Week # 05

→ Invertible Matrices (Inversion of Matrices)

→ Minor , Cofactors and Determinants

- **Identity Matrices:-**

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Multiplication of Identity Matrices:-**

- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$Sol: -$

$$A.I_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A.I_2 = \begin{bmatrix} 1+0 & 2+0 \\ 3+0 & 0+4 \end{bmatrix}$$

$$A.I_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Multiplication of Identity matrix with other matrix Always obey commutative Law.

$$\boxed{A.I_2 = I_2.A}$$

- Find Invertible Matrix.
- **Note: -** Two Matrices will be invertible if their Multiplication is Identity Matrix otherwise not.

1. $A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

Show that A and B are invertible.

$Sol: -$

$Find\ A.B = ?\ \&\ B.A = ?$

$$A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} . \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 6-5 & 10-10 \\ -3+3 & -5+6 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{.......... (i)}$$

Now $B.A = ?$

$$B.A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} . \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$B.A = \begin{bmatrix} 6-5 & -15+15 \\ 2-2 & -5+6 \end{bmatrix}$$

$$B.A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{.......... (ii)}$$

From (i) & (ii) it is proved that A and B are Invertible

2. $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3/2 \\ 1 & -1 \end{bmatrix}$

Show that $A.B = B.A = I$

$Sol: -$

$Find\ A.B = ?\ \&\ B.A = ?$

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix} B = \begin{bmatrix} -1 & 3/2 \\ 1 & -1 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix} . \begin{bmatrix} -1 & 3/2 \\ 1 & -1 \end{bmatrix}$$

$$A.B = \begin{bmatrix} -2+3 & 2 \times 3/2 - 3 \\ -2+2 & 2 \times 3/2 - 2 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 1 & \not{2} \times \frac{3}{\not{2}} - 3 \\ 0 & \not{2} \times \frac{3}{\not{2}} - 2 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 1 & 3-3 \\ 0 & 3-2 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{.......... (i)}$$

Now $B.A = ?$

$$B.A = \begin{bmatrix} -1 & 3/2 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$$

$$B.A = \begin{bmatrix} -2 + 3/2 \times 2 & -3 + 3/2 \times 2 \\ 2-2 & 3-2 \end{bmatrix}$$

$$B.A = \begin{bmatrix} -2 + \frac{3}{\not{2}} \times \not{2} & -3 + \frac{3}{\not{2}} \times \not{2} \\ 0 & 1 \end{bmatrix}$$

$$B.A = \begin{bmatrix} -2+3 & -3+3 \\ 0 & 1 \end{bmatrix}$$

$$B.A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{.......... (ii)}$$

From (i) & (ii) it is proved that A and B are Invertible

$$\boxed{A.B = B.A = I}$$

- **Minor:-**

1. $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$

$M_{11} = 7$, $M_{12} = 4$, $M_{21} = 3$, $M_{22} = 2$

2. $B = \begin{bmatrix} 3 & 2 & 7 \\ 4 & 2 & 5 \\ 3 & -1 & 10 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$

$$M_{11} = \begin{vmatrix} 2 & 5 \\ -1 & 10 \end{vmatrix} = 20 + 5 = 25$$

$$M_{12} = \begin{vmatrix} 4 & 5 \\ 3 & 10 \end{vmatrix} = 40 - 15 = 25$$

$$M_{13} = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = -4 - 6 = -10$$

$$M_{21} = \begin{vmatrix} 2 & 7 \\ -1 & 10 \end{vmatrix} = 20 + 7 = 27$$

$$M_{22} = \begin{vmatrix} 3 & 7 \\ 3 & 10 \end{vmatrix} = 30 - 21 = 9$$

$$M_{23} = \begin{vmatrix} 3 & 2 \\ 3 & -1 \end{vmatrix} = -3 - 6 = -9$$

$$M_{31} = \begin{vmatrix} 2 & 7 \\ 2 & 5 \end{vmatrix} = 10 - 14 = -4$$

$$M_{32} = \begin{vmatrix} 3 & 7 \\ 4 & 5 \end{vmatrix} = 15 - 28 = -13$$

$$M_{33} = \begin{vmatrix} 3 & 2 \\ 4 & 2 \end{vmatrix} = 6 - 8 = -2$$

Minors of Matrix A are

$$A = \begin{bmatrix} 25 & 25 & -10 \\ 27 & 9 & -9 \\ -4 & -13 & -2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

