Week # 02

→ Solution to System of Linear Equation

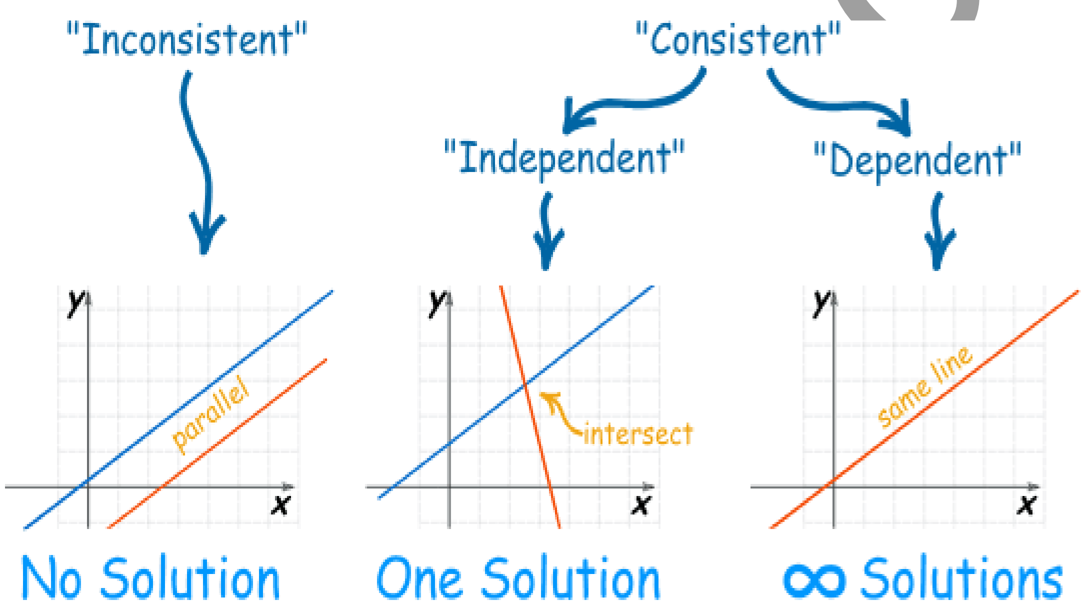
➢ Solution to System of Linear Equation:-

There are three types of systems of linear equations in two variables, and three types of solutions.

i. An independent system has exactly one solution pair (x, y).The point where the two lines intersect is the only solution.

ii. An inconsistent system has no solution. Notice that the two lines are parallel and will never intersect.

iii. A dependent system has infinitely many solutions. They are the same line, so every coordinate pair on the line is a solution to both equations.



- **Questions:-**

i. Find the solution of $x + 2y = 8$ and $3x - 4y = 4$ by graph.

$Sol: -$

$x + 2y = 8 \longrightarrow$ (i)

$3x - 4y = 4 \longrightarrow$ (ii)

$x + 2y = 8$	$3x - 4y = 4$
For x-intercept $y = 0$	For x-intercept $y = 0$
$x + 2(0) = 8$	$3x - 4(0) = 4$

