Recap lecture 10

- Definition of GTG, examples of GTG accepting the languages of strings:containing aa or bb, beginning with and ending in same letters, beginning with and ending in different letters, containing aaa or bbb,
- Nondeterminism, Kleene's theorem (part I, part II, part III), proof of Kleene's theorem part I

Kleene's Theorem continued …

Proof(Kleene's Theorem Part II)

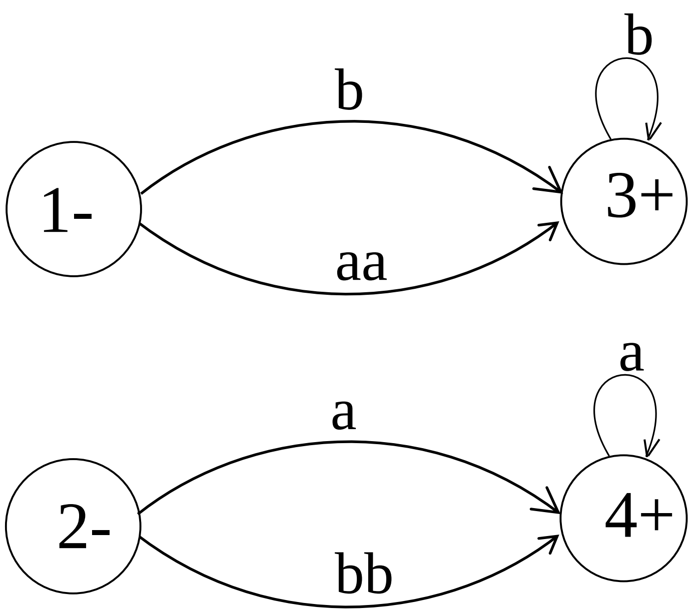
To prove part II of the theorem, an algorithm consisting of different steps, is explained showing how a RE can be obtained corresponding to the given TG. For this purpose the notion of TG is changed to that of GTG *i.e.* the labels of transitions are corresponding REs.

Kleene's Theorem part II continued …

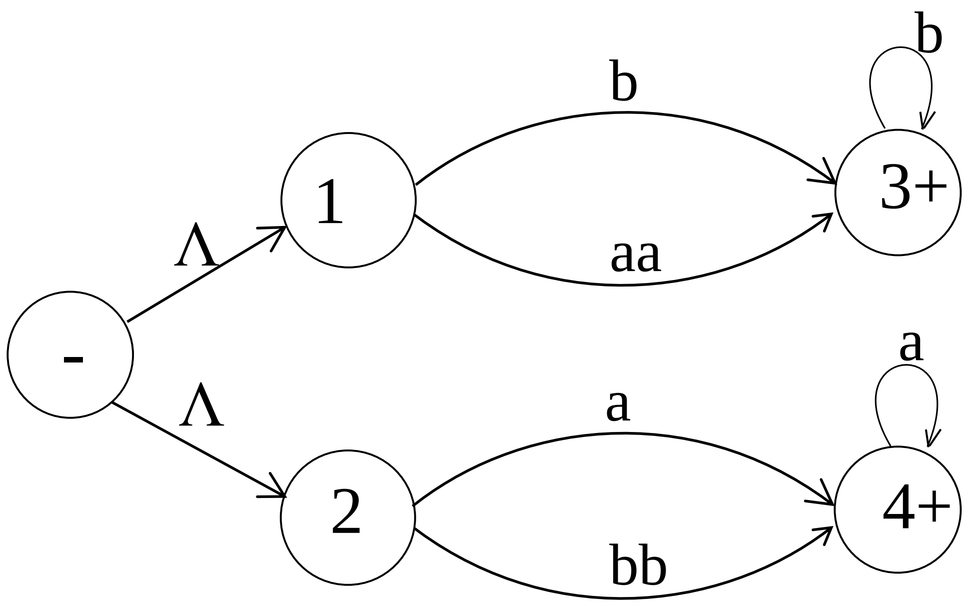
Basically this algorithm converts the given TG to GTG with one initial state along with a single loop, or one initial state connected with one final state by a single transition edge. The label of the loop or the transition edge will be the required RE.

