Week # 4

Line - Integral and surface Integral.

Definition: Let D be a set in R^2 (a plane region).
A vector field on R^2 is a function F that assigns to each point (x, y) in D a two dimensional vector $F(x, y)$

$$F(x, y) = P(x, y)\hat{i} + Q(x, y)\hat{j}$$

Note:
that P and Q are scalar function of two variables

Definition: Let E be a subset of R^3.
A vector field on R^3 is a function F that assigns to each point (x, y, z) in E
three dimensional vector ~~field~~
$F(x, y, z)$

$$F_3(x, y, z) = P(x, y, z)\hat{i} + Q(x, y, z)\hat{j} + R(x, y, z)\hat{k}$$

Gradient Fields:

if f is a scalar function of two variables

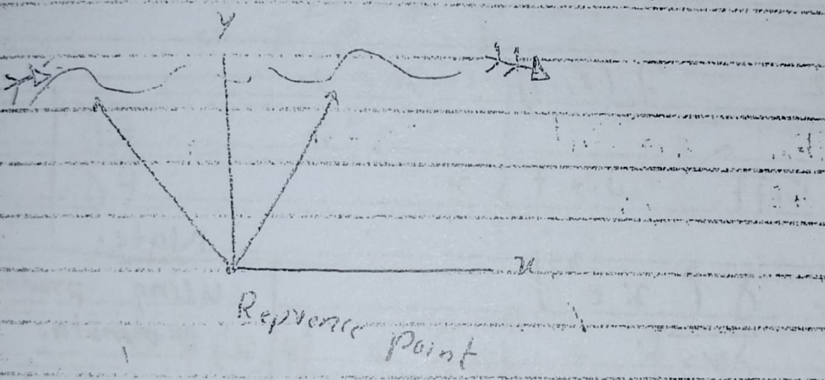
* represented by ∇ (Neble) mean operator

$$\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)$$ For two dimensional

$$\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$ For three dimensional.

$$\nabla F = (x,y) = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j}$$

$$\nabla F = (x,y,z) = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$



Note

∇ = Scalar convert to vector.

operator = work to change occure.

Q # 1 Find the gradiant vector field of f.

$$f(x,y) = x^2y - y^3$$

Diff w.r.t x

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2y - y^3)$$

$$= 2xy - 0$$

$$\boxed{\frac{\partial f}{\partial x} = 2xy \longrightarrow (1)}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial y}(x^2 y - y^3)$$

$$\boxed{\frac{\partial z}{\partial y} = x^2 - 3y^2}$$

$$\nabla = \Delta z(x,y) = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j}$$

$$\nabla z(x,y) = (2xy)\hat{i} + (x^2 - 3y^2)\hat{j}$$

Find the gradient vector field of z:

Q # 2 $\quad z(x,y) \quad xe^{xy}$

Diff w.r.t x.

$$\frac{\partial z}{\partial x} = \frac{\partial (xe^{xy})}{\partial x}.$$

$$= x\frac{\partial}{\partial x}e^{xy} + e^{xy}\frac{\partial}{\partial x}x$$

$$= xe^{xy}\frac{\partial}{\partial x}xy + e^{xy}.1$$

$$= xe^{xy}(1.y) + e^{xy}$$

$$= xye^{xy} + e^{xy}$$

$$\boxed{\frac{\partial z}{\partial x} = e^{xy}(xy+1)} \longrightarrow ①$$

Note.
Using product Formula.

$$\frac{d}{dx}(a.b) =$$

$$a\frac{d}{dx}b + b\frac{d}{dx}a$$

$$\frac{\partial z}{\partial y} = \frac{\partial\, x e^{xy}}{\partial y} \quad \text{w.r.t } y$$

$$= x\frac{\partial}{\partial y} e^{xy} + e^{xy}\frac{\partial x}{\partial y}$$

$$= x e^{xy} \frac{\partial\, xy}{\partial y} + e^{xy}\frac{\partial x}{\partial y}$$

$$= x e^{xy}\, x \cdot 1 + e^{xy}(0)$$

$$= x^2 e^{xy} + 0$$

$$\boxed{\frac{\partial z}{\partial y} = x^2 e^{xy}} \longrightarrow (2)$$

$$\nabla z(x,y) = z_x \hat{i} + z_y \hat{j}$$

$$\boxed{\nabla z(x,y) = e^{xy}(xy+1)\hat{i} + (x^2 e^{xy})\hat{j}}$$

OR

$$\nabla z(x,y) = e^{xy}\left[(xy+1)\hat{i} + (x^2)\hat{j}\right]$$ Ans

Q # 3 $f(x,y) = \tan(3x - 4y)$

Sol: w.r.t x

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\tan(3x - 4y))$$

$$= \sec^2(3x - 4y)\,\frac{\partial}{\partial x}(3x - 4y)$$

$$= \sec^2(3x - 4y) \cdot 3 - 0$$

$$\boxed{\frac{\partial f}{\partial x} = 3\sec^2(3x - 4y)} \longrightarrow (i)$$

w.r.t y

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\tan(3x - 4y))$$

$$= \sec^2(3x - 4y)\,\frac{\partial}{\partial y}(3x - 4y)$$

$$= \sec^2(3x - 4y)\ 0 - 4$$

$$\boxed{\frac{\partial f}{\partial y} = -4\sec^2(3x - 4y)} \longrightarrow (ii)$$

$$\nabla f(x,y) = f_x\hat{i} + f_y\hat{j}$$

$$\boxed{\nabla f(x,y) = 3\sec^2(3x - 4y)\,\hat{i} - 4\sec^2(3x - 4y)\,\hat{j}}$$

Answer

OR

$$\boxed{\nabla f(x,y) = \sec^2(3x-4y)\,(3\hat{i}-4\hat{j})}$$ Ans

Q # 4: $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

Sol:

w.r.t x.

$$f(x,y,z) = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2}$$

$$= \frac{\partial}{\partial x}(x^2+y^2+z^2)^{1/2}$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{1/2-1}\frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{-1/2}\cdot 2x+0+0$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}} \longrightarrow (i)$$

w.r.t y.

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\sqrt{x^2+y^2+z^2}$$

$$= \frac{\partial}{\partial y}(x^2+y^2+z^2)^{1/2}$$

$$= \frac{\partial}{\partial y}$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{-1/2}\frac{\partial}{\partial y}(x^2+y^2+z^2)$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(0+2y+0)$$

$$= \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}(\cancel{2}y)$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}} \longrightarrow (2)$$

w.r.t z.

