Kleene Star Closure

- Given Σ, then the Kleene Star Closure of the alphabet Σ, denoted by Σ^*, is the collection of all strings defined over Σ, including Λ.
- It is to be noted that Kleene Star Closure can be defined over any set of strings.

Examples

- If $\Sigma = \{x\}$

  Then $\Sigma^* = \{\Lambda, x, xx, xxx, xxxx, \ldots.\}$
- If $\Sigma = \{0,1\}$

  Then $\Sigma^* = \{\Lambda, 0, 1, 00, 01, 10, 11, \ldots.\}$
- If $\Sigma = \{aaB, c\}$ d

  Then $\Sigma^* = \{\Lambda, aaB, c, aaBaaB, aaBc, caaB, cc, \ldots.\}$

Note

- Languages generated by Kleene Star Closure of set of strings, are infinite languages. (By infinite language, it is supposed that the language contains infinite many words, each of finite length).

Task

⌘Q)

1) Let S={ab, bb} and T={ab, bb, bbbb} Show that $S^* = T^*$ [Hint $S^* \subseteq T^*$ and $T^* \subseteq S^*$]
2) Let S={ab, bb} and T={ab, bb, bbb} Show that $S^* \neq T^*$ But $S^* \subset T^*$
3) Let S={a, bb, bab, abaab} be a set of strings. Are abbabaabab and baabbbabbaabb in S^*? Does any word in S^* have odd number of b's?

Task

- Q) 2: **Solution**

Since $S \subset T$, so every string belonging to S^* , also belongs to T^* but bbb is a string belongs to T^* but does not belong to S^*

Task

- Q) 3: **Solution**

Since abbabaabab can be grouped as a, bb, abaab, ab, which shows that the last member of the group does not belong to S, so abbabaabab is not in S^* , while baabbbabbaabb can not be grouped as members of S, hence baabbbabbaabb is not in S^* . Since each string in S has even number of b's so there is no possibility of any string with odd number of b's to be in S^*

PLUS Operation ($^+$)

- Plus Operation is same as Kleene Star Closure except that it does not generate Λ (null string), automatically.

Example:

- If $\Sigma = \{0,1\}$

  Then $\Sigma^+ = \{0, 1, 00, 01, 10, 11, ...\}$
- If $\Sigma = \{aab, c\}$

  Then $\Sigma^+ = \{aab, c, aabaab, aabc, caab, cc, ...\}$

TASK

Q1)Is there any case when S^+ contains Λ? If yes then justify your answer.

Q2) Prove that for any set of strings S

i. $(S^+)^* = (S^*)^*$
ii. $(S^+)^+ = S^+$
iii. Is $(S^*)^+ = (S^+)^*$

Remark

⌘ It is to be noted that Kleene Star can also be operated on any string *i.e.* a^* can be considered to be all possible strings defined over {a}, which shows that a^* generates

Λ, a, aa, aaa, …

It may also be noted that a^+ can be considered to be all possible non empty strings defined over {a}, which shows that a^+ generates

a, aa, aaa, aaaa, …

Defining Languages Continued…

- **Recursive definition of languages**

  The following three steps are used in recursive definition

1. Some basic words are specified in the language.
2. Rules for constructing more words are defined in the language.
3. No strings except those constructed in above, are allowed to be in the language.

Example

- **Defining language of INTEGER**

Step 1:

1 is in **INTEGER**.

Step 2:

If x is in **INTEGER** then x+1 and x-1 are also in **INTEGER**.

Step 3:

No strings except those constructed in above, are allowed to be in **INTEGER**.

Example

- **Defining language of EVEN**

Step 1:

2 is in **EVEN**.

Step 2:

If x is in **EVEN** then x+2 and x-2 are also in **EVEN**.

Step 3:

No strings except those constructed in above, are allowed to be in **EVEN**.

Example

- **Defining the language factorial**

<u>Step 1:</u>

As 0!=1, so 1 is in **factorial**.

<u>Step 2:</u>

n!=n*(n-1)! is in **factorial**.

<u>Step 3:</u>

No strings except those constructed in above, are allowed to be in **factorial**.

- **Defining the language PALINDROME, defined over** $\Sigma = \{a,b\}$

<u>Step 1:</u>

a and b are in **PALINDROME**

<u>Step 2:</u>

if x is palindrome, then s(x)Rev(s) and xx will also be palindrome, where s belongs to Σ^*

<u>Step 3:</u>

No strings except those constructed in above, are allowed to be in palindrome

- **Defining the language $\{a^n b^n\}$, n=1,2,3,... , of strings defined over Σ={a,b}**

<u>Step 1:</u>

ab is in $\{a^n b^n\}$

<u>Step 2:</u>

if x is in $\{a^n b^n\}$, then axb is in $\{a^n b^n\}$

<u>Step 3:</u>

No strings except those constructed in above, are allowed to be in $\{a^n b^n\}$

- **Defining the language L, of strings ending in a , defined over $\Sigma=\{a,b\}$**

Step 1:

a is in L

Step 2:

if x is in L then s(x) is also in **L,** where s belongs to Σ^*

Step 3:

No strings except those constructed in above, are allowed to be in **L**

- **Defining the language L, of strings beginning and ending in same letters , defined over Σ={a, b}**

<u>Step 1:</u>

a and b are in L

<u>Step 2:</u>

(a)s(a) and (b)s(b) are also in **L,** where s belongs to Σ^*

<u>Step 3:</u>

No strings except those constructed in above, are allowed to be in **L**

- **Defining the language L, of strings containing aa or bb , defined over Σ={a, b}**

Step 1:

aa and bb are in L

Step 2:

s(aa)s and s(bb)s are also in **L,** where s belongs to Σ^*

Step 3:

No strings except those constructed in above, are allowed to be in **L**

- **Defining the language L, of strings containing exactly aa, defined over Σ={a, b}**

Step 1:

aa is in L

Step 2:

s(aa)s is also in **L,** where s belongs to b^*

Step 3:

No strings except those constructed in above, are allowed to be in **L**

Summing Up

- Kleene Star Closure, Plus operation, recursive definition of languages, INTEGER, EVEN, factorial, PALINDROME, {$\mathbf{a^n b^n}$}, languages of strings (i) ending in a, (ii) beginning and ending in same letters, (iii) containing aa or bb (iv)containing exactly aa,