Step 1 If a TG has more than one start states, then introduce a new start state connecting the new state to the old start states by the transitions labeled by Λ and make the old start states the non-start states. This step can be shown by the following example

Example



Example Continued ...



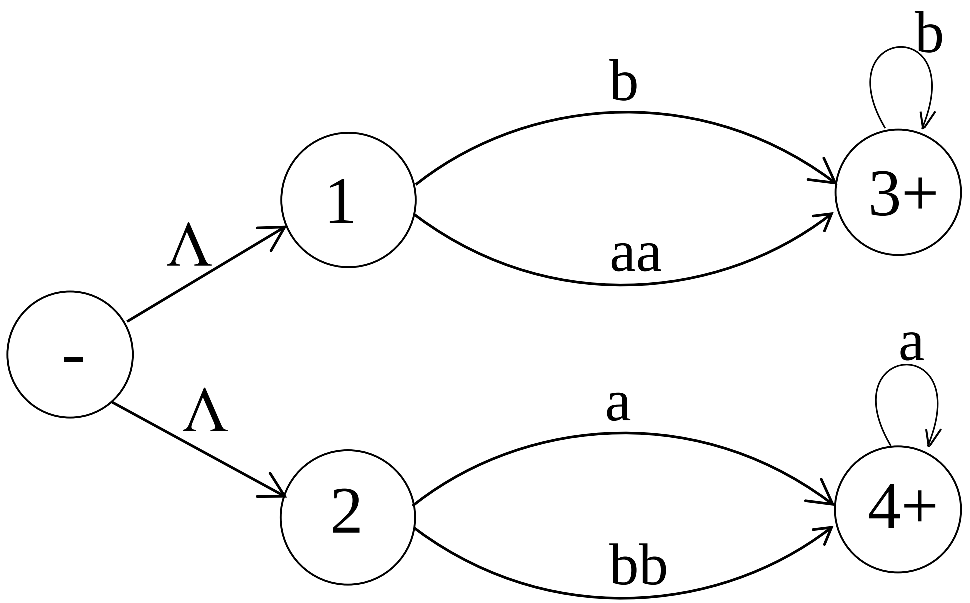
Kleene's Theorem part II continued …

Step 2:

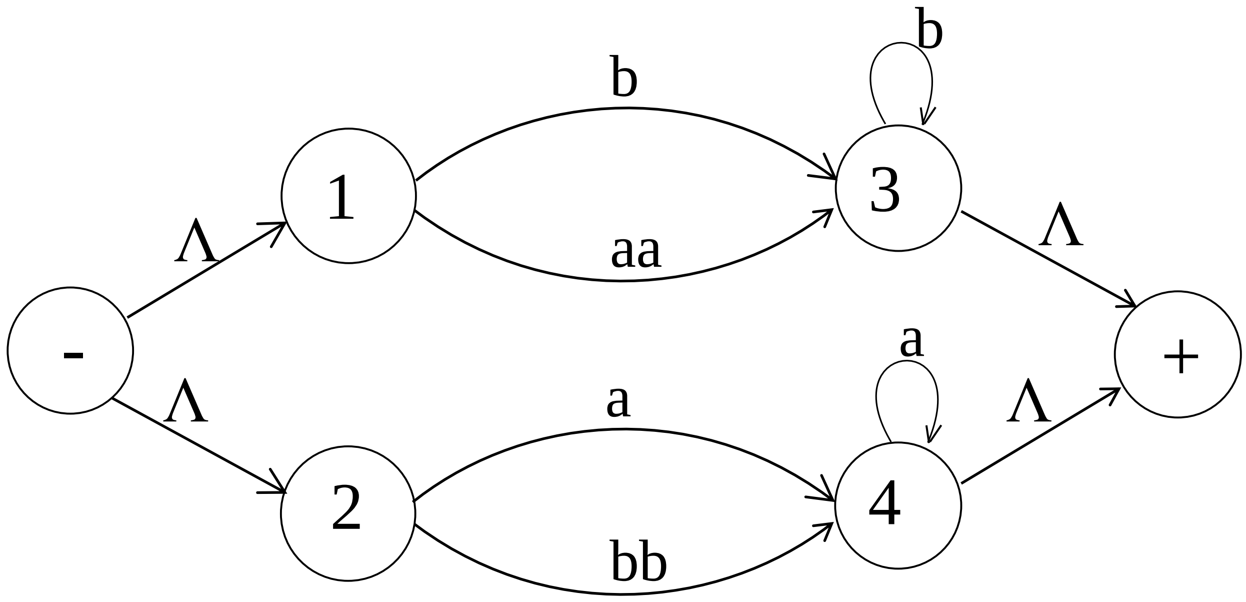
If a TG has more than one final states, then introduce a new final state, connecting the old final states to the new final state by the transitions labeled by Λ.