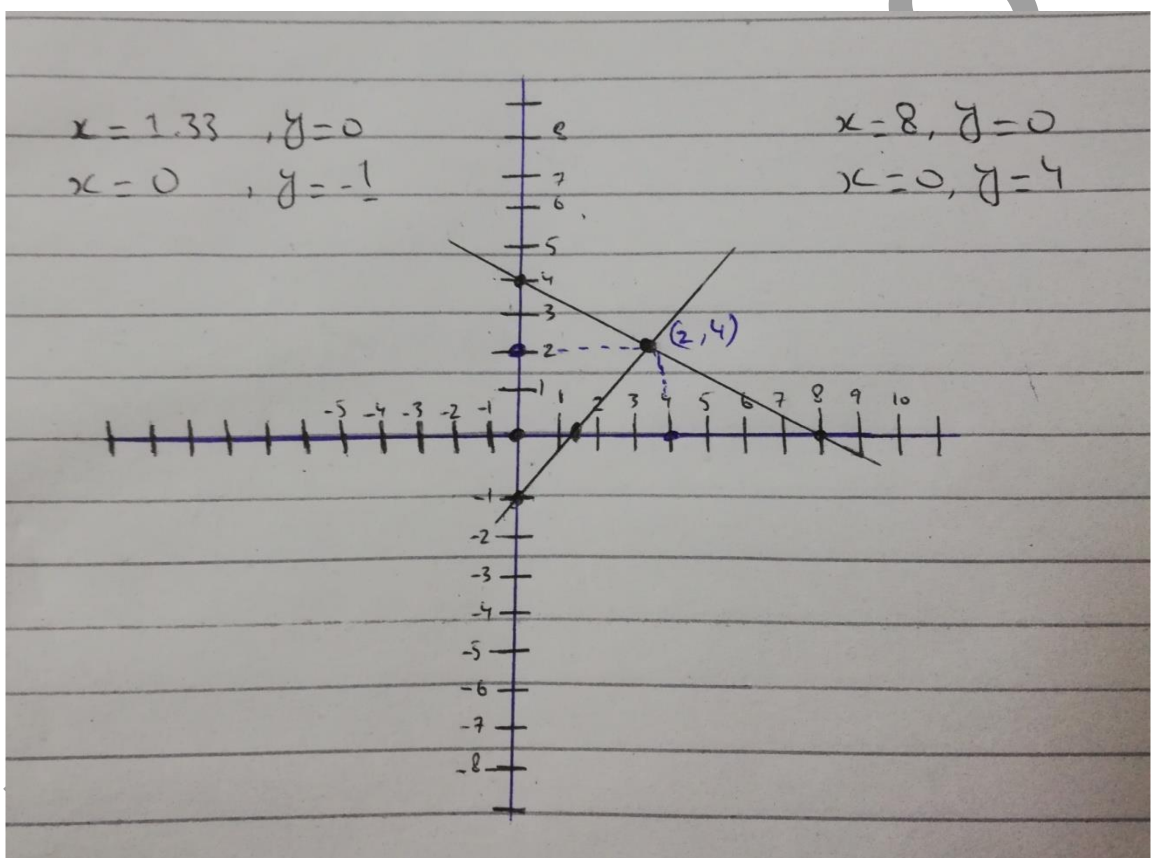
$x = 8$

For y-intercept $x = 0$

$0 + 2y = 8$
$y = 4$
$(8,0), (0,4)$

$x = \frac{3}{4}$
$x = 1.333$

For y-intercept $x = 0$
$3(0) - 4y = 4$
$0 - 4y = 4$
$y = -1$
$(1.333, 0), (0, -1)$



The graph shows that the two lines intersect each other at points $(2\ , 4)$.It means the points $(2,4)$ are the unique solution set. The lines of the unique solution will always intersect each other.

ii. Find the solution of $x + y = 5$ and $3x + 3y = 10$,also draw the graph.

$Sol{:}-$
$x + y = 5$ ⟶ (i)
$3x + 3y = 10$ ⟶ (ii)

Multiplying → (i) by 3

$3x + 3y = 15$ → (ii)

Subtract → (iii) from → (ii)

$$\not{3x} + \not{3y} = 15$$
$$\pm \not{3x} \pm \not{3y} = \pm 10$$
$$0 + 0 = 5$$

$0 = 5$ → (A)

→ (A) Show that it has no solution.