3. $A = \begin{bmatrix} 0.3 & 0.5 & 0.7 \\ 1/2 & -5/7 & 3/4 \\ -0.9 & -0.1 & 0.2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$

$$M_{11} = \begin{vmatrix} -5/7 & 3/4 \\ -0.1 & 0.2 \end{vmatrix} = -5/7 \times 0.2 - (-0.1 \times 3/4)$$

$$=> -1/7 - (-0.3/4) => -1/7 + 0.1/4)$$

$$=> -0.14 - (-0.075) \quad => -0.14 + 0.075$$

$$=> -0.065$$

$$M_{12} = \begin{vmatrix} 1/2 & 3/4 \\ -0.9 & 0.2 \end{vmatrix} = 1/7 \times 0.2 - (-0.9 \times 3/4)$$

$$=> 0.2/7 - (-2.7/4) => 0.1 - (-0.675)$$

$$=> 0.1 + 0.675 => 0.775$$

$$M_{13} = \begin{vmatrix} 1/2 & -5/7 \\ -0.9 & -0.1 \end{vmatrix} = -0.1 \times 1/2 - (-0.9 \times -5/7)$$

$$=> -0.1/2 - \left(4.5/7\right) => -0.05 - 0.642$$

$$=> -0.692$$

$$M_{21} = \begin{vmatrix} 0.5 & 0.7 \\ -0.1 & 0.2 \end{vmatrix} = 0.2 \times 0.5 - (-0.1 \times 0.7)$$

$$=> 0.1 + 0.07 \quad => \quad 0.17$$

$M_{22} = \begin{vmatrix} 0.3 & 0.7 \\ -0.9 & 0.2 \end{vmatrix} = 0.2 \times 0.3 - (-0.9 \times 0.7)$

$=> 0.06 - (-0.63) => 0.06 + 0.63 => 0.69$

$M_{23} = \begin{vmatrix} 0.3 & 0.5 \\ -0.9 & -0.1 \end{vmatrix} = -0.1 \times 0.3 - (-0.9 \times 0.5)$

$=> -0.03 - (-0.45) => -0.03 + 0.45 => 0.42$

$M_{31} = \begin{vmatrix} 0.5 & 0.2 \\ -5/7 & 3/4 \end{vmatrix} = 3/4 \times 0.5 - (-5/7 \times 0.2)$

$=> 1.5/4 - (-1/7) => 0.375 - (-0.142)$

$=> 0.375 + 0.142 => 0.517$

$M_{32} = \begin{vmatrix} 0.3 & 0.7 \\ 1/2 & 3/4 \end{vmatrix} = 3/4 \times 0.3 - (1/2 \times 0.7)$

$=> 0.9/4 \times -(0.7/2) => 0.225 - 0.35 => -0.125$

$M_{33} = \begin{vmatrix} 0.3 & 0.5 \\ 1/2 & -5/7 \end{vmatrix} = -5/7 \times 0.3 - (1/2 \times 0.5)$

$=> -1.5/7 - \left(0.5/2\right) => -0.214 - 0.25$

$=> -0.464$

Minors of Matrix A are

$$A = \begin{bmatrix} -0.065 & 0.775 & -0.692 \\ 0.17 & 0.69 & 0.42 \\ 0.517 & -0.125 & -0.464 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

- Inverse of a Matrix:-

$$A^{-1} = \frac{\text{Adj}\mathbf{A}}{|\mathbf{A}|}$$

- Questions:-

Find the Inverse of the Matrices.

