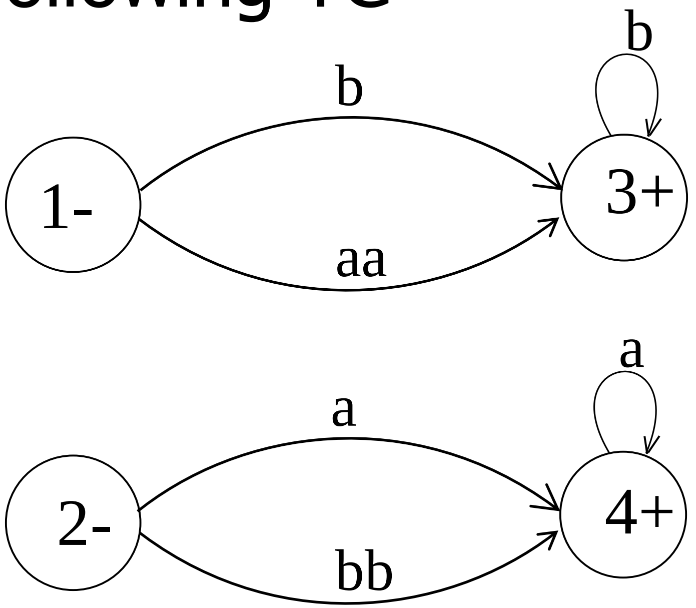
Recap lecture 11

- Proof of Kleene's theorem part II (method with different steps), particular examples of TGs to determine corresponding REs.

