Week # 01

## → Introduction to System of Linear Equation

### ➢ Linear Equation:-

An equation of the type $ax = b$ expressing the variable $'b'$ in terms of the variable $'x'$ and the constant $'a'$ is called a linear equation .In linear equation $a \neq 0$.

OR

A linear equation in one unknown is an equation of the form $ax = b$, where '$a$' and '$b$' are constants and '$x$' is an unknown that we wish to solve for.

- **Examples:-**

1. $3x = 2$
   It is linear equation because it is in the form of $ax = b$
   here $a = 3$ , $b = 2$

2. $5x = 10$
   It is linear equation because it is in the form of $ax = b$
   here $a = 5$ , $b = 10$

3. $-2x = -70$
   It is linear equation because it is in the form of $ax = b$
   here $a = -2$ , $b = -70$

### ➢ Linear Equation of One Variable:-

A linear equation in one variable is an equation that can be written in the form of $ax = b$ .where '$a$' and '$b$' are real numbers, $a \neq 0$. The answer of the linear equation of variable will be one.

- **Examples:-**

$2x + 7 = 19$

$Sol: -$

Subtract 7 from both sides

$2x + 7 - 7 = 19 - 7$

$2x = 12$

Divide both sides by 2

$\boxed{x = 6. \text{Ans}}$

### ➢ Linear Equation of two Variables:-

If $a, b$ and $r$ are real numbers and ($a \neq 0, b \neq 0$) then $ax + by = \text{r}$ is called a linear equation in two variables. The two variables are'$x$' and '$y$'.The numbers $a$ and $b$ are called the coefficients of the equation $ax + by = \text{r}$ .The number $r$ is called constant of the equation $ax + by = \text{r}$ . Linear equation of two variables can also be written in the form of $a_1x_1 + a_2x_2 = b$.

- **Examples:-**

$10x - 3y = 5$ And $-2x - 4y = 7$ are linear equations in two variables.

### ➢ Linear Equation of three Variables:-

The equation $ax + by + cz = k$ are linear equation in three variables .In order to solve equations in three variables, are known as three-by-three systems. The equation of linear in three variables can also be written in the form of$a_1x_1 + a_2x_2 + a_3x_3 = b$.

Similarly the equation

$a_1x_1 + a_2x_2 + a_3x_3$ ——— $a_nx_n = b$. Expressing $'b'$ in terms of the variables $x_1, x_2, x_3$ ——— $x_n$ and the known constants $a_1, a_2, a_3$ ——— $a_n$ is called linear equation.

- **Examples:-**

Solution of $\mathrm{n}^{\mathrm{th}}$ equation

$2x + 5y = 10$

$x = 5, \quad y = 0$

$x = 0, \quad y = 2$

$x = 10, \quad y = -2$

$x_1 + x_2 + x_3 + x_4 = 10$

$1 + 2 + 3 + 4$

$2 + 1 + 3 + 4$

$3 + 1 + 2 + 4$

$10 + 0 + 0 + 0$

$5 + 5 + 0 + 0$

These all are many Infinite solutions.

## ➢ Systems of linear equations:-

$a_{11}x_{1+}a_{12}x_2 + a_{13}x_3$ ——— $a_{1n}x_n = b_1$

$a_{21}x_{1+}a_{22}x_2 + a_{23}x_3$ ——— $a_{2n}x_n = b_2$

$a_{31}x_{1+}a_{32}x_2 + a_{33}x_3$ ——— $a_{3n}x_n = b_3$

$\vdots$

$a_{n1}x_{1+}a_{n2}x_2 + a_{n3}x_3$ ——— $a_{nn}x_n = b_n$

- **Questions:-**

Solve the system of linear equation by using method of elimination.

1. $x + 2y = 8$

$3x - 4y = 4$

$Sol: -$

$x + 2y = 8$ ⟶ (i)

$3x - 4y = 4$ ⟶ (ii)

Multiplying ⟶ (i) by 3

$3x + 6y = 24$ ⟶ (iii)

Subtract ⟶ (ii) from ⟶ (iii)

$\cancel{3x} + 6y = 24$

$\pm\cancel{3x} \mp 4y = \pm 4$

$0 + 10y = 20$

$10y = 20$

$y = \frac{20}{10}$

2. $x_1 - 2x_2 = -1$

$-x_1 + 3x_2 = 3$

$Sol: -$

$x_1 - 2x_2 = -1$ ⟶ (i)

$-x_1 + 3x_2 = 3$ ⟶ (ii)

Adding ⟶ (i) and (ii)

$\cancel{x_1} - 2x_2 = -1$

$\cancel{-x_1} + 3x_2 = 3$

$0 + x_2 = 2$

$\boxed{x_2 = 2}$

Put $x_2 = 2$ in ⟶ (i) or (ii)

