Regular Expression

- As discussed earlier that a^* generates
  Λ, a, aa, aaa, …
  and a^+ generates a, aa, aaa, aaaa, …, so the language $L_1 = \{\Lambda, a, aa, aaa, \ldots\}$ and $L_2 = \{a, aa, aaa, aaaa, \ldots\}$ can simply be expressed by a^* and a^+, respectively.
  a^* and a^+ are called the regular expressions (RE) for L_1 and L_2 respectively.
  Note: a^+, aa^* and a^*a generate L_2.

Recursive definition of Regular Expression(RE)

Step 1: Every letter of Σ including Λ is a regular expression.

Step 2: If r_1 and r2 are regular expressions then

1. (r_1)

2. $r_1\ r_2$

3. $r_1 + r_2$ and

4. r_1^*

are also regular expressions.

Step 3: Nothing else is a regular expression.

Defining Languages (continued)…

- **Method 3 (Regular Expressions)**
  - Consider the language $L=\{\Lambda, x, xx, xxx, \ldots\}$ of strings, defined over $\Sigma = \{x\}$.

    We can write this language as the Kleene star closure of alphabet Σ or $L=\Sigma^*=\{x\}^*$

    this language can also be expressed by the regular expression x^*.
  - Similarly the language $L=\{x, xx, xxx, \ldots\}$, defined over $\Sigma = \{x\}$, can be expressed by the regular expression x^+.

- Now consider another language L, consisting of all possible strings, defined over $\Sigma = \{a, b\}$. This language can also be expressed by the regular expression

$$(a + b)^*.$$

- Now consider another language L, of strings having exactly double a, defined over $\Sigma = \{a, b\}$, then it's regular expression may be

$$b^*aab^*$$

- Now consider another language L, of even length, defined over $\Sigma = \{a, b\}$, then it's regular expression may be

$$((a+b)(a+b))^*$$

- Now consider another language L, of odd length, defined over $\Sigma = \{a, b\}$, then it's regular expression may be

$$(a+b)((a+b)(a+b))^* \text{ or}$$

$$((a+b)(a+b))^*(a+b)$$

Remark

- It may be noted that a language may be expressed by more than one regular expressions, while given a regular expression there exist a unique language generated by that regular expression.

- Example:
  - Consider the language, defined over $\Sigma=\{a, b\}$ of words having at least one a, may be expressed by a regular expression $(a+b)^*a(a+b)^*$.
  - Consider the language, defined over $\Sigma = \{a, b\}$ of words having at least one a and one b, may be expressed by a regular expression

$(a+b)^*a(a+b)^*b(a+b)^* + (a+b)^*b(a+b)^*a(a+b)^*$.

- Consider the language, defined over $\Sigma=\{a, b\}$, of words starting with double a and ending in double b then its regular expression may be $aa(a+b)^*bb$
- Consider the language, defined over $\Sigma=\{a, b\}$ of words starting with a and ending in b OR starting with b and ending in a, then its regular expression may be $a(a+b)^*b+b(a+b)^*a$

TASK

- Consider the language, defined over $\Sigma=\{a, b\}$ of **words beginning with a**, then its regular expression may be $a(a+b)^*$

- Consider the language, defined over $\Sigma=\{a, b\}$ of **words beginning and ending in same letter**, then its regular expression may be $(a+b)+a(a+b)^*a+b(a+b)^*b$

TASK

- Consider the language, defined over $\Sigma=\{a, b\}$ of **words ending in b**, then its regular expression may be $(a+b)^*b$.
- Consider the language, defined over $\Sigma=\{a, b\}$ of **words not ending in a**, then its regular expression may be $(a+b)^*b + \Lambda$. It is to be noted that this language may also be expressed by $((a+b)^*b)^*$.

SummingUP Lecture 3

RE, Recursive definition of RE, defining languages by RE, $\{ x\}^*$, $\{ x\}^+$, $\{a+b\}^*$, Language of strings having **exactly one aa**, Language of strings of **even length**, Language of strings of **odd length**, RE defines unique language (as Remark), Language of strings having **at least one a**, Language of strings havgin **at least one a and one b,** Language of strings **starting with aa and ending in bb,** Language of strings **starting with and ending in different letters**.