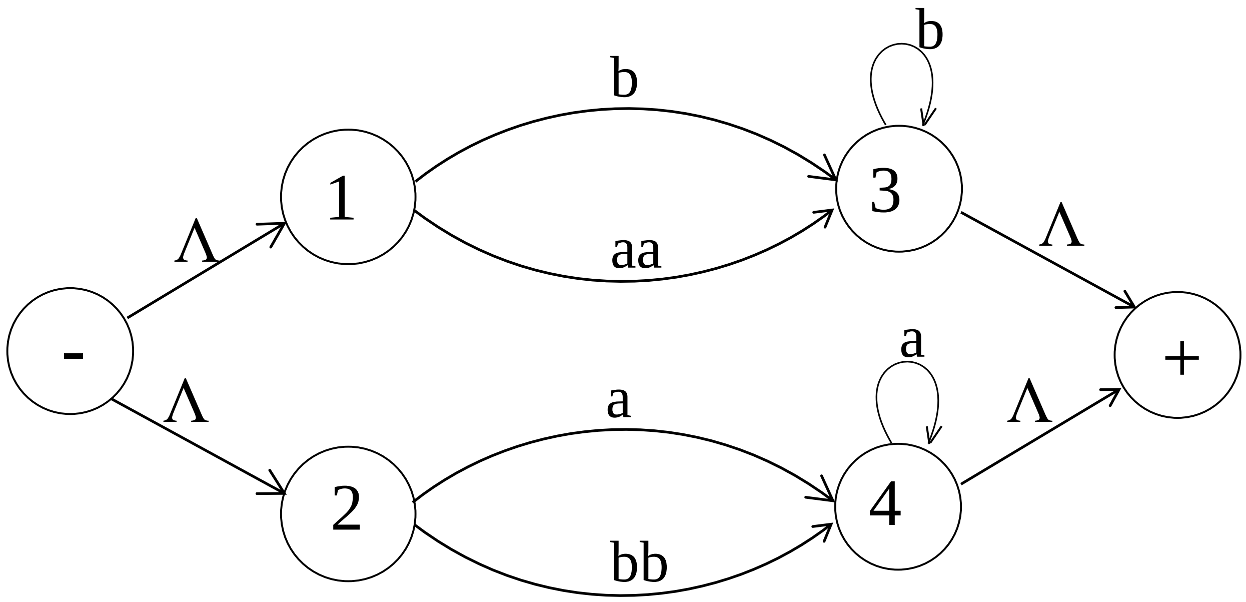
Example

⌘Consider the following TG



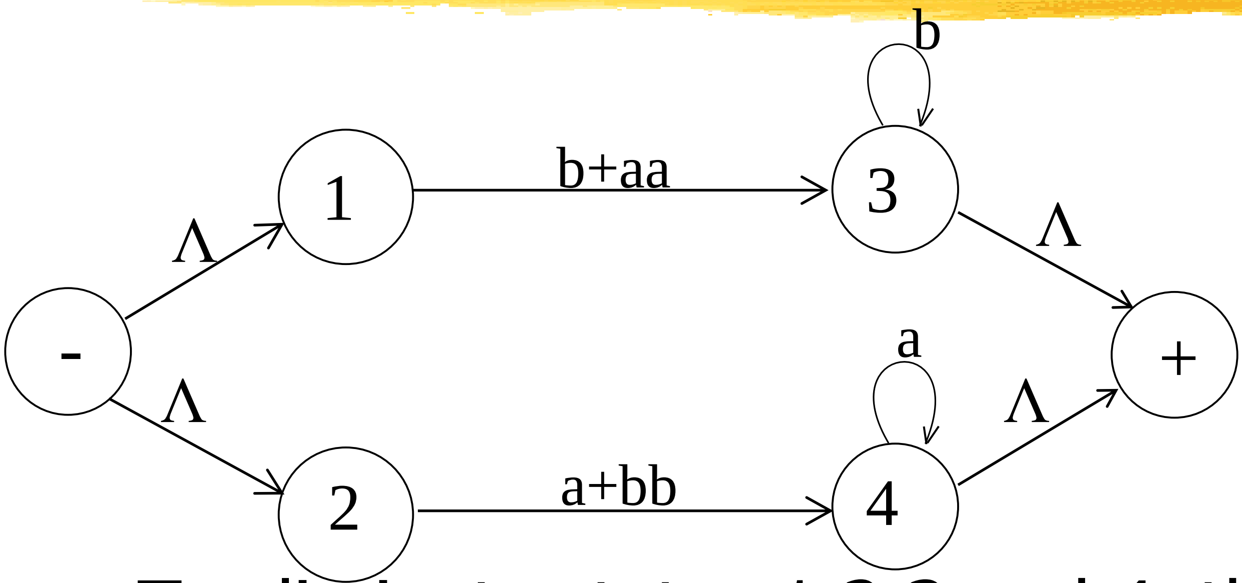
To have single initial and single final state the above TG can be reduced to the following

Example continued …

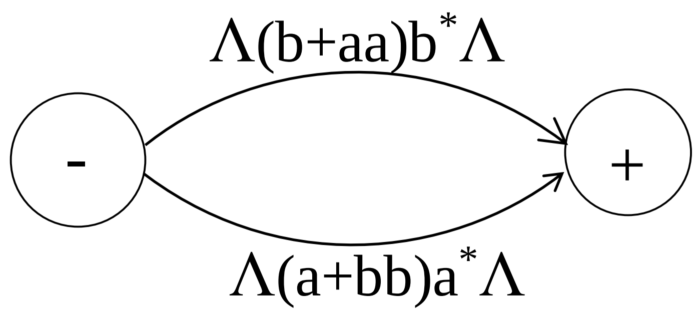


To obtain single transition edge between 1 and 3; 2 and 4, the above can be reduced to the following