This step can be shown by the previous example of TG, where the step 1 has already been processed

Example



Example continued ...



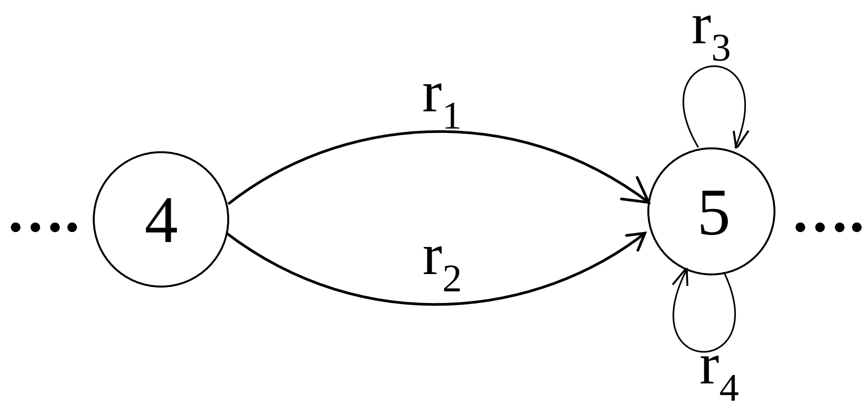
Kleene's Theorem part II continued ...

Step 3:

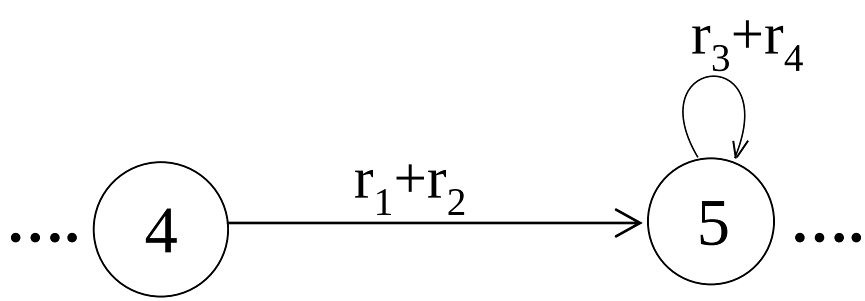
If a state has two (more than one) incoming transition edges labeled by the corresponding REs, from the same state (including the possibility of loops at a state), then replace all these transition edges with a single transition edge labeled by the sum of corresponding REs.