1. $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

$Sol:-$

$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

First find |A|

$|A| = \begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} = 7 \times 2 - 3 \times 5 => 14 - 15$

$|A| = -1$

Taking $Adjoint$

$Adj\mathbf{A} = \begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix}$

Now Know that $A^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$

$A^{-1} = \frac{\begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix}}{-1}$

$$\boxed{A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}}$$

2. $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$

i. Show that $A.A^{-1} = A^{-1}.A = I$

ii. $(A^{-1})^t = (A^t)^{-1}$

Sol: –

$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$

$A.A^{-1} = A^{-1}.A = I$

Taking L-H-S

$A^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$

$|A| = \begin{vmatrix} 1 & 3 \\ -2 & 6 \end{vmatrix} = 6 \times 1 - (-2 \times 3)$

$|A| = 6 + 6 => 12$

Taking *Ajdjoint*

$\text{Adj}\mathbf{A} = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$

$A^{-1} = \frac{\begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}}{|12|}$

$A^{-1} = \begin{bmatrix} 6/12 & -3/12 \\ 2/12 & 1/12 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \cancel{6}^1/\cancel{12}^2 & \cancel{-3}^1/\cancel{12}^4 \\ \cancel{2}^1/\cancel{12}^6 & 1/12 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$ .... (A)

$A.A^{-1} = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix} . \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$

$A.A^{-1} = \begin{bmatrix} 1 \times \frac{1}{2} + 3 \times \frac{1}{6} & 1 \times \frac{-1}{4} + 3 \times \frac{1}{12} \\ -2 \times \frac{1}{2} + 6 \times \frac{1}{6} & -2 \times \frac{-1}{4} + 6 \times \frac{1}{12} \end{bmatrix}$

$A.A^{-1} = \begin{bmatrix} \frac{1}{2} + \cancel{3}^1 \times \frac{1}{\cancel{6}^2} & \frac{-1}{4} + \cancel{3}^1 \times \frac{1}{\cancel{12}^4} \\ \cancel{-2}^{-1} \times \frac{1}{\cancel{2}^2} + \cancel{6}^1 \times \frac{1}{\cancel{6}^1} & \cancel{-2}^{-1} \times \frac{-1}{\cancel{4}^2} + \cancel{6}^1 \times \frac{1}{\cancel{12}^2} \end{bmatrix}$

$$A.A^{-1} = \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & \frac{-1}{4}+\frac{1}{4} \\ -1+1 & \frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$A.A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{......... (i)}$$

Taking R-H-S

From .... (A) A^{-1} Is

$$A^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$$

$$A^{-1}.A = \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix} . \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$$

$$A^{-1}.A = \begin{bmatrix} \frac{1}{2}\times1+(-\frac{1}{4_2}\times -2^{-1}) & \frac{1}{2}\times3+(-\frac{1}{4_2}\times 6^{3}) \\ \frac{1}{6}\times1+\frac{1}{12_6}\times -2^{-1} & \frac{1}{6_2}\times 3^{1}+\frac{1}{12_6}\times 6^{1} \end{bmatrix}$$

$$A^{-1}.A = \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & \frac{3}{2}-\frac{3}{2} \\ \frac{1}{6}-\frac{1}{6} & \frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$A^{-1}.A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{......... (ii)}$$

From (i) And (ii) it is proved that

$$\boxed{A.A^{-1} = A^{-1}.A = I}$$

ii. $(A^{-1})^t = (A^t)^{-1}$

$Sol: -$

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$$

We know that

$$A^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$$

$$|A| = \begin{vmatrix} 1 & 3 \\ -2 & 6 \end{vmatrix} = 6\times1-(-2\times3)$$

$$|A| = 6+6 => 12$$

Now Taking $Adjoint$

$$\text{Adj}\mathbf{A} = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}}{12}$$

$$A^{-1} = \begin{bmatrix} 6/12 & -3/12 \\ 2/12 & 1/12 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{6^1}{12^2} & \frac{-3^1}{12^4} \\ \frac{2^1}{12^6} & \frac{1}{12} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$$