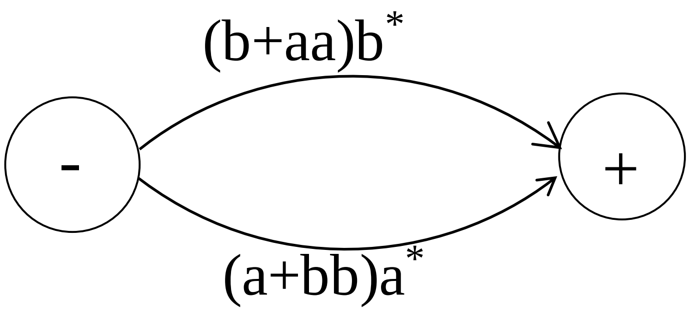
Example continued …



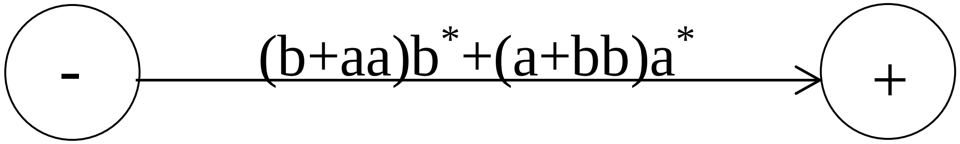
To eliminate states 1,2,3 and 4, the above TG can be reduced to the following TG



Example continued …



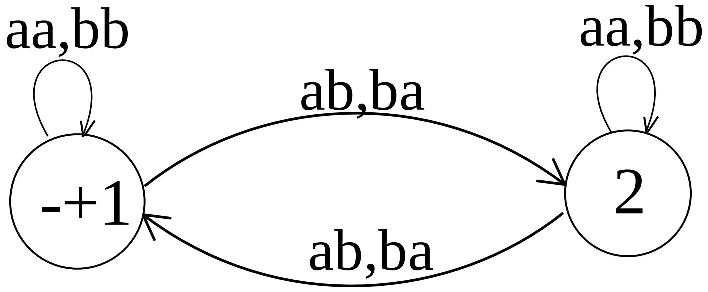
To connect the initial state with the final state by single transition edge, the above TG can be reduced to the following



Hence the required RE is $(b+aa)b^*+(a+bb)a^*$

Example

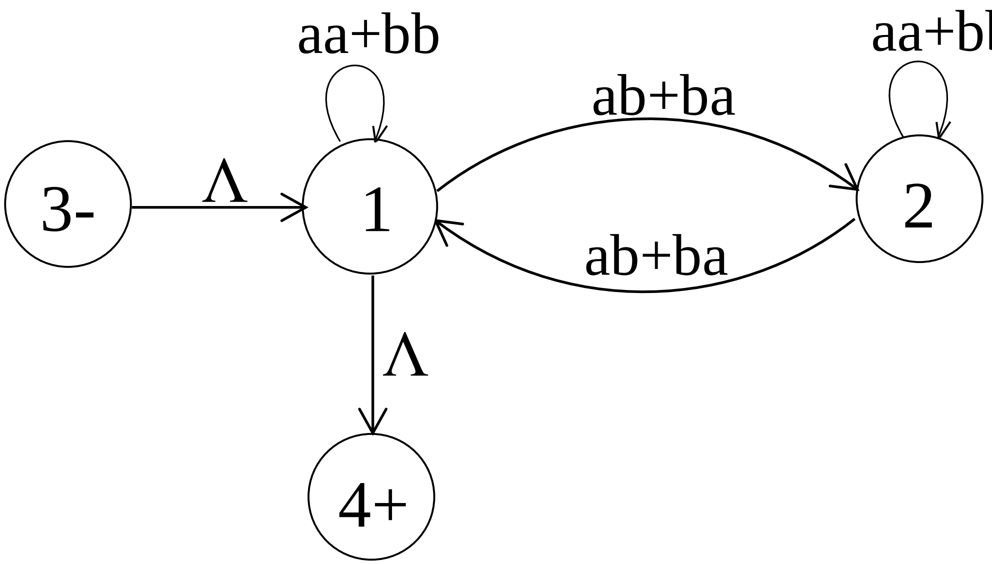
Consider the following TG, accepting EVEN-EVEN language