By Graph

$x + y = 5$

For x-intercept $y = 0$

$x + (0) = 5$

$x = 5$

For y-intercept $x = 0$

$0 + y = 5$

$y = 5$

$(5 , 0), (5 , 0)$

$3x + 3y = 10$

For x-intercept $y = 0$

$3x + 3(0) = 10$

$x = \frac{10}{3}$

$x = 3.33$

For y-intercept $x = 0$

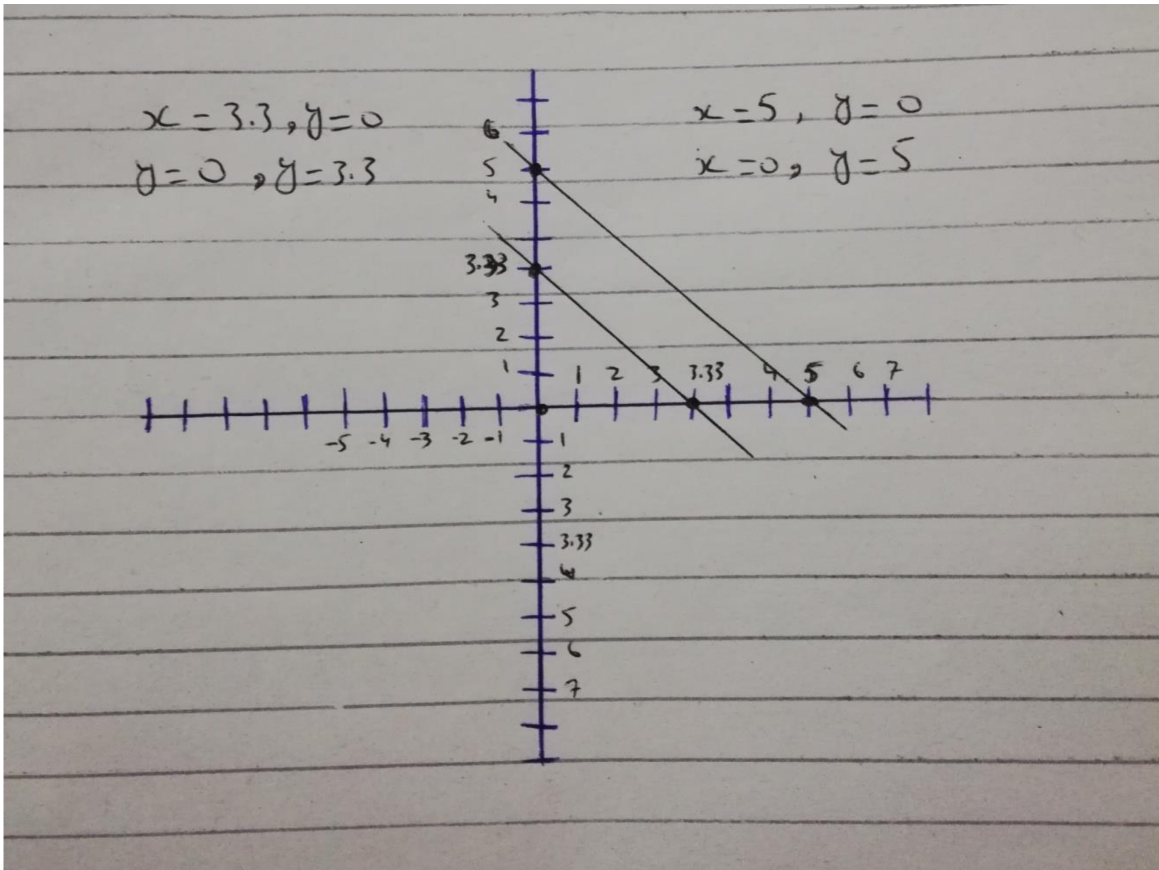
$3(0) + 3y = 10$

$0 + 3y = 10$

$x = \frac{10}{3}$

$x = 3.33$

$(3.33 ,\ 0), (0 ,\ 3.33)$



The two lines in the graph show that it has no solution. The lines in the graph of no solution will be always parallel to each other.

iii. Find the solution set of $x - 2y = -1$ and $-x + 2y = 3$

$Sol: -$

$x - 2y = -1$ ⟶ (i)
$-x + 2y = 3$ ⟶ (ii)

Adding ⟶ (i) & ⟶ (ii)

$$\not{x} - \not{2y} = -1$$
$$-\not{x} + \not{2y} = 3$$
$$0 + 0 = 2$$

$0 = 2$ ⟶ (A)

⟶ (A) Shows that it has no solution.

iv. Find the solution set of $x_1 - 2x_2 = -1$ and $-x_1 + 2x_2 = 1$, also draw graph.

$Sol:-$

$x_1 - 2x_2 = -1 \longrightarrow$ (i)

$-x_1 + 2x_2 = 1 \longrightarrow$ (ii)

Adding $\longrightarrow$ (i) & $\longrightarrow$ (ii)