This step can be shown by a part of TG in the following example

Example

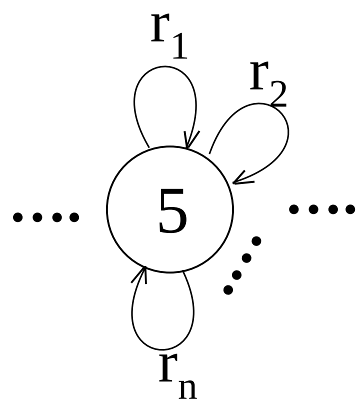


The above TG can be reduced to

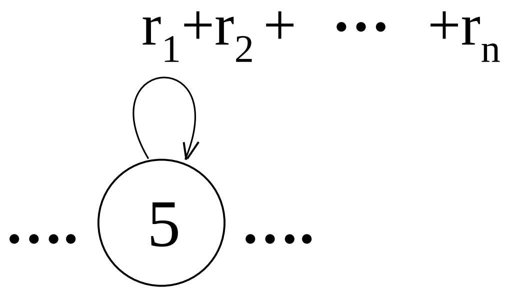


Note

- The step 3 can be generalized to any finite number of transitions as shown below



The above TG can be reduced to

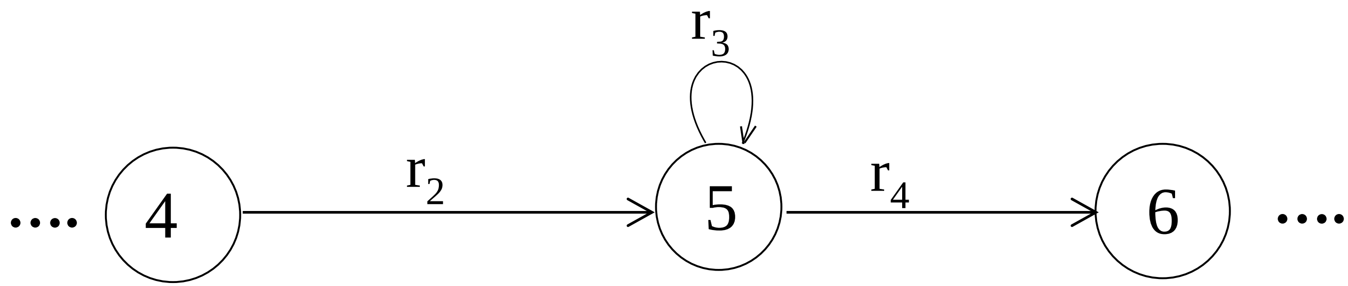


Kleene's Theorem part II continued ...

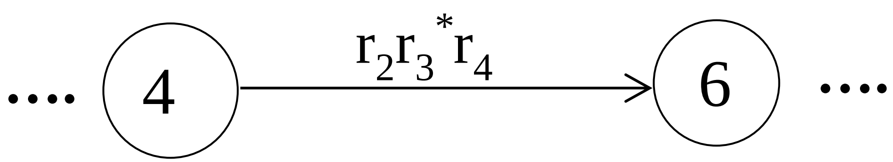
Step 4 (bypass and state elimination)

If three states in a TG, are connected in sequence then eliminate the middle state and connect the first state with the third by a single transition (include the possibility of circuit as well) labeled by the RE which is the concatenation of corresponding two REs in the existing sequence. This step can be shown by a part of TG in the following example

Example



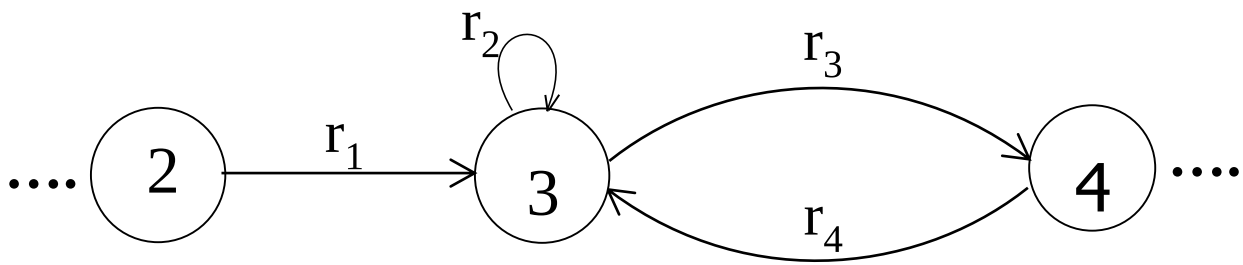
To eliminate state 5 the above can be reduced to