Example continued …

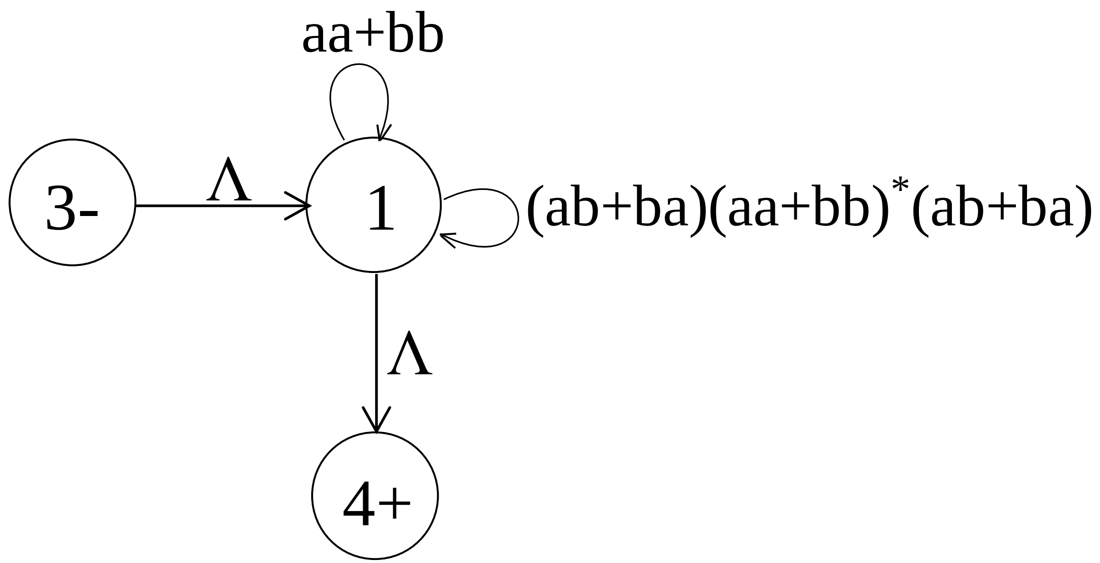
It is to be noted that since the initial state of this TG is final as well and there is no other final state, so to obtain a TG with single initial and single final state, an additional initial and a final state are introduced as shown in the following TG

Example continued …



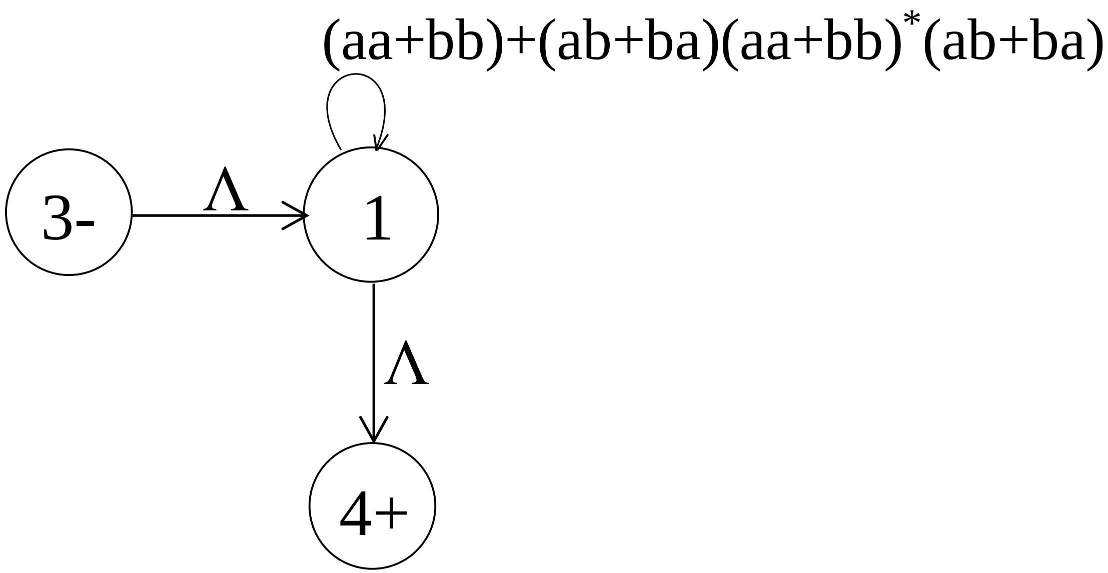
- To eliminate state 2, the above TG may be reduced to the following