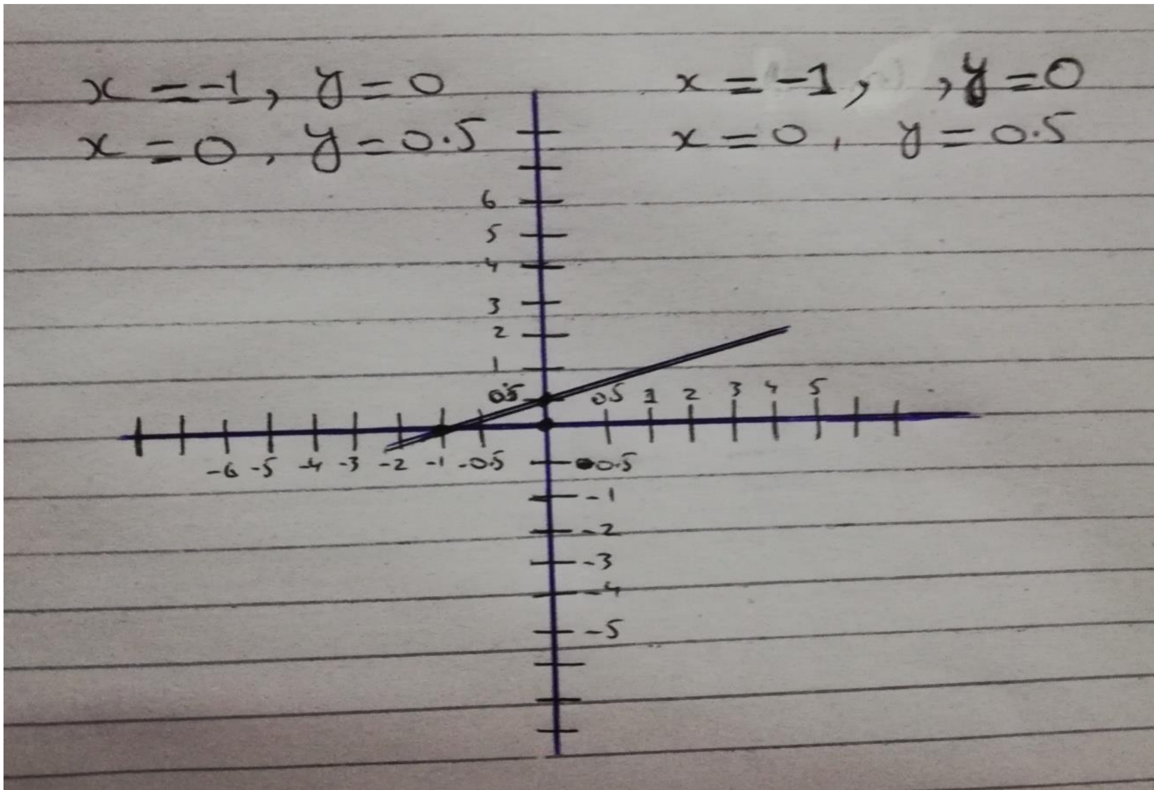
$$\cancel{x_1} - \cancel{2x_2} = -1$$
$$\cancel{x_1} + \cancel{2x_2} = 1$$
$$0 + 0 = 0$$

$0 = 0 \longrightarrow$ (A)

$\longrightarrow$ (A) Show that it has infinite solution

By Graph

$x_1 - 2x_2 = -1$	$-x_1 + 2x_2 = 1$
For x_1-intercept $x_2 = 0$	For x_1-intercept $x_2 = 0$
$x_1 - 2(0)_2 = -1$	$-x_1 + 2(0)_2 = 1$
$x_1 = -1$	$x_1 = -1$
For x_2-intercept $x_1 = 0$	For x_2-intercept $x_1 = 0$
$0 - 2x_2 = -1$	$-(0)_1 + 2x_2 = 1$
$x_2 = \frac{-1}{-2}$	$x_2 = \frac{1}{2}$
$x_2 = 0.5$	$x_2 = 0.5$
$(-1, 0), (0, 0.5)$	$(-1, 0), (0.5, 0)$



The two Lines in graph lies on each other .It shows the infinite solution .The lines in graph of infinite solution will always be same.

v. Find the solution set of

$2x + 4y + 6z = -12$
$2x - 3y - 4z = 15$
$3x + 4y + 5z = -8$

$Sol:-$

$2x + 4y + 6z = -12 \longrightarrow$ (i)
$2x - 3y - 4z = 15 \longrightarrow$ (ii)

$3x + 4y + 5z = -8 \longrightarrow$ (iii)

Subtracting $\longrightarrow$ (iii) from $\longrightarrow$ (i)

$2x + \cancel{4y} + 6z = -12$
$\pm 3x \pm \cancel{4y} \pm 5z = \mp 8$
$-x + z = -4$

$-x + z = -4 \longrightarrow$ (iv)

Multiplying $\longrightarrow$ (ii) by 4

vi. Find the solution set of

$x + y + 3z = 12$
$2x + 2y + 6z = 6$

$Sol:-$

$x + y + 3z = 12 \longrightarrow$ (i)

$2x + 2y + 6z = 6 \longrightarrow$ (ii)

Multiplying $\longrightarrow$ (i) by 2

$2x + 2y + 6z = 24 \longrightarrow$ (iii)