$\boxed{y = 2}$

Put $y = 2$ in → (i) or (ii)

$x + 2y = 8$

$x + 2(2) = 8$

$x + 4 = 8$

$x = 8 - 4$

$\boxed{x = 4}$

$solution\ set = \{4, \quad 2\}$

3. $2x - 3y + 4z = -12$
$x - 2y + z = -5$
$3x + y + 2z = 1$
$Sol:-$

$2x - 3y + 4z = -12$ → (i)

$x - 2y + z = -5$ → (ii)

$3x + y + 2z = 1$ → (iii)

Multiplying → (ii) by 2

$2x - 4y + 2z = -10$ → (iv)

Multiplying → (ii) by 3

$3x - 6y + 3z = -15$ → (v)

Subtract → (i) from → (iv)

$\cancel{2x} - 4y + 2z = -10$

$\underline{\pm \cancel{2x} \mp 3y \pm 4z = \mp 12}$

$0 - y - 2z = 2$

$-y - 2z = 2$

$y + 2z = -2$ → (vi)

Subtract → (iii) from → (v)

$\cancel{3x} - 6y + 3z = -15$

$\underline{\pm \cancel{3x} \pm y \pm 2z = \pm 1}$

$0 - 7y + z = -16$

$x_1 - 2x_2 = -1$

$x_1 - 2(2) = -1$

$x_1 - 4 = -1$

$x_1 = -1 + 4$

$x_1 = 3$

$\boxed{x_1 = 3}$

$S.S = \{3, \ 2\}$

4. $3x + 2y + z = 2$
$4x + 2y + 2z = 8$
$x - y + z = 4$
$Sol:-$

$3x + 2y + z = 2$ → (i)

$4x + 2y + 2z = 8$ → (ii)

$x - y + z = 4$ → (iii)

Multiplying → (iii) by 4

$4x - 4y + 4z = 16$ → (iv)

Multiplying → (iii) by 3

$3x - 3y + 3z = 12$ → (v)

Subtract → (iv) from → (ii)

$\cancel{4x} - 4y + 4z = 16$

$\underline{\pm \cancel{4x} \pm 2y \pm 2z = \pm 8}$

$0 - 6y + 2z = 8$

$-6y + 2z = 8$ → (vi)

Subtract → (i) from → (v)

$\cancel{3x} - 3y + 3z = 12$

$\underline{\pm \cancel{3x} \pm 2y \pm z = \pm 2}$

$0 - 5y + 2z = 10$

$-7y + z = -16 \longrightarrow$ (vii)

$y + 2z = -2 \longrightarrow$ (vi)

Multiplying $\longrightarrow$ (vi) by 7

$7y + 14z = -14 \longrightarrow$ (viii)

Add $\longrightarrow$ (vii) & $\longrightarrow$ (viii)

$$\begin{array}{l} \cancel{7y} + 14z = -14 \\ \cancel{-7y} + 1z = -16 \\ \hline 0 + 15z = -30 \end{array}$$

$15z = -30$

$z = \frac{\cancel{-30}^{2}}{\cancel{15}^{1}}$

$\boxed{z = -2}$

Put $z = -2$ $in$ $\longrightarrow$ (vi)

$y + 2z = -2$

$y + 2(-2) = -2$

$y - 4z = -2$

$y = -2 + 4$

$\boxed{y = 2}$

Put $y = 2\,, z = -2$ $\longrightarrow$ (ii)

$x - 2(2) + (-2) = -5$

$x - 4 - 2 = -5$

$x - 6 = -5$

$x = -5 + 6$

$x = 1$

$\boxed{x = 1}$

$S.S = \{1, \quad 2, \quad -2\}$

$-5y + 2z = 10 \longrightarrow$ (vii)

Subtract $\longrightarrow$ (vii) from $\longrightarrow$ (vi)

$$\begin{array}{l} -5y + \cancel{2z} = 10 \\ \mp 6y \pm \cancel{2z} = \pm 8 \\ \hline y + 0 = 2 \end{array}$$

$\boxed{y = 2}$

put $y = 2$ in $\longrightarrow$ (vi)

$-6y + 2z = 8$

$-6(2) + 2z = 8$

$-12 + 2z = 8$

$2z = 8 + 12$

$2z = 20$

$z = \frac{\cancel{20}^{10}}{\cancel{2}^{1}}$

$\boxed{z = 10}$

Put $y = 2\,, z = 10$ $\longrightarrow$ (i)

$3x + 2y + z = 2$

$3x + 2(2) + 10 = 2$

$3x + 4 + 10 = 2$

$3x = 2 - 14$

$3x = -12$

$x = \frac{\cancel{-12}^{4}}{\cancel{3}^{1}}$

$\boxed{x = -4}$

$S.S = \{-4, \quad 2, \quad 10\}$

**The End of Week #02**