$$\frac{\partial z}{\partial z} = \frac{\partial}{\partial z}\sqrt{x^2+y^2+z^2}$$

$$= \frac{\partial}{\partial z}(x^2+y^2+z^2)^{1/2}$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{1/2-1}\frac{\partial}{\partial z}(x^2+y^2+z^2)$$

$$\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(0+0+2z)$$

$$\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}(\cancel{2}z)$$

$$\boxed{\frac{\partial z}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}} \longrightarrow (3)$$

$$\nabla z(x,y,z) = z_x\hat{i} + z_y\hat{j} + z_z\hat{k}$$

$$= \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)\hat{i} + \left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)\hat{j} + \left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)\hat{k}$$

$$\boxed{\nabla z(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}$$

Ans

Q # 5 $\quad z(x, y, z) = x\cos(y/z)$

Sol: w.r.t x

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} x\cos(y/z)$$

$$\frac{\partial z}{\partial x} = 1 \cdot \cos(y/z)$$

$$\boxed{\frac{\partial z}{\partial x} = \cos(y/z)} \rightarrow (1)$$

$z(x, y, z) = x\cos(y/z)$

Diff w.r.t y

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y} x\cos(y/z)$$

$$= x(-\sin(y/z)) \frac{\partial}{\partial y}(y/z)$$

$$= -x\sin(y/z) \frac{1}{z} \cdot 1$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-x}{z}\sin(y/z)} \rightarrow (2)$$

written by
Ahmad Ali

$f(x, y, z) = x \cos(y/z)$

Diff w.r.t z

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} x \cos(y/z)$$

$$= x\left(-\sin(y/z)\right) \frac{\partial}{\partial z}(y/z)$$

$$= -x \sin(y/z) \frac{\partial}{\partial z}(y \cdot z^{-1})$$

$$= -x \sin(y/z) \cdot y \; -z^{-2}$$

$$= x \sin(y/z)\, y\, z^{-2}$$

$$= \frac{xy \sin(y/z)}{z^2}$$

$$\boxed{\frac{\partial f}{\partial z} = \frac{xy \sin(y/z)}{z^2}} \longrightarrow (\mathrm{I})$$

$$\nabla f(x, y, z) = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$$

$$= \cos(y/z)\hat{i} - \frac{x}{z} \sin(y/z)\hat{j} + \frac{xy}{z^2} \sin(y/z)\hat{k}$$

$$\boxed{\nabla f(x,y,z) = \cos(y/z)\hat{i} - \frac{x}{z} \sin(y/z)\hat{j} + \frac{xy}{z^2} \sin(y/z)\hat{k}}$$

Answer

D.R
9/4/18

FINAL

LINE INTEGRAL:-

We define an integral that is similar to a single Integral except that instead of Integrating over an interval [a,b], we Integrate over a curve C.

Such Integrals are called line Integrals, although " curve Integrals, would be better terminology.
They were invented in the early 19th century to solve problems involving fluid flow forces electricities magnetism.

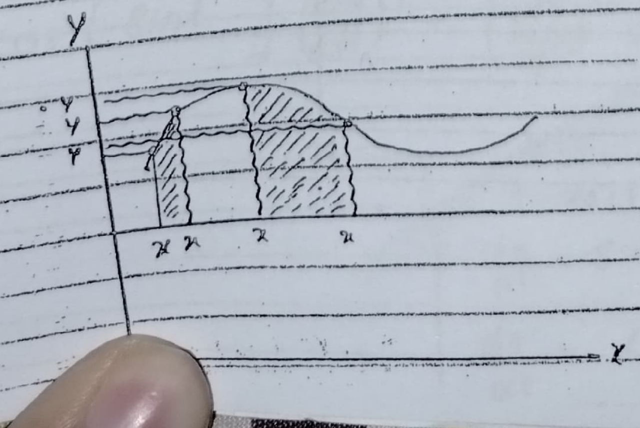
We start with a plane curve c given by the parametric equation.

$x = x(t)$, $y = y(t)$ $\quad a \leq t \leq b$

or

equivalently by the vector equation

$r(t) = x(t)\hat{i} + y(t)\hat{j}$ and we assume that c is a smoth curve.



Definition: if f is defined on a smooth curve c then the line Integral of f along c is

$$\int_c f(x,y)\,ds = \int_a^b f(x(t),\,y(t))\cdot\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$$

Note
S = Length

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$$

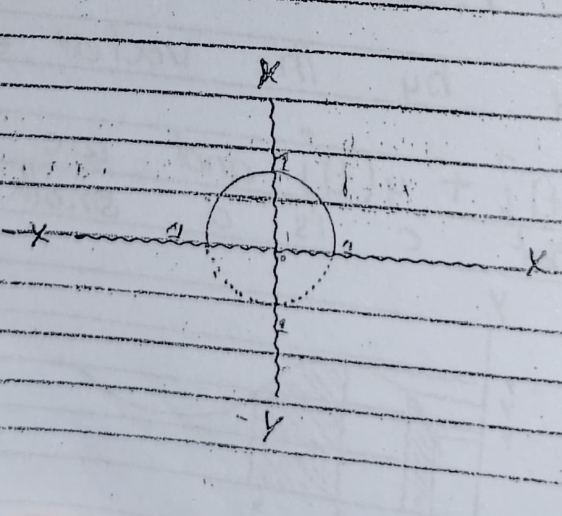
$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\cdot dt$$

Q# 01

Evaluate $\int_c (2+x^2y)\,dx$ where c is the upper half of the unit circle.

$$x^2+y^2=1$$

Note
unit circle
Radius is
one.



3

$x = \cos t, \quad y = \sin t$

$x^2 + y^2 = 1$

$(\cos t)^2 + (\sin t)^2 = 1$

$1 = 1$ Satisfied the given equation.

Sol: Recall that the unit circle can be parametrized by means of the equation.

$x = \cos t, \quad y = \sin t$

and the upper half of the circle is discribed by the parameter Interval.

$0 \leq t \leq \pi$

$2 + x^2 y = 2t + \cos^2 t \cdot \sin t \rightarrow (i)$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\int_a^b f(x)(t), y(t)\, ds = \int_a^b f(x)(t), y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_0^\pi (2 + \cos^2 t \sin t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Note

$x = \cos t, \; y = \sin t$

$\frac{dx}{dt} = -\sin t$

$\frac{dy}{dt} = \cos t$

$$\int_0^{\pi} ((2+\cos^2 t\,\sin t)\sqrt{(-\sin t)^2+(\cos t)^2}\,dt$$

$$\int_0^{\pi} (2+\cos^2 t\cdot \sin t)\,dt$$

$$\int_0^{\pi} 2\,dt - \int_0^{\pi} (-\cos t)^2 + (-\sin t)^2\,dt$$

$$2t\Big|_0^{\pi} - \frac{(\cos t)^3}{3}\Big|_0^{\pi}$$

$$= [2(\pi)-2(0)] - \frac{1}{3}\left[(\cos\pi)^3 - (\cos(0))^3\right]$$

$$= 2\pi - 0 - 1/3\left[(-1)^3 - 1\right]$$

$$2\pi - 1/3[-2]$$

$$\boxed{2\pi + 2/3}$$ Ans

LINE Integral in Space:

We now suppose that c is a smooth space curve given by the parametric equation.

$$x = x(t),\ y = y(t),\ z = z(t) \quad (a \le t \le b)$$

$$\int_c f(x,y,z)\,ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

$$\int_a^b f(r(t)) \left| r'(t) \right| dt$$

$$r(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$r'(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$|r'(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

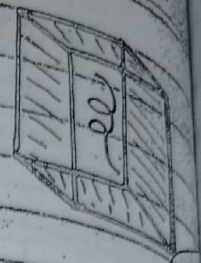
Q# 02 Evaluate Integral

$$\int_c y \sin z\, ds.$$

Where c is circular helix given by the equations.

$$x = \cos t,\quad y = \sin t,\quad z = t$$
$$0 \le t \le 2\pi$$

$f(x,y,z) = y \sin z$



ooth
etric

$f(x(t), y(t), z(t)) = \sin t \cdot \sin t =$

$= \sin^2 t \rightarrow (1)$

$+\left(\frac{dz}{dt}\right)^2 dt$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \rightarrow (A)$$

put the Note value in (A)

$$ds = \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2}$$

$$ds = \sqrt{\sin^2 t + \cos^2 t + 1}$$

$$ds = \sqrt{1 + 1}$$

$$ds = \sqrt{2} \rightarrow (2)$$

$$\int_C f(x,y,z) = \int_0^{2\pi} \sin^2 t \sqrt{2}\, dt$$

$$\sqrt{2} \int_0^{2\pi} \sin^2 t \, dt$$

we use half angle formula

Note

$ds =$ magn

$\frac{dx}{dt} = \cos t$

$\frac{dx}{dt} = -\sin t$

$\frac{dy}{dt} = \sin t$

$\frac{dy}{dt} = \cos t$

$\frac{dz}{dt} = t$

$\frac{dz}{dt} = 1$

$\sin^2 t + \cos^2 t =$

Half angle formula.