Subtracting $\longrightarrow$ (iii) from $\longrightarrow$ (ii)

$\cancel{2x} + \cancel{2y} + \cancel{6z} = 24$
$\pm \cancel{2x} \pm \cancel{2y} \pm \cancel{6z} = \pm 6$
$0 + 0 = 18$

$0 = 18 \longrightarrow$ (A)

$\longrightarrow$ (A) Show that it has no solution

$8x - 12y - 16z = 60$ ⟶ (v)

Multiplying ⟶ (iii) by 3

$9x + 12y + 15z = -24$ ⟶ (vi)

Adding ⟶ (v) & ⟶ (vi)

$$\begin{array}{l} 8x - \cancel{12y} - 16z = 60 \\ \underline{9x + \cancel{12y} + 15z = -24} \\ 17x + 0 - z = 36 \end{array}$$

$17x - z = 36$ ⟶ (vii)

Adding ⟶ (iv) & ⟶ (vii)

$$\begin{array}{l} -x - \cancel{+z} = -4 \\ \underline{17x \cancel{-z} = 36} \\ 16x + 0 = 32 \end{array}$$

$$x = \frac{32}{16}$$

$\boxed{x = 2}$

Putting $x = 2$ in ⟶ (iv)

$-x + z = -4$

$-2 + z = -4$

$z = -4 + 2$

$\boxed{z = -2}$

Putting $x = 2$ & $z = -2$ ⟶ in (i)

$2x + 4y + 6z = -12$ ⟶ (i)

$2(2) + 4y + 6(-2) = -12$

$4 + 4y - 12 = -12$

$4y = \cancel{-12} \cancel{+12} - 4$

$$y = \frac{\cancel{-4}}{\cancel{4}}$$

$\boxed{y = -1}$

$S.s = \{\, 2, -1, -2 \,\}$

- **Solution of Three Variables And Two Equations:-**

vii. $x + y - 2z = 5$

$2x + 3y + 4z = 2$

$Sol: -$

$x + y - 2z = 5 \longrightarrow$ (i)

$2x + 3y + 4z = 2 \longrightarrow$ (ii)

Multiplying $\longrightarrow$ (i) by 2

$x + y - 2z = 5$

$2x + 2y - 4z = 10 \longrightarrow$ (iii)

Subtract $\longrightarrow$ (ii) From $\longrightarrow$ (iii)

$\cancel{2x} + 2y - 4z = 10$

$\pm \cancel{2x} \pm 3y \pm 4z = \pm 2$

$0 - y - 8z = 8$

$-y - 8z = 8$

$-y = 8 + 8z$

$\boxed{y = -8 - 8z}$

Put $z = r$ $\quad => r = Any\ Real\ Number$

$y = -8 - 8(r)$

$\boxed{y = -8 - 8r}$

Now Using $\longrightarrow$ (i)

Put $y = -8 - 8r$ & $z = r$

$x + y - 2z = 5$

$x + (-8 - 8r) - 2(r) = 5$

$x - 8 - 8r - 2r = 5$

$x = 8 + 5 + 10r$

$\boxed{x = 13 + 10r}$

viii. $x + 4y - z = 12$

$3x + 8y - 2z = 4$

$Sol: -$

$x + 4y - z = 12 \longrightarrow$ (i)

$3x + 8y - 2z = 4 \longrightarrow$ (ii)

Multiplying $\longrightarrow$ (i) by 3

$x + 4y - z = 12$

$3x + 12y - 3z = 36 \longrightarrow$ (iii)

Subtract $\longrightarrow$ (iii) From $\longrightarrow$ (ii)

$\cancel{3x} + 12y - 3z = 36$

$\pm \cancel{3x} \pm 8y \mp 2z = \pm 4$

$0 + 4y - z = 32$

$4y = 32 + z$

$$y = \frac{32 + z}{4}$$

Put $z = r$ $\quad => r = Any\ Real\ Number$

$$\boxed{y = \frac{32 + r}{4}}$$

Now Using $\longrightarrow$ (i)

Put $y = \frac{32+r}{4}$ & $z = r$

$x + 4y - z = 12$

$x + 4(\frac{32+r}{4}) - r = 12$

$x + \cancel{4}(\frac{32 + r}{\cancel{4}}) - r = 12$

$x + 32 + \cancel{r} - \cancel{r} = 12$

$x = 12 - 32$

$\boxed{x = -20}$

ix. $3x + 4y - z = 8$
$6x + 8y - 2z = 3$

$Sol: -$

$3x + 4y - z = 8 \longrightarrow$ (i)
$6x + 8y - 2z = 3 \longrightarrow$ (ii)

