FAs)