$\sin t = \sqrt{\frac{1-\cos}{2}}$

$\sin^2 t = \left(\frac{1-\cos}{2}\right.$

$$= \sqrt{2}\int_0^{2\pi}\left(\frac{1-\cos 2t}{2}\right)dt$$

$$= \frac{\sqrt{2}}{2}\int_0^{2\pi}(1-\cos 2t)\,dt$$

use difference vule.

$$= \frac{\sqrt{2}}{2}\left[\int_0^{2\pi}1\,dt - \int_0^{2\pi}\cos 2t\,dt\right]$$

$$= \frac{\sqrt{2}}{2}\left[t\Big|_0^{2\pi} - \frac{\sin 2t}{2}\Big|_0^{2\pi}\right]$$

$$= \frac{\sqrt{2}}{2}\left[(2\pi - 0) - \frac{1}{2}\left[\sin 2(2\pi) - \sin 2(0)\right]\right]$$

$$= \frac{\sqrt{2}}{2}\left[2\pi\right] - \frac{1}{2}\left[\sin 4\pi - \sin(0)\right]$$

$$= \frac{\sqrt{2}}{\cancel{2}}\left[\cancel{2}\pi\right] - \frac{1}{2}\left[0 - 0\right]$$

$$= \sqrt{2}\pi - \frac{1}{2}[0]$$

$$= \sqrt{2}\pi - 0$$

$$= \boxed{\sqrt{2}\pi \quad \text{Answer}}$$

Note

$\sin 2\pi = 0$

$\sin 4\pi = 0$

$2\pi = 360$

$4\pi = 720$

Ex # 16.2 Page # 1043

1-16 Evaluate the line Integral where C is given curve.

1) $\int_C y^3\, ds \qquad C: x = t^3,\ y = t \quad 0 \le t \le 2$

$$\int_C f(x,y)\, ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$f(x,y) = y^3$

$f(x(t), y(t)) = t^3 \longrightarrow (i)$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Note.

$\frac{dx}{dt} = t^3$

$\frac{dx}{dt} = 3t^2$

$\frac{dy}{dt} = t$

$\frac{dy}{dt} = 1$

put the Note value in ds.

$$ds = \sqrt{(3t^2)^2 + (1)^2}$$

$$ds = \sqrt{9t^4 + 1}$$

$$\int_0^2 (t^3) \cdot (9t^4+1)^{1/2}\, dt$$

$$\int_0^2 (9t^4+1)^{1/2} (t^3)\, dt$$

xing & ÷ing by 36

$$\frac{1}{36}\int_0^2 (9t^4+1)^{1/2}\,(36t^3)\,dt$$

$$\frac{1}{36}\frac{(9t^4+1)^{1/2+1}}{1/2+1}\Big|_0^2$$

$$\frac{1}{36}\frac{(9t^4+1)^{3/2}}{3/2}\Big|_0^2$$

$$\frac{2}{36}\frac{(9t^4+1)^{3/2}}{3}\Big|_0^2$$

18

$$\frac{1}{18}\frac{(9t^4+1)^{3/2}}{3}\Big|_0^2$$

$$\frac{(9t^4+1)^{3/2}}{54}\Big|_0^2$$

$$\frac{1}{54}\left[(9(2)^4+1)^{3/2}-(9(0)^4+1)^{3/2}\right]$$

$$\frac{1}{54}\left[(144+1)^{3/2}-(0+1)^{3/2}\right]$$

$$\frac{1}{54}\left[(145)^{3/2}-(1)^{3/2}\right]$$

$$\boxed{\frac{1}{54}\left[(145)^{3/2}-1\right]}$$ Ans

Note

$$\frac{d}{dt}(9t^4+1)$$
$$=36t^3$$

$$\int g(t)^n\, g'(t)\,dt$$

Note

$\frac{d}{dt}(9t^4+1)$

$= 36t^3$

$f(t)^n \cdot f'(t)\,dt$

Q # 2:

$$\int_c xy\,ds = \quad C: x = t^2,\ y = 2t,\ 0 \le t \le$$

Sol:-

$$\int_c f(x,y)\,ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dx}\right)^2}\,dt$$

$$f(x,y) = xy$$

$$f(x(t), y(t)) = t^2 \cdot 2t$$

$$f(x(t), y(t)) = 2t^3 \longrightarrow (1)$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Note

$\frac{dx}{dt} = t^2$

$\frac{dx}{dt} = 2t$

$\frac{dy}{dt} = 2t$

$\frac{dy}{dt} = 2$

put the Note value in above.

$$ds = \sqrt{(2t)^2 + (2)^2}$$

$$ds = \sqrt{4t^4 + 4}$$

$$ds = \sqrt{4(t^4 + 1)}$$

$$ds = \sqrt{4} \cdot \sqrt{t^4 + 1}$$

$ds = 2\sqrt{t^2+1} \rightarrow (ii)$

put the value of i & ii in "A"

$$\int_0^1 2t^3 \cdot 2\sqrt{t^2+1}\, dt$$

$$2\int_0^1 2t^3 \cdot \sqrt{t^2+1}\, dt$$

$$2\int 2t \cdot t^2 \sqrt{t^2+1}\, dt$$

$$2\int t^2 \sqrt{t^2+1}\; 2t\, dt \rightarrow Z$$

Suppose

$t^2+1 = v \rightarrow 1$

$t^2 = v-1 \rightarrow (2)$

$2t = \frac{dv}{dt} \rightarrow (3)$

$2t\,dt = dv \rightarrow 4$

put all 1, 2, 3 & 4 in Z

Note

$t^2 = v-1$

Derivative both side w.r.t t

$2t = \frac{dv}{dt}$

$2t\,dt = dv$

$$2\int_0^1 (v-1)\cdot v^{1/2}\,dv$$

$$2\int_0^1 v^1\cdot v^{1/2} - v^{1/2}\,dv$$

$$2\int_0^1 v^{1/2+1} - v^{1/2}\,dv$$

$$2\int_0^1 v^{3/2} - v^{1/2}\,dv$$

$$2\left[\int_0^1 v^{3/2}\,dv - \int_0^1 v^{1/2}\,dv\right]$$

$$2\left[\left.\frac{v^{3/2+1}}{3/2+1}\right|_0^1 - \left.\frac{v^{1/2+1}}{1/2+1}\right|_0^1\right]$$

$$2\left[\left.\frac{v^{5/2}}{5/2}\right|_0^1 - \left.\frac{v^{3/2}}{3/2}\right|_0^1\right]$$

$$2\left[\left.\frac{2}{5}v^{5/2}\right|_0^1 - \left.\frac{2}{3}v^{3/2}\right|_0^1\right]$$

$$2\left[\frac{2}{5}(1)^{5/2} - (0)^{5/2} - \frac{2}{3}(1)^{3/2} - (0)^{3/2}\right]$$

Note

$x\cdot x = x$

base same power add.