Taking Transpose

$$\boxed{(A^{-1})^t = \begin{bmatrix} 1/2 & 1/6 \\ -1/4 & 1/12 \end{bmatrix}} \text{ .... (i)}$$

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$$

$$(A^t)^{-1} = ?$$

Taking Transpose

$$(A)^t = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$$

We know that

$$(A^t)^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 3 & 6 \end{vmatrix} = 6\times1-(3\times-2)$$

$$|A| = 6+6 => 12$$

Now Taking $Adjoint$

$$\text{Adj}\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -3 & 1 \end{bmatrix}$$

$$(A^t)^{-1} = \frac{\begin{bmatrix} 6 & 2 \\ -3 & 1 \end{bmatrix}}{12}$$

$$(A^t)^{-1} = \begin{bmatrix} 6/12 & 2/12 \\ -3/12 & 1/12 \end{bmatrix}$$

$$(A^t)^{-1} = \begin{bmatrix} \cancel{6}^{1}/\cancel{12}^{2} & \cancel{2}^{1}/\cancel{12}^{6} \\ \cancel{-3}^{1}/\cancel{12}^{4} & 1/12 \end{bmatrix}$$

$$\boxed{(A^t)^{-1} = \begin{bmatrix} 1/2 & 1/6 \\ -1/4 & 1/12 \end{bmatrix}} \text{ .... (ii)}$$

From (i) And (ii) it is prove that

$$\boxed{(A^{-1})^t = (A^t)^{-1}}$$

- Questions:-

1. Using inverse property

Show that $A = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ is non-singular.

$Sol: -$

Let $A = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A.\ A^{-1} = I$$

$$\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a + c & 2b + d \\ -2a + 3c & -2b + 3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Compare equations

$2a + c = 1$ .... (i)

$2b + d = 0$ .... (ii)

$-2a + 3c = 0$ .... (iii)

$-2b + 3d = 1$ .... (iv)

Adding (i) and (iii)

$$\cancel{2a} + c = 1$$
$$\cancel{-2a} + 3c = 0$$
$$4c = 1$$

s $\boxed{c = \frac{1}{4}}$

Put $c = \frac{1}{4}$ in (i)

$$2a + c = 1 \quad => \quad 2a + \frac{1}{4} = 1$$

$$2a = 1 - \frac{1}{4} \; => \; 2a = \frac{4-1}{4} \quad => \; 2a = \frac{3}{4}$$

$$\boxed{a = \frac{3}{8}}$$

Adding (ii) and (iv)

$$\cancel{2b} + d = 0$$
$$\cancel{-2b} + 3d = 1$$
$$4d = 1$$

$$\boxed{d = \frac{1}{4}}$$

Put $d = \frac{1}{4}$ in (ii)

$$2b + d = 0 \; => 2b + \frac{1}{4} = 0 \; => 2b = -\frac{1}{4}$$

$$\boxed{b = -\frac{1}{8}}$$

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/8 & -1/8 \\ 1/4 & 1/4 \end{bmatrix}$$

2. Using inverse property

Show that $A = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$ is non-singular.

$Sol: -$

Let $A = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A.\ A^{-1} = I$$

$$\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix} . \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a + c & 2b + d \\ -4a + 2c & -4b + 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Compare equations

$2a + c = 1$ .... (i)

$2b + d = 0$ .... (ii)

$-4a + 2c = 0$ .... (iii)

$-4b + 2d = 1$ .... (iv)

Using (i) and (iii)

$2a + c = 1$ .... (i)

$-4a + 2c = 0$ .... (iii)

Multiplying 2 with (i)

$4a + 2c = 2$......... (v)

Add (i) and (v)

$\cancel{4a} + 2c = 2$

$\cancel{-4a} + 2c = 0$

$4c = 2$

$\boxed{c = \frac{1}{2}}$

Put $c = \frac{1}{2}$ in (i)