Example continued …



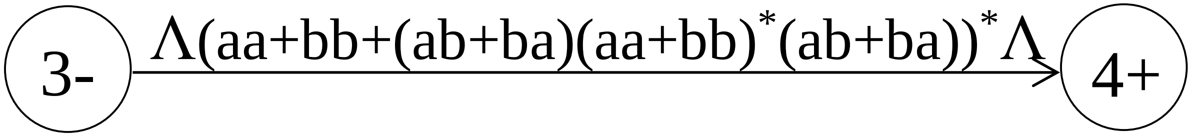
To have single loop at state 1, the above TG may be reduced to the following

Example continued …



To eliminate state 1, the above TG may be reduced to the following

Example continued …



Hence the required RE is

$(aa+bb+(ab+ba)(aa+bb)^*(ab+ba))^*$

Kleene's Theorem Part III

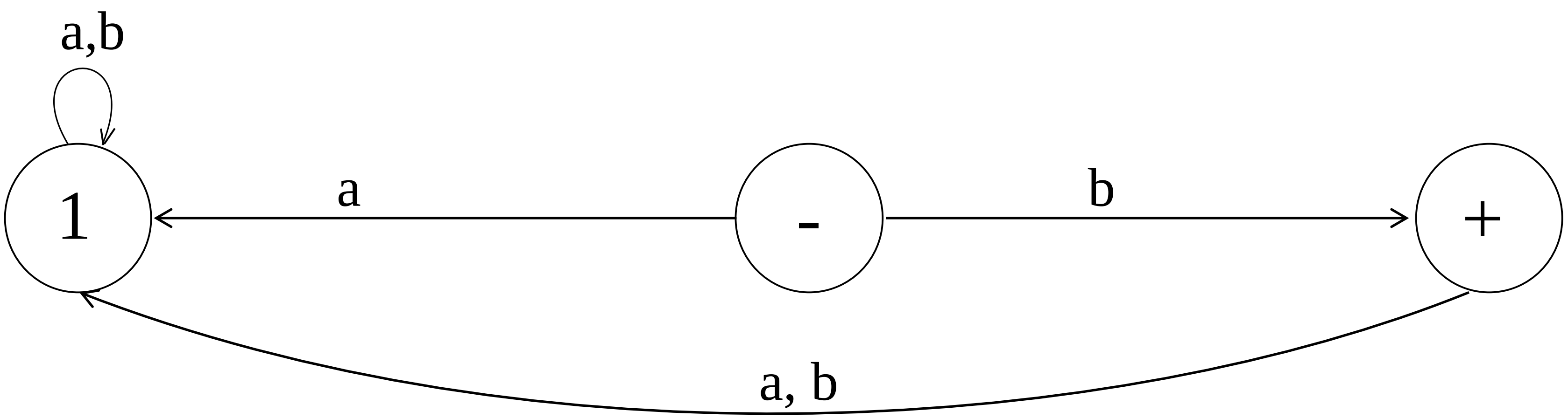
Statement:

If the language can be expressed by a RE then there exists an FA accepting the language.

A) As the regular expression is obtained applying addition, concatenation and closure on the letters of an alphabet and the Null string, so while building the RE, sometimes, the corresponding FA may be built easily, as shown in the following examples

Example

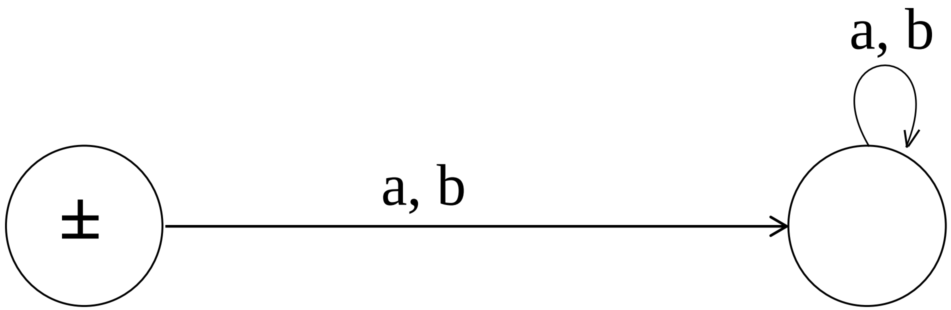
- Consider the language, defined over Σ={a,b}, **consisting of only b**, then this language may be accepted by the following FA



which shows that this FA helps in building an FA accepting only one letter

Example

- Consider the language, defined over $\Sigma=\{a,b\}$, **consisting of only** Λ, then this language may be accepted by the following FA