$\frac{\frac{x}{y}}{z} = x$

$\frac{x^2}{y}$

e

$v-1$

true side.

$+ t$

$= \frac{dv}{dt} - 0$

$= dv$

$$2\left[\frac{2}{5}(1-0)-\frac{2}{3}(1-0)\right]$$

$$2\left[\frac{2}{5}-\frac{2}{3}\right] \quad \text{L.C.M}$$

$$2\left[\frac{6-10}{15}\right]$$

$$2\left[\frac{-4}{15}\right] = \boxed{\frac{-8}{15}}$$

Answer

Q # 3

$$\int_c xyz\, ds = \quad C: x = 2\sin t,\ y = t$$
$$z = -2\cos t,$$
$$0 \le t \le \pi$$

$$\int_c f(x,y,z)\, ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$f(x,y,z) = x.y.z$$

$$= 2\sin t \cdot t \cdot -2\cos t$$

$$\boxed{f(x(t), y(t), z(t)) = -4t\sin t\cos t} \longrightarrow (1)$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \longrightarrow (A)$$

put the Note value in (A)

$$ds = \sqrt{(2\cos t)^2 + (1)^2 + (-2\sin t)^2}$$

$$= \sqrt{4\cos^2 t + 1 + 4\sin^2 t}$$

$$= \sqrt{4\cos^2 t + 4\sin^2 t + 1}$$

$$= \sqrt{4(\cos^2 t + \sin^2 t) + 1}$$

$$= \sqrt{4(\cos^2 t + \sin^2 t) + 1}$$

$$= \sqrt{4(1) + 1}$$

$$= \sqrt{4 + 1}$$

$$\boxed{ds = \sqrt{5}}$$

Note

$\frac{dx}{dt} = 2\sin$

$\frac{dx}{dt} = 2\cos$

$\frac{dy}{dt} = t$

$\frac{dy}{dt} = 1$

$\frac{dz}{dt} = -2\cos$

$\frac{dz}{dt} = +2\sin$

$\cos^2 t + \sin^2 t =$

$$\int_0^{\pi} -4t\, \sin t \cos t \cdot \sqrt{5}$$

$$-4\sqrt{5} \int_0^{\pi} t \sin t \cos t \quad dt$$

xing & ÷ing by 2

$$-\frac{4\sqrt{5}}{2} \int_0^{\pi} t \; 2\sin t \cos t \quad dt$$

$$-2\sqrt{5} \int_0^{\pi} t \sin 2t \quad dt$$

$$-2\sqrt{5}\left[t \int_0^{\pi} \sin 2t \, dt - \int_0^{\pi} \frac{dt}{dt} \int \sin 2t \, dt \; dt \right.$$

Note
$2\sin t \cos t =$
$\sin 2t$

$$-2\sqrt{5}\left[t \left(-\frac{\cos 2t}{2}\right)\Big|_0^{\pi} - \int_0^{\pi} 1 \cdot \left(-\frac{\cos 2t}{2}\right) dt \right.$$

$$-2\sqrt{5}\left[-t\frac{\cos 2t}{2}\Big|_0^{\pi} - \int_0^{\pi} -\frac{\cos 2t}{2} \, dt \right]$$

$$-2\sqrt{5}\left[-t\frac{\cos 2t}{2}\Big|_0^{\pi} + \frac{\sin 2t}{4}\Big|_0^{\pi} \right]$$

$$-2\sqrt{5}\left[\left(\frac{-\pi\cos 2\pi}{2}+\frac{0\cos 2\,0}{2}\right)+\left(\frac{\sin 2\pi}{4}-\sin\right.\right.$$

$$-2\sqrt{5}\left[\left(\frac{-\pi(1)}{2}+0\right)+(0-0)\right]$$

$\cos 2\pi$

$$-2\sqrt{5}\left[\frac{-\pi}{2}+0\right]$$

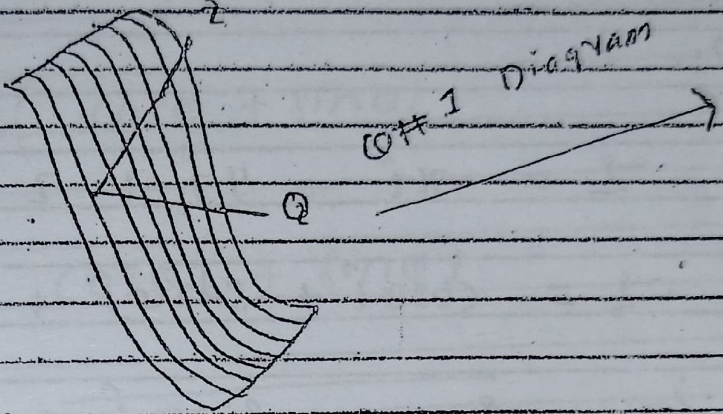
$$-2\sqrt{5}\left(\frac{-\pi}{2}\right)$$

$$+\frac{\cancel{2}\pi\sqrt{5}}{\cancel{2}}$$

$$=\boxed{\pi\sqrt{5}}\ \text{Ans}$$

Surface Integral:

Introduction:→ The relationship Between Surface integrals and Surface area is much the same as the relationship between line Integrals and arc length.



$$\iint_S f(x,y,z)\, dS = \iint_D f(\vec{r}(u,v))\, |\vec{r}_u \times \vec{r}_v|\, dA$$

Where $\vec{r}_u = \frac{\partial x}{\partial u}\hat{i} + \frac{\partial y}{\partial u}\hat{j} + \frac{\partial z}{\partial u}\hat{k}$

$\vec{r}_v = \frac{\partial x}{v}\hat{i} + \frac{\partial}{\partial}\hat{j} + \frac{\partial z}{\partial u}\hat{k}$

where $\vec{r}_u + \vec{r}_v$ $\begin{vmatrix} i & j & k \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}$

Q # 01:-

$\iint_S z\, ds$ where S is the surface whose sides $S1$ are given by the cylinder

$x^2 + y^2 = 1$ perametric equation

$x = \cos\theta, \quad y = \sin\theta \quad z = z$

$0 \leq \theta \leq 2\pi \qquad 0 \leq z \leq 1 + x$

$\qquad\qquad\qquad\qquad 0 \leq z \;\; 1 + \cos\theta$

Sol:-

$r = x\hat{i} + y\hat{j} + z\hat{k}$

$r = \cos\theta\hat{i} + \sin\theta\hat{j} + z\hat{k}$

$\frac{dr}{d\theta} = r_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j} + 0\cdot\hat{k}$

$r = \cos\theta\hat{i} + \sin\theta\hat{j} + z\hat{k}$

$\frac{dr}{dz} = r_z = 0\cdot\hat{i} + 0\cdot\hat{j} + 1\hat{k}$

$$r_\theta \quad r_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$(-1)^{3+2}$ ①

Expand by R_3

$r_\theta \quad r_z = 0$

$$0\begin{vmatrix} 1 & 1 \\ \cos\theta & 0 \end{vmatrix} - 0\begin{vmatrix} \cancel{\sin\theta}\,1 & 1 \\ -\sin\theta & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$0 - 0 + 1(\cos\theta + \sin\theta)$