$2a + c = 1 => 2a + \frac{1}{2} = 1$

$=> 2a = 1 - \frac{1}{2} => 2a = \frac{2-1}{2}$

$=> 2a = \frac{1}{2}$ $\boxed{a = \frac{1}{4}}$

Using (ii) and (iv)

$2b + d = 0$ .... (ii)

$-4b + 2d = 1$ .... (iv)

Multiplying 2 with (ii)

$4b + 2d = 0$ .... (vi)

Add (iv) and (vi)

$\cancel{4b} + 2d = 0$

$\cancel{-4b} + 2d = 1$

$4d = 1$

$\boxed{d = \frac{1}{4}}$

Put $d = \frac{1}{4}$ in (ii)

$2b + d = 0$

$=> 2b + \frac{1}{4} = \quad => 2b = -\frac{1}{4}$

$\boxed{b = -\frac{1}{8}}$

$$A^{-1} = \begin{bmatrix} 1/4 & -1/8 \\ 1/2 & 1/4 \end{bmatrix}$$

3. Using inverse property

Show that $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix}$ is non-singular.

$Sol{:}-$

Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$A.\ A^{-1} = I$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix} . \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a + 2d - g & b + 2e - h & c + 2f - i \\ 3a + 2d + 3g & 3b + 2e + 3h & 3c + 2f + 3i \\ 2a + 2d + g & 2b + 2e + h & 2c + 2f + i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Compare equations

$a + 2d - g$
$b + 2e - h$
$c + 2f - i$
$3a + 2d + 3g$
$3b + 2e + 3h$
$3c + 2f + 3i$
$2a + 2d + g$
$2b + 2e + h$
$2c + 2f + i$

4. $A = \begin{bmatrix} 3 & 5 \\ 10 & 10 \end{bmatrix}$ Show that $(A^{-1})^{-1} = A$

$Sol:-$

$$A = \begin{bmatrix} 3 & 5 \\ 10 & 10 \end{bmatrix}$$

Taking Inverse

$$A^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$$

$$|A| = \begin{vmatrix} 3 & 5 \\ 10 & 10 \end{vmatrix} = 10 \times 3 - 10 \times 5$$
$$|A| = 30 - 50$$
$$|A| = -20$$

Taking *Adjont*

$$Adj\mathbf{A} = \begin{bmatrix} 10 & -5 \\ -10 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\begin{bmatrix} 10 & -5 \\ -10 & 3 \end{bmatrix}}{-20}$$

$$A^{-1} = \begin{bmatrix} \frac{10^1}{-20^2} & \frac{-5^1}{-20^4} \\ \frac{-10^1}{-20^2} & \frac{3}{-20} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/-2 & 1/4 \\ 1/2 & 3/-20 \end{bmatrix}$$

Taking Again Inverse

$$(A^{-1})^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$$

$$|A| = \begin{vmatrix} 1/-2 & 1/4 \\ 1/2 & 3/-20 \end{vmatrix} = \frac{3}{-20} \times \frac{1}{-2} - \frac{1}{2} \times \frac{1}{4}$$

$$|A| = \frac{3}{40} - \frac{1}{8} => \frac{3-1}{40}$$

$$|A| = \frac{2^1}{40^2}$$

$$|A| = \frac{1}{20}$$

Taking *Adjont*

$$\text{Adj } \mathbf{A^{-1}} = \begin{bmatrix} 3/-20 & -1/4 \\ -1/2 & 1/-2 \end{bmatrix}$$

$$(A^{-1})^{-1} = \frac{\text{Adj } \mathbf{A}}{|\mathbf{A}|}$$

$$(A^{-1})^{-1} = \frac{\begin{bmatrix} 3/-20 & -1/4 \\ -1/2 & 1/-2 \end{bmatrix}}{1/20}$$

$$(A^{-1})^{-1} = 20 \begin{bmatrix} 3/-20 & -1/4 \\ -1/2 & 1/-2 \end{bmatrix}$$