Kleene's Theorem Part III Continued …

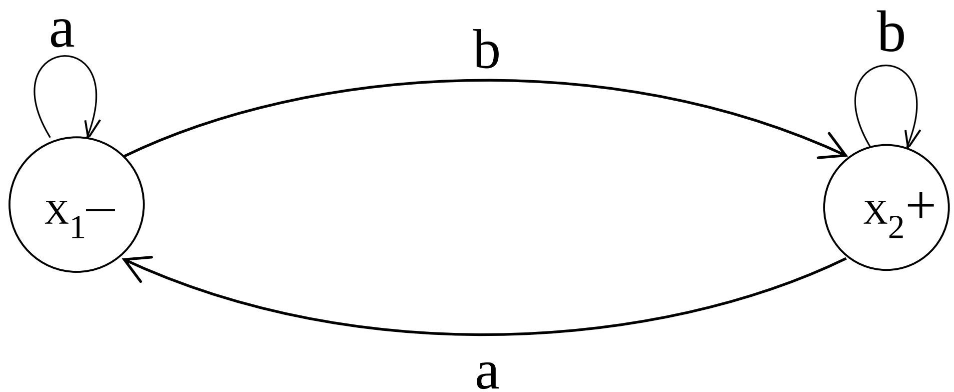
B) As, if r_1 and r_2 are regular expressions then their sum, concatenation and closure are also regular expressions, so an FA can be built for any regular expression if the methods can be developed for building the FAs corresponding to the sum, concatenation and closure of the regular expressions along with their FAs. These three methods are explained in the following discussions

Kleene's Theorem Part III Continued …

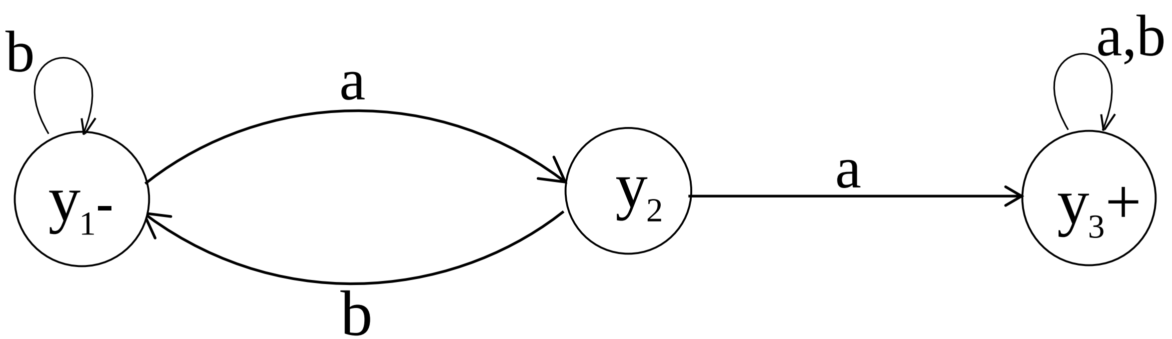
- **<u>Method1 (Union of two FAs):</u>** Using the FAs corresponding to r_1 and r_2 an FA can be built, corresponding to $r_1 + r_2$. This method can be developed considering the following examples