$r_\theta + r_z = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$|r_\theta + r_z| = \sqrt{(\cos\theta)^2 + (\sin\theta)^2}$

$|r_\theta + r_z| = \sqrt{\cos^2\theta + \sin^2\theta}$

Note $\cos^2\theta + \sin^2\theta = 1$

$\boxed{|r_\theta + r_z| = 1}$

$\iint_S z(x,y,z)\, ds = \iint_D z\, |r_\theta \times r_z|\, dA$

$\int_0^{2\pi}\int_0^{1+\cos\theta} z \cdot 1\, dz\, d\theta$

$$\int_0^{2\pi}\left[\int_0^{1+\cos\theta} z\,dz\right]d\theta$$

$$\int_0^{2\pi} \frac{z^2}{2}\Big|_0^{1+\cos\theta}\, d\theta$$

$$\int_0^{2\pi}\left[\frac{(1+\cos\theta)^2}{2} - \frac{(0)^2}{2}\right]d\theta$$

$$\int_0^{2\pi}\left[\frac{(1+\cos\theta)^2}{2} - 0\right]d\theta$$

$$\int_0^{2\pi}\left[\frac{(1+\cos\theta)^2}{2}\right]d\theta$$

$$\frac{1}{2}\int_0^{2\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta$$

$$\frac{1}{2}\left[\int_0^{2\pi} 1\,d\theta + \int_0^{2\pi} 2\cos\theta\,d\theta + \int_0^{2\pi}\cos^2\theta\,d\theta\right.$$

$$\frac{1}{2}\left[\theta\Big|_0^{2\pi} + 2\sin\theta\Big|_0^{2\pi} + \int_0^{2\pi}\left(\frac{1+\cos 2\theta}{2}\right)\right.$$

$$\frac{1}{2}\left[(2\pi - 0) + (2\sin 2\pi - 2\sin(0)) + \frac{1}{2}\int_0^{2\pi} 1 + \cos 2\theta \, d\theta\right]$$

$$\frac{1}{2}\left[(2\pi) + (0-0) + \frac{1}{2}\left[\int_0^{2\pi} 1\, d\theta + \int_0^{2\pi} \cos 2\theta \, d\theta\right]\right]$$

$$\frac{1}{2}\left[2\pi + \frac{1}{2}\left(\theta\Big|_0^{2\pi} + \frac{\sin 2\theta}{2}\Big|_0^{2\pi}\right)\right]$$

$$\frac{1}{2}\left[2\pi + \frac{1}{2}(2\pi - 0) + \frac{1}{2}(\sin 2(2\pi) - \sin 2(0))\right]$$

$$\frac{1}{2}\left[2\pi + \frac{1}{\cancel{2}}(\cancel{2}\pi) + \frac{1}{2}(0-0)\right]$$

$$\frac{1}{2}\left[2\pi + \pi + 0\right]$$

$$\frac{1}{2}\left[3\pi\right]$$

$$\boxed{\frac{3\pi}{2}}$$ Ans

Q # 2 Compute the surface Integral

$$\iint_S x^2 \, dS, \text{ where } \begin{aligned} x &= \cos\theta \\ y &= \sin\theta \\ z &= z \end{aligned}$$

$$0 \le \theta \le 2\pi$$
$$0 \le z \le 1 + \cos\theta$$

Sol:-

$$r(\theta, z) = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} + z\hat{k}$$

$$\frac{\partial r}{\partial \theta} = r_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} + 0\hat{k}$$

$$\frac{\partial r}{\partial z} = r_z = 0\hat{i} + 0\hat{j} + \hat{k}$$

$$r_\theta \times r_z = \begin{vmatrix} \hat{j} & \hat{i} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \text{exp by } R_3$$

$$r_\theta \times r_z = 0\begin{vmatrix} \hat{i} & \hat{k} \\ \cos\theta & 0 \end{vmatrix} - 0\begin{vmatrix} \hat{i} & \hat{k} \\ -\sin\theta & 1 \end{vmatrix} + 1\begin{vmatrix} \hat{i} & \hat{j} \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$= 0 - 0 + \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$r_\theta \times r_z = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$|r_\theta \times r_z| = \sqrt{\cos^2\theta + \sin^2\theta}$$

$$|r_\theta \times r_z| = \sqrt{1}$$

Note
$\cos^2\theta + \sin^2\theta = 1$

$$\boxed{|r_\theta \times r_z| = 1}$$

$$\iint_S f(x,y,z)\, dS = \iint_D f\, |r_\theta \times r_z|\, dA$$

$$\int_0^{2\pi}\int_0^{1+\cos\theta} x^2 \cdot 1 \, dz\, d\theta$$

$$\int_0^{1+\cos\theta} x^2\, dz$$

Note.
~~I only~~
x^2 or z^2

I called z^2 instead of x^2.

$$\cancel{\frac{z^3}{3}\Big|_0^{1+\cos\theta}}$$

$$\cancel{\left[\frac{(1+\cos\theta)^3}{3} - \frac{(0)^3}{3}\right]}$$

$$= \int_0^{1+\cos\theta} x^2 \, dz$$

$$= x^2 z \Big|_0^{1+\cos\theta}$$

$$= x^2 (1+\cos\theta - 0)$$

$$= x^2 (1+\cos\theta)$$

$$= x^2 + x^2\cos\theta$$

$$= \int_0^{2\pi} x^2 + x^2\cos\theta \, d\theta$$

$$= \int_0^{2\pi} x^2 \, d\theta + \int_0^{2\pi} x^2\cos\theta \, d\theta$$

$$= x^2\theta \Big|_0^{2\pi} + x^2 \sin\theta \Big|_0^{2\pi}$$

$$= x^2(2\pi - 0) + x^2(\sin 2\pi - \sin(0))$$

$$= 2x^2\pi + x^2(0-0)$$

$$= 2x^2\pi + 0$$

$\boxed{2\pi^2 \pi}$ ans

Q # 03 Compute the surface Integrals:

$$\iint_S x^2\, dS \quad \text{where} \quad x = \sin\phi \cos\theta$$

$$y = \sin\phi \sin\theta$$

$$z = \cos\phi$$

$$0 \le \phi \le \pi$$

$$0 \le \theta \le 2\pi.$$

Sol:-

$$r(\phi, \theta) = \sin\phi\cos\theta\,\hat{i} + \sin\phi\sin\theta\,\hat{j} + \cos\phi\,\hat{k}$$

$$\frac{dr}{d\phi} - r(\phi) = \cos\phi\cos\theta\,\hat{i} + \cos\phi\sin\theta\,\hat{j} - \sin\phi\,\hat{k}$$

$$\frac{dr}{d\theta} - r(\theta) = -\sin\phi\sin\theta\,\hat{i} + \sin\phi\cos\theta\,\hat{j} + 0\hat{k}$$

$$|r\phi \times r\theta| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ -\sin\phi\sin\theta & \sin\phi\cos\theta & 0 \end{vmatrix}$$