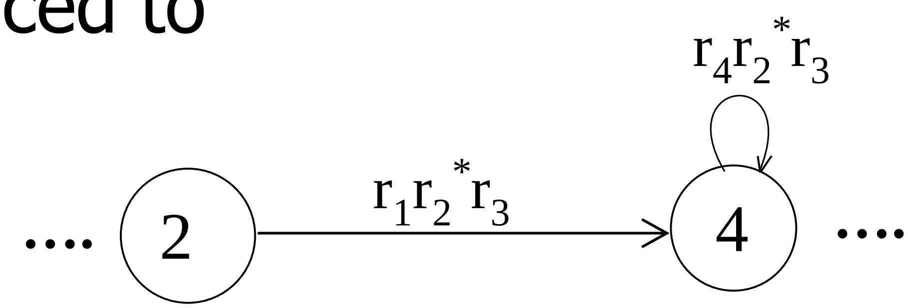
Consider the following example containing a circuit

Example

Consider the part of a TG, containing a circuit at a state, as shown below

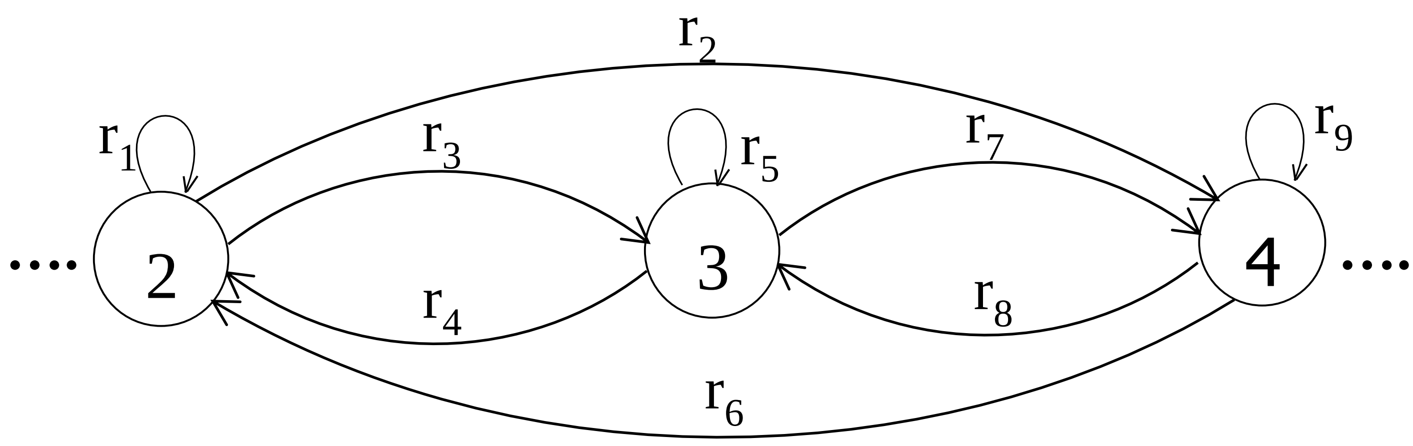


To eliminate state 3 the above TG can be reduced to



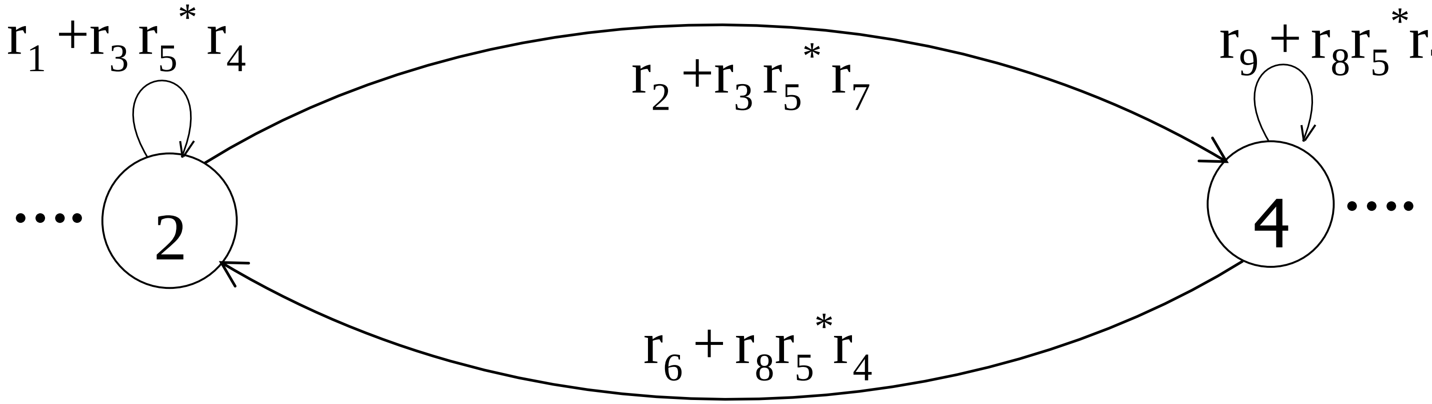
Example

Consider a part of the following TG



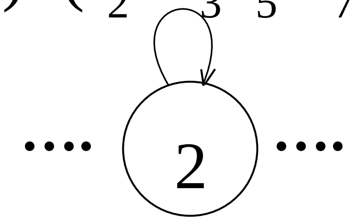