Example

Let $r_1=(a+b)^*b$ defines L_1 and the FA_1 be



and $r_2 = (a+b)^*aa(a+b)^*$ defines L_2 and FA_2 be



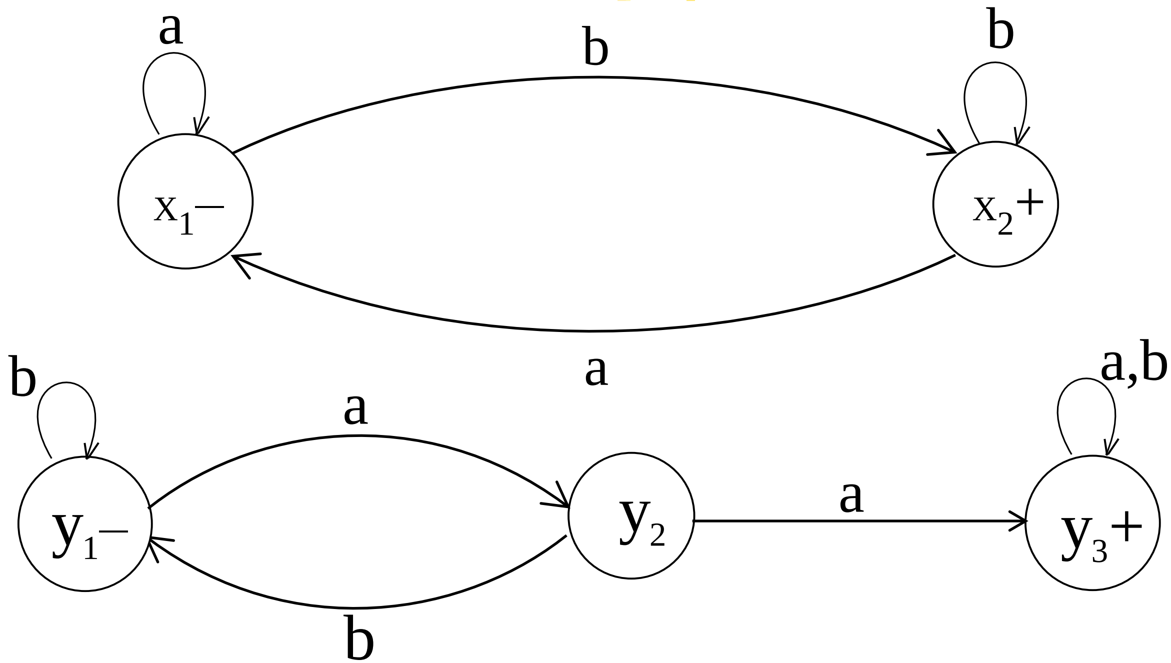
Sum of two FAs Continued …

Let FA_3 be an FA corresponding to $r_1 + r_2$, then the initial state of FA_3 must correspond to the initial state of FA_1 or the initial state of FA_2. Since the language corresponding to $r_1 + r_2$ is the union of corresponding languages L_1 and L_2, consists of the strings belonging to L_1 or L_2 or both, therefore a final state of FA_3 must correspond to a final state of FA_1 or FA_2 or both.

Sum of two FAs Continued …

Since, in general, FA_3 will be different from both FA_1 and FA_2, so the labels of the states of FA_3 may be supposed to be z_1,z_2, z_3, …, where z_1 is supposed to be the initial state. Since z_1 corresponds to the states x_1 or y_1, so there will be two transitions separately for each letter read at z_1. It will give two possibilities of states either z_1 or different from z_1. This process may be expressed in the following transition table for all possible states of FA_3.