Exp by R-1

$$\hat{i}\begin{vmatrix} \cos\phi\sin\alpha & -\sin\phi \\ \sin\phi\cos\alpha & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} \cos\phi\cos\alpha & \\ -\sin\phi\sin\alpha & \end{vmatrix}$$

$$+\hat{k}\begin{vmatrix} \cos\phi\cos\alpha & \cos\phi\sin\alpha \\ -\sin\phi\sin\alpha & \sin\phi\cos\alpha \end{vmatrix}$$

$$= \hat{i}\,(0 + \sin^2\phi\cos\alpha) - \hat{j}\,(0 - \sin^2\phi\sin\alpha)$$

$$+\hat{k}\,(\sin\phi\cos\phi\cos^2\alpha + \cos\phi\sin\phi\sin^2\alpha)$$

$$= (\sin^2\phi\cos\alpha) + (\sin^2\phi\sin\alpha) +$$

$+\cos\phi\,\hat{k}$

$\sin\phi\,\hat{k}$

$0\hat{k}$

Exp by

R.I

$$|r_\phi \times r_\theta| = \sin\phi$$

$$\iint_S x^2\, dS = \int_0^{2\pi}\int_0^{\pi} (\sin\phi\cos\theta)^2\, |r_\phi \times r_\theta|\, dA$$

$$\int_0^{2\pi}\int_0^{\pi} \sin^2\phi\cos^2\theta\, \sin\phi \cdot d\phi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi} \sin^3\phi\, \cos^2\theta \, d\phi\, d\theta$$

$$= \int_0^{2\pi} \cos^2\theta \left[\int_0^{\pi} \sin^3\phi\, d\phi\right] d\theta$$

$$= \int_0^{2\pi} \cos^2\theta \left[\int_0^{\pi} \sin\phi \cdot \sin^2\phi\, d\phi\right] d\theta$$

Note: $\sin^3\phi = \sin\phi \cdot \sin^2\phi = \sin\phi(1-\cos^2\phi)$

$$= \int_0^{2\pi} \cos^2\theta \left[\int_0^{\pi} \sin\phi\,(1-\cos^2\phi)\, d\phi\right] d\theta$$

$$\int_0^{2\pi} \cos^2\theta\, d\theta \left[\int_0^{\pi} \sin\phi\, d\phi - \int_0^{\pi} \cos^2\phi \cdot \sin\phi\, d\phi\right]$$

$$\int_0^{2\pi} \left(\frac{1+\cos 2\theta}{2}\right) d\theta \left[-\cos\phi \Big|_0^{\pi} + \frac{\cos^3}{3} \Big|_0^{\pi}\right]$$

dA

$$\frac{1}{2}\left[\int_0^{2\pi} 1\, d\theta + \int_0^{2\pi} \cos 2\theta\, d\theta\right]\left[-[\cos\pi - \cos(0)] + \frac{1}{3}[\cos^3\pi - \cos(0)]\right]$$

$$\left[\frac{1}{2}\left[\theta \Big|_0^{2\pi} + \frac{\sin 2\theta}{2} \Big|_0^{2\pi}\right]\right]\left(-(-1-1) + \tfrac{1}{3}(-1-\right.$$

$$\frac{1}{2}(2\pi - 0 + 0)\left(2 + \tfrac{1}{3}(-2)\right)$$

$$\frac{1}{\cancel{2}}(\cancel{2}\pi)\left(2 + \left(-\tfrac{2}{3}\right)\right)$$

$$\pi\left(2 - \tfrac{2}{3}\right)$$

$^2\phi =$

$\cos^2\phi)$

$$\pi\left(\frac{6-2}{3}\right)$$

θ

$$\pi\left(\frac{4}{3}\right) = \boxed{\frac{4\pi}{3}}$$

$d\phi]$

Note

$\sin^2\phi =$

$(1-\cos^2\phi)$

Week # 6 19/J/2018.

Curl :→

If $F = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field on R^3 and the partial derivatives of P, Q and R all exist, then the curl of F is the vector field on R^3 defined by.

$$\text{curl } F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\hat{j} - \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

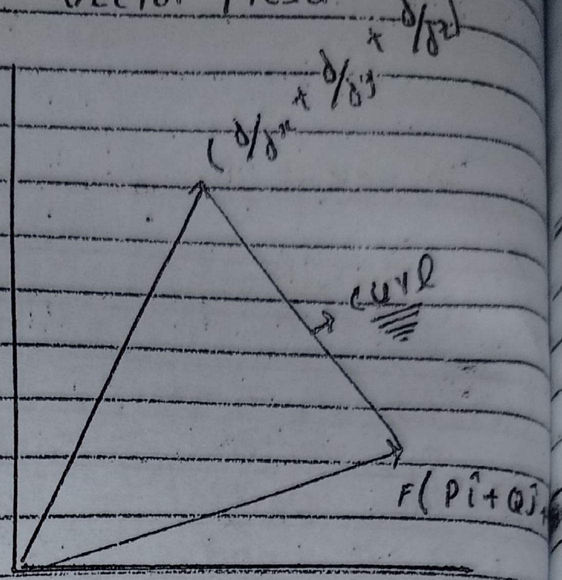
Exp by R_1

$$\text{Curl } F = \vec{\nabla} \times \vec{F} = \hat{i}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - \hat{j}\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)$$

$$+ \hat{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

Note

$R = F = P\hat{i} + Q\hat{j} + R\hat{k}$ = vector Field.



Q#1 if $F(x,y,z) = xz\,\hat{i} + xyz\,\hat{j} - y^2 k$

Find curl F ?

Sol:→ we know that

$$\text{curl } F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & xyz & -y^2 \end{vmatrix}$$ Exp by R1

$$\hat{i}\left(\frac{\partial}{\partial y}(-y^2) - \frac{\partial}{\partial z}(xyz)\right)$$

$$-\hat{j}\left(\frac{\partial}{\partial x}(-y^2) - \frac{\partial}{\partial z}(xz)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial x}(xyz) - \frac{\partial}{\partial y}(xz)\right)$$

$$(-2y - xy)\hat{i} - (0 - x)\hat{j} + (yz - 0)\hat{k}$$

$$\text{curl } F = (-2y - xy)\hat{i} + x\hat{j} + yz\hat{k} \quad \text{ans}$$

Q # 2 Find curl F = ?

$$F(x, y, z) = y^2 z^3 \hat{i} + 2xyz^3 \hat{j} + 3xy^2 z^2 \hat{k}$$

Sol:→

$$\text{curl } F = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3xy^2 z^2 \end{vmatrix} \quad \text{exp by R1}$$

$$= \vec{\nabla} \times \vec{F} = \hat{i}\left(\frac{\partial}{\partial y}(3xy^2z^2) - \frac{\partial}{\partial z}(2xyz^3)\right)$$

$$- \hat{j}\left(\frac{\partial}{\partial x}(3xy^2z^2) - \frac{\partial}{\partial z}(y^2z^3)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial x}(2xyz^3) - \frac{\partial}{\partial y}(y^2z^3)\right)$$

$$\vec{\nabla} \times \vec{F} = \hat{i}(6\cancel{x}yz^2 - 6\cancel{x}yz^2)$$

$$- \hat{j}(3\cancel{y^2}z^2 - \cancel{3}y^2z^2)$$

$$\hat{k}(2\cancel{y}z^3 - \cancel{2}yz^3)$$

Note

$$\boxed{\vec{\nabla} \times \vec{F} = 0\hat{i} + 0\hat{j} + 0\hat{k}}$$

$-0 = 0$

This is called conservative vector field.

Find curl vector $F = ?$ Home work.