To eliminate state 3 the above TG can be reduced to

Example continued ...



To eliminate state 4 the above TG can be reduced to

$$(r_1 + r_3 r_5^* r_4) + (r_2 + r_3 r_5^* r_7)(r_9 + r_8 r_5^* r_7)^*(r_6 + r_8 r_5^* r_4)$$

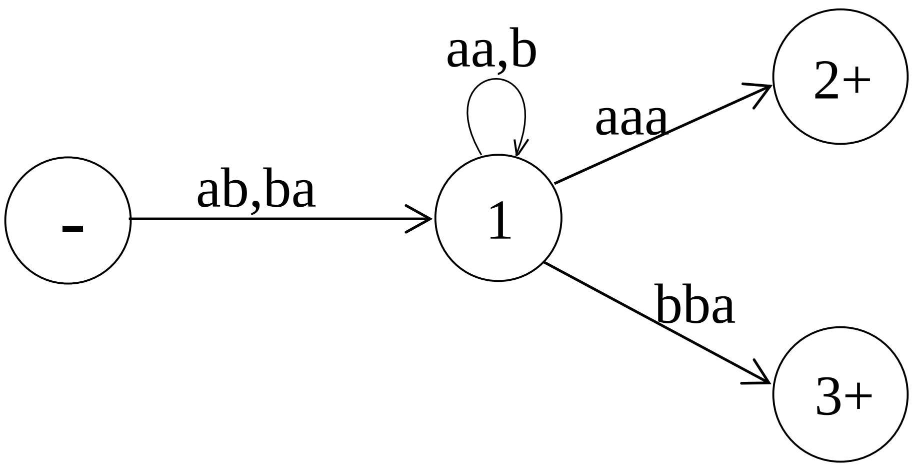


Note

⌘ It is to be noted that to determine the RE corresponding to a certain TG, four steps have been discussed. This process can be explained by the following particular examples of TGs