Example continued …

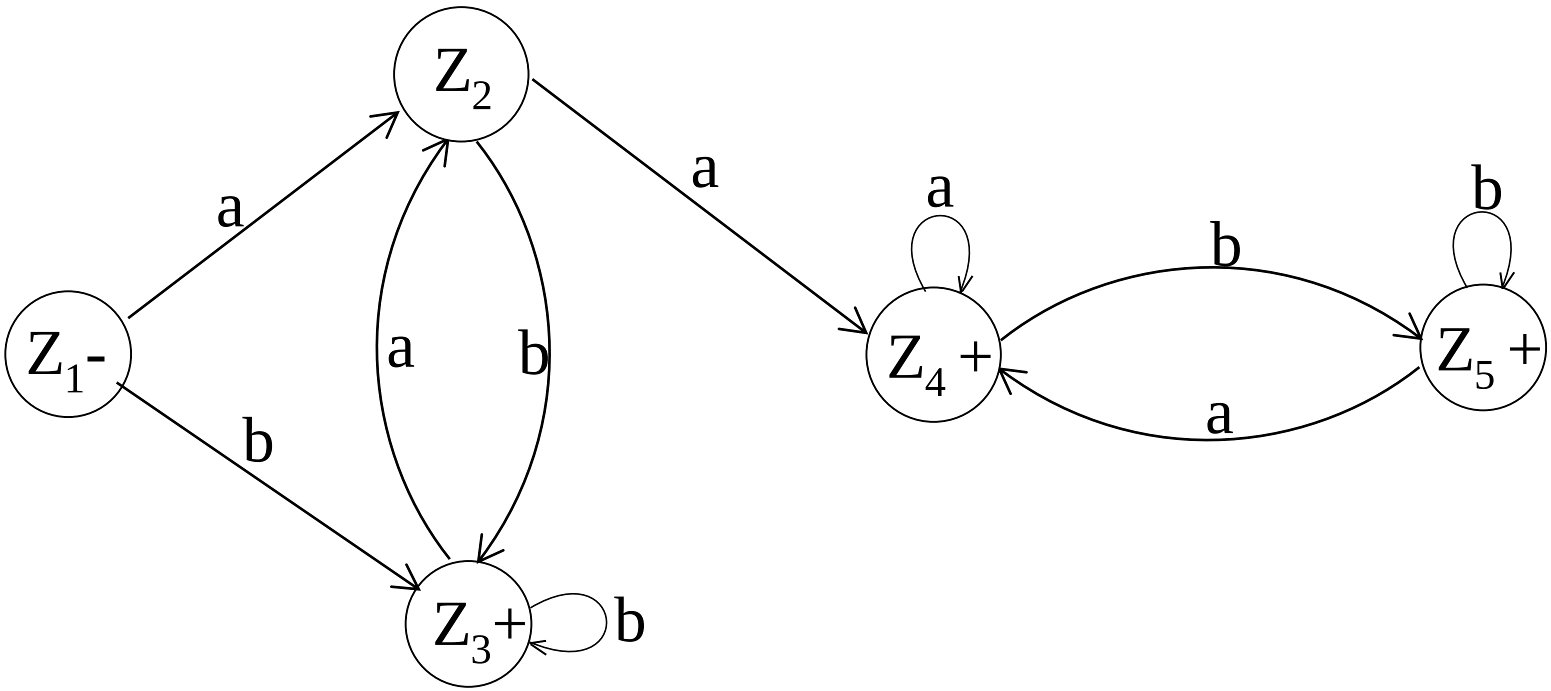


Old states	New states after reading	
	a	b
$z_1 - \cong (x_1, y_1)$	$(x_1, y_2) \cong z_2$	$(x_2, y_1) \cong z_3$

Example continued …

Old States	New States after reading	
	a	b
$z_2 \cong (x_1,y_2)$	$(x_1,y_3) \cong z_4$	$(x_2,y_1) \cong z_3$
$z_3+ \cong (x_2,y_1)$	$(x_1,y_2) \cong z_2$	$(x_2,y_1) \cong z_3$
$z_4+ \cong (x_1,y_3)$	$(x_1,y_3) \cong z_4$	$(x_2,y_3) \cong z_5$
$z_5+ \cong (x_2,y_3)$	$(x_1,y_3) \cong z_4$	$(x_2,y_3) \cong z_5$

Example continued …



RE corresponding to the above FA may be

$$r_1+r_2 = (a+b)^*b + (a+b)^*aa(a+b)^*$$

Summing Up

- Examples of writing REs to the corresponding TGs, RE corresponding to TG accepting EVEN-EVEN language, Kleene’s theorem part III (method 1:union of FAs), examples of FAs corresponding to simple REs, example of Kleene’s theorem part III (method 1) continued