1) $F(x,y,z) = xyz\hat{i} + 0\hat{j} - x^2y\hat{k}$
2) $F(x,y,z) = x^2yz\hat{i} + xy^2z\hat{j} + xyz^2\hat{k}$
3) $F(x,y,z) = \hat{i} + (x+yz)\hat{j} + (xy - \sqrt{z})\hat{k}$
4) $F(x,y,z) = \cos xz\,\hat{j} - \sin xy\,\hat{k}$.

Q # 1:→

Sol:→ $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & 0 & -x^2y \end{vmatrix}$ Exp by R

$$= \hat{i}\left(\frac{\partial}{\partial y}(-x^2y) - \frac{\partial}{\partial z}(0)\right)$$

$$- \hat{j}\left(\frac{\partial}{\partial x}(-x^2y) - \frac{\partial}{\partial z}(xyz)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(xyz)\right)$$

$$= (-x^2 - 0)\hat{i} - (-2xy - xy)\hat{j} + (0 - xz)\hat{k}$$

$$\text{Curl } F = -x^2\hat{i} + 3xy\,\hat{j} - xz\,\hat{k}$$ Ans

Q# 2 : $x^2yz\,\hat{i} + xy^2z\,\hat{j} + xyz^2\,\hat{k}$

$$\text{Curl } F = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & xy^2z & xyz^2 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{F} = \hat{i}\left(\frac{\partial}{\partial y}(xyz^2) - \frac{\partial}{\partial z}(xy^2z)\right)$$

$$-\hat{j}\left(\frac{\partial}{\partial x}(xyz^2) - \frac{\partial}{\partial z}(x^2yz)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial x}(xy^2z) - \frac{\partial}{\partial y}(x^2yz)\right)$$

$$\text{Curl } F = (xz^2 - xy^2)\,\hat{i} - (yz^2 - x^2y)\,\hat{j} + (y^2z - x^2z)\,\hat{k}$$

Ans

Q # 3: $F(x,y,z) = \hat{i} + (x+yz)\,\hat{j} + (xy - \sqrt{z})\,\hat{k}$

Find curl F ?

Sol: $\operatorname{curl} F = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & (x+yz) & xy-\sqrt{z} \end{vmatrix}$

$$= \hat{i}\left(\frac{\partial}{\partial y}(xy-\sqrt{z}) - \frac{\partial}{\partial z}(x+yz)\right)$$

$$- \hat{j}\left(\frac{\partial}{\partial x}(xy-\sqrt{z}) - \frac{\partial}{\partial z}(1)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial x}(x+yz) - \frac{\partial}{\partial y}(1)\right)$$

$$\cancel{= \left(y - \frac{1}{2} z^{-1/2} - (0+y)\right)\hat{i}}$$

$$\cancel{(y}$$

$$= \hat{i}\,(x - 0 - 0 + y)$$

$$- \hat{j}\,(y - 0 - 0)$$

$$\hat{k}\,(1 + 0 - 0)$$

$$\operatorname{curl} F = (x+y)\hat{i} - y\hat{j} + \hat{k}$$ Ans

Q# 4 $F(x,y,z) = \cos xz\,\hat{j} - \sin xy\,\hat{k}$

$= 0\hat{i} + (\cos xz)\hat{j} - \sin xy\,\hat{k}$

Sol:

$$\text{Curl } F = \vec{\nabla} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & \cos xz & -\sin xy \end{vmatrix}$$ Exp by R_1

$$\hat{i}\left(\frac{\partial}{\partial y}(-\sin xy) - \frac{\partial}{\partial z}(\cos xz)\right)$$

$$-\hat{j}\left(\frac{\partial}{\partial x}(-\sin xy) - \frac{\partial}{\partial z}(0)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial x}(\cos xz) - \frac{\partial}{\partial y}(0)\right)$$

$$= (-x\cos xy + x\sin xz)\hat{i}$$

$$-(-y\cos xy - 0)\hat{j}$$

$$(-z\sin xz - 0)\hat{k}$$

$$\text{Curl } F = (-x\cos xy + x\sin xz)\hat{i} + y\cos xy\,\hat{j} - z\sin x$$

Ans

Divergence of F

if $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector on IR^3 and

$\frac{\partial P}{\partial x}$, $\frac{\partial Q}{\partial y}$, and $\frac{\partial R}{\partial z}$ exist, then

the divergence of F is the function of three variables defined by

$$\text{div}F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\text{div}\vec{F} = \vec{\nabla}\cdot\vec{F}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(P\hat{i} + Q\hat{j} + R\hat{k}\right)$$

$$\boxed{\vec{\nabla}\cdot\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}}$$

Note

$\hat{i}\cdot\hat{i} = 1$

$\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{i} = 0$

Q#1 Find divergence of F

$F(x,y,z) = xyz\hat{i} + 0\hat{j} - x^2y\hat{k}$

Sol:

$$\text{div}F = \vec{\nabla}\cdot\vec{F}$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(xyz\hat{i} + 0\hat{j} - x^2y\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(-x^2y)$$

$$\boxed{\text{div}F = yz}$$ Ans

Q#02 $F(x,y,z) = y^2z^3\hat{i} + 2xyz^3\hat{j} + 3xy^2z^2\hat{k}$

$\text{Div}\vec{F} = \vec{\nabla}\cdot\vec{F}$

Sol:

$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(y^2z^3\hat{i} + 2xyz^3\hat{j} + 3xy^2z^2\hat{k}\right)$

$= \frac{\partial}{\partial x}(y^2z^3) + \frac{\partial}{\partial y}(2xyz^3) + \frac{\partial}{\partial z}(3xy^2z^2)$

$= 0 + 2xz^3 + 6xy^2z$

$$\boxed{\text{Div}F = 2xz^3 + 6xy^2z}$$

Find diversion = ? Home work.

Q#1) $F(x,y,z) = xyz^2\hat{i} + x^2yz^2\hat{j} + 3xy^2z^2\hat{k}$

Sol:-

$\text{Div}\vec{F} = \vec{\nabla}\cdot\vec{F}$

$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(xyz^2\hat{i} + x^2yz^2\hat{j} + 3xy^2z^2\hat{k}\right)$

$= \frac{\partial}{\partial x}(xyz^2) + \frac{\partial}{\partial y}(x^2yz^2) + \frac{\partial}{\partial z}(3xy^2z^2)$

$= yz^2 + x^2z^2 + 6xy^2z$

$\text{Div}\vec{F} = yz^2 + x^2z^2 + 6xy^2z$ Ans

Q#

So

$= \left(\frac{\partial}{\partial x}\hat{i}\right.$

$= \frac{\partial}{\partial x}($

$= 2$

D

Q#3

Sol

$\left(\frac{\partial}{\partial x}\right.$

=

=

D

Q#2 $= 2xy\hat{i} + (x^2+2yz)\hat{j} + y^2\hat{k}$

Sol:-

$$\text{Div}\vec{F} = \vec{\nabla}\cdot\vec{F}$$

$$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\cdot\left(2xy\hat{i} + [x^2+2yz]\hat{j} + y^2\hat{k}\right)$$

$$= \frac{d}{dx}(2xy) + \frac{d}{dy}(x^2+2yz) + \frac{d}{dz}(y^2)$$

$$= 2y + 0 + 2z + 0$$

$$\text{Div}\vec{F} = 2y + 2z \quad \text{Ans}$$

Q#3 $F(x,y,z) = e^x\hat{i} + \hat{j} + xe^z\hat{k}$

Sol:-

$$\text{Div}\vec{F} = \vec{\nabla}\cdot\vec{F}$$

$$\left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\cdot\left(e^x\hat{i} + \hat{j} + xe^z\hat{k}\right)$$

$$= \frac{d}{dx}e^x + \frac{d}{dy}(1) + \frac{d}{dz}(xe^z)$$

$$= e^x + 0 + x\cdot e^z$$

$$\text{Div}\vec{F} = e^x + xe^z \quad \text{Ans}$$

Q# 4 $F(x,y,z) = ye^{-x}\hat{i} + e^{x}\hat{j} + 2z\hat{k}$

Sol:→

$\text{Div}\vec{F} = \vec{\nabla}\cdot\vec{F}$

$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(ye^{-x}\hat{i} + e^{x}\hat{j} + 2z\hat{k}\right)$