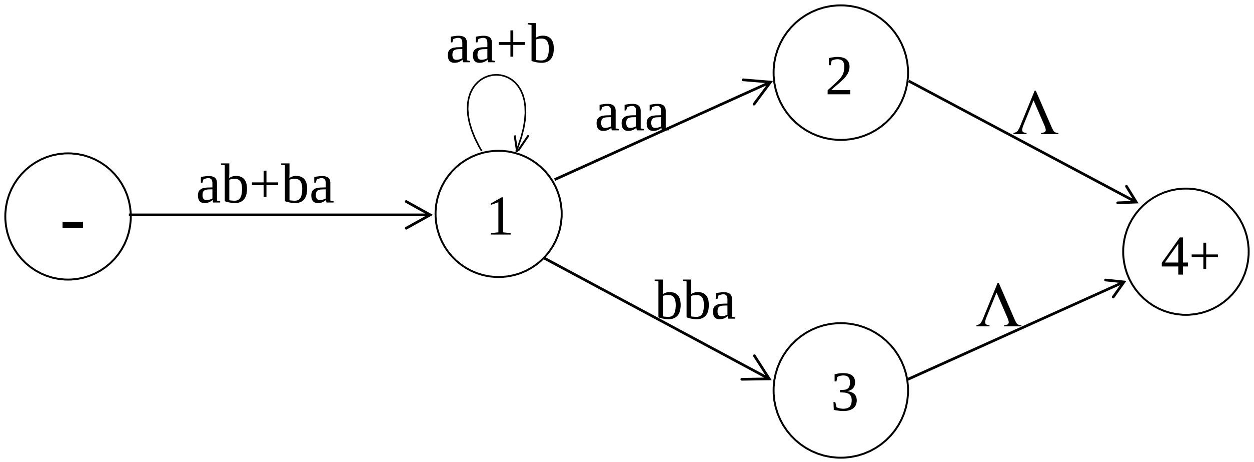
Example

- Consider the following TG

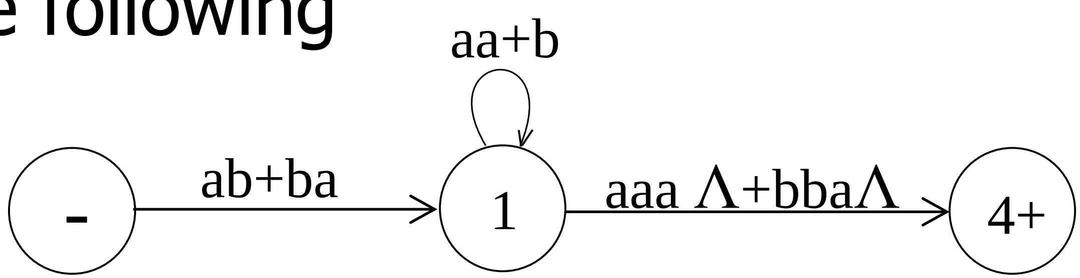


To have single final state, the above TG can be reduced to the following

Example continued …



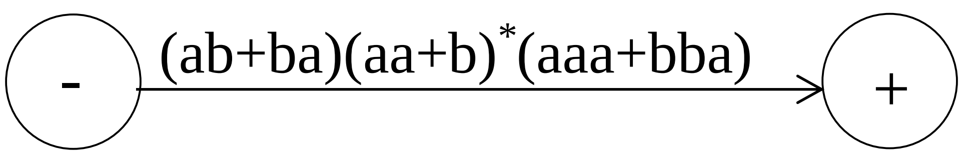
To eliminate states 2 and 3, the above TG can be reduced to the following



The above TG can be reduced to the following

Example continued …

To eliminate state 1 the above TG can be reduced to the following



Hence the required RE is

$(ab+ba)(aa+b)^{*}(aaa+bba)$

Summing Up

- proof of Kleene's theorem part II (method with different steps), particular examples of TGs to determine corresponding Res.