Multiplying $\longrightarrow$ (i) by 2
$6x + 8y - 2z = 16 \longrightarrow$ (iii)

Subtract $\longrightarrow$ (iii) From $\longrightarrow$ (i)

$$\begin{aligned} \cancel{6x} + \cancel{8y} - \cancel{2z} &= 16 \\ \pm\cancel{6x} \pm \cancel{8y} \mp \cancel{2z} &= \pm 3 \\ \hline 0 + 0 + 0 &= 13 \end{aligned}$$

$0 = 13 \longrightarrow$ (A)

$\longrightarrow$ (A) Show that it has no solution.

- **Solution of Two Variables And Three Equations:-**

x. $2x + 3y = 13$
$x - 2y = 3$
$5x + 2y = 27$

$Sol: -$

$2x + 3y = 13 \longrightarrow$ (i)
$x - 2y = 3 \longrightarrow$ (ii)
$5x + 2y = 27 \longrightarrow$ (iii)

Using $\longrightarrow$ (i) & $\longrightarrow$ (ii)
$2x + 3y = 13 \longrightarrow$ (i)
$x - 2y = 3 \longrightarrow$ (ii)

Multiplying $\longrightarrow$ (ii) by 2
$2x - 4y = 6 \longrightarrow$ (iv)
Subtract $\longrightarrow$ (i) From $\longrightarrow$ (iv)

$$\begin{aligned} \cancel{2x} - 4y &= 6 \\ \pm\cancel{2x} \pm 3y &= \pm 13 \\ \hline 0 - 7y &= -7 \end{aligned}$$

xi. $x - 5y = 6$
$3x + 2y = 1$
$5x + 2y = 2$

$Sol: -$

$x - 5y = 6 \longrightarrow$ (i)
$3x + 2y = 1 \longrightarrow$ (ii)
$5x + 2y = 2 \longrightarrow$ (iii)

Using $\longrightarrow$ (i) & $\longrightarrow$ (ii)
$x - 5y = 6 \longrightarrow$ (i)
$3x + 2y = 1 \longrightarrow$ (ii)

Multiplying $\longrightarrow$ (i) by 3
$3x - 15y = 18 \longrightarrow$ (iv)
Subtract $\longrightarrow$ (ii) From $\longrightarrow$ (iv)

$$\begin{aligned} \cancel{3x} - 15y &= 18 \\ \pm\cancel{3x} \pm 2y &= \pm 1 \\ \hline 0 - 17y &= 17 \end{aligned}$$

$-7y = -7$

Divide both sides by -7

$\frac{\cancel{-7}y}{\cancel{-7}} = \frac{\cancel{-7}}{\cancel{-7}}$

$\boxed{y = 1}$

Put $y = 1$ in $\longrightarrow$ (i)

$2x + 3y = 13 \longrightarrow$ (i)

$2x + 3(1) = 13$

$2x = 13 - 3$

$2x = 10$

$x = \frac{\cancel{10}^5}{\cancel{2}^1}$

$\boxed{x = 5}$

Put $x = 5$ & $y = 1$ in $\longrightarrow$ (iii)

$5x + 2y = 27 \longrightarrow$ (iii)

$5(5) + 2(1) = 27$

$25 + 2 = 27$

$\boxed{27 = 27}$

When the Values of $'x'$ and 'y' satisfied the equation, it means that it is a unique solution.

$-17y = 17$

Divide both sides by -17

$\frac{\cancel{-17}y}{\cancel{-17}} = \frac{\cancel{17}}{-\cancel{17}}$

$\boxed{y = -1}$

Put $y = -1$ in $\longrightarrow$ (i)

$x - 5y = 6 \longrightarrow$ (i)

$x - 5(-1) = 6$

$x + 5 = 6$

$x + 5 = 6 - 5$

$\boxed{x = 1}$

Put $x = 1$ & $y = -1$ in $\longrightarrow$ (iii)

$5x + 2y = 2 \longrightarrow$ (iii)

$5(1) + 2(-1) = 2$

$5 - 2 = 2$

$3 \neq 2$

When the values of $'x' and 'y'$ does not satisfied the equation, it means that it has no solution.

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The End of Week #02