$\frac{\partial}{\partial x}(ye^{-x}) + \frac{\partial}{\partial y}(e^{x}) + \frac{\partial}{\partial z}(2z)$

$= -ye^{-x} + 0 + 2$

$\text{Div}\vec{F} = -ye^{-x} + 2$ Ans

Q# 5 $F(x,y,z) = y\cos xy\hat{i} + x\cos xy\hat{j} - \sin z\hat{k}$

Sol:

$\text{Div}\vec{F} = \vec{\nabla}\cdot\vec{F}$

$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(y\cos xy\hat{i} + x\cos xy\hat{j} - \sin z\hat{k}\right)$

$= \frac{\partial}{\partial x}y(\cos xy) + \frac{\partial}{\partial y}(x\cos xy) + \frac{\partial}{\partial z}(-\sin z)$

$= -y\cdot y\sin xy + -x\cdot x\sin xy - \cos z$

$= -y^2\sin xy - x^2\sin xy - \cos z$

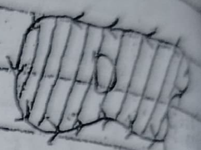
$\text{Div}\vec{F} = -y^2\sin xy - x^2\sin xy - \cos z$

Ans

Green's Theorem

Greens theorem gives the relationship between a line integral around a simple closed curve C and a double Integral over the plane region D bounded C.

Green's theorem: Let C be a positively, piecewise-smooth simple closed curve in the plane and let D be the region bounded by C.



if P and Q have continous partial derivatives on an open region that contain D then.

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$F(x,y) = P(x,y)\hat{i} + Q(x,y)\hat{j}$$

Q # 01 $F(x,y) = (x^2 - y^2)\hat{i} + (2xy)\hat{j}$

$0 \le x \le 1 \qquad 2x^2 \le y \le 2x$

using the green's theorem. Find area under the closed curve C.

Sol:

$$F(x,y) = P(x,y)\hat{i} + Q(x,y)\hat{j}$$

$$F(x,y) = (x^2 - y^2)\hat{i} + 2xy\hat{j}$$

$$P = (x^2 - y^2) \qquad Q = 2xy$$

$$\frac{d}{dy}(x^2 - y^2) \qquad \frac{d}{dx}(2xy)$$

$$\frac{dP}{dy} = 0 - 2y \qquad \frac{dQ}{dx} = 2y$$

$$\frac{dP}{dy} = -2y \qquad \frac{dQ}{dx} = 2y$$

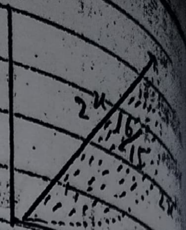
$$\iint_0 \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA = \int_0^1 \int_{2x^2}^{2x} (2y - (-2y)) \, dy\, dx$$

$$= \int_0^1 \int_{2x^2}^{2x} 2y + 2y \; dy\, dx$$

$$= \int_0^1 \left[\int_{2x^2}^{2x} 4y \; dy\right] dx \rightarrow (A)$$

$$= \int_{2x^2}^{2x} 4y \; dy$$

Note



integral used for curve area

$$= \frac{{}^{2}\cancel{4}y^2}{\cancel{2}_{1}}\Bigg|_{2x^2}^{2x}$$

$$= 2\left((2x)^2 - (2x^2)^2\right)$$

$$= 2\left(4x^2 - 4x^4\right)$$

$$= 8x^2 - 8x^4$$ put in "A"

$$= \int_0^1 8x^2 - 8x^4\, dx$$

$$= \int_0^1 8x^2\, dx - \int_0^1 8x^4\, dx$$

$\int dydx$

$$= \frac{8x^3}{3}\Bigg|_0^1 - \frac{8x^5}{5}\Bigg|_0^1$$

$$= \frac{8}{3}\left((1)^3 - (0)^3\right) - \frac{8}{5}\left((1)^5 - (0)^5\right)$$

$$= \frac{8}{3}(1+0) - \frac{8}{5}(1-0)$$

$$= 8/3 - 8/5$$

$$= \frac{40-24}{15} = \boxed{\frac{16}{15}}$$ Ans

Q#02 Using Green's theorem calculate

$F(x,y) = (xy - y^2 + e^{x^2})\hat{i} + (xy + \sin y^3)\hat{j}$

under the boundary condition.

$-1 \le x \le 1$

$x^2 \le y \le 1$

Sol:

$F(x,y) = (xy - y^2 + e^{x^2})\hat{i} + (xy + \sin y^3)\hat{j}$

$= P(x,y)\,\hat{i} + Q(x,y)\,\hat{j}$

$P(x,y) = xy - y^2 + e^{x^2}$

$Q(x,y) = xy + \sin y^3$

$$\frac{\partial P}{\partial y} = \frac{d}{dy}(xy - y^2 + e^{x^2}) = x - 2y + 0$$

$$\frac{dQ}{dx} = \frac{d}{dx}(xy + \sin y^3) = y + 0$$

$$\iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA = \int_{-1}^{1}\int_{x^2}^{1} y - (x - 2y)\, dy\, dx$$

$$\int_{-1}^{1}\int_{x^2}^{1} (y - x + 2y)\, dy\, dx$$

te

$$= \int_{-1}^{1}\int_{x^2}^{1} (3y - x)\, dy\, dx \rightarrow \text{"A"}$$

$$= \int_{-x^2}^{1} (3y - x)\, dy$$

$$= \int_{-x^2}^{1} 3y\,dy - \int_{-x^2}^{1} x\, dy$$

$$= \frac{3y^2}{2}\Big|_{-x^2}^{1} - xy\Big|_{-x^2}^{1}$$

$$= \frac{3}{2}\left((1)^2 - (-x^2)^2\right) - x\left((1) - (-x^2)\right)$$

$$= 3/2\,(1 - x^4) \quad - x(1 + x^2)$$

$$= 3/2 - \frac{3}{2}x^4 - x - x^3$$

$$= -\frac{3}{2}x^4 - x^3 - x + 3/2 \quad \text{put in A}$$

$$= \int_{-1}^{1}\left(-\frac{3}{2}x^4 - x^3 - x + 3/2\right) dx$$

$$\int_{-1}^{1} -\frac{3}{2}x^4\,dx - \int_{-1}^{1} x^3\,dx - \int_{-1}^{1} x\,dx + \int_{-1}^{1} \frac{3}{2}\,dx$$

$$-\frac{3}{2}\frac{x^5}{5}\Big|_{-1}^{1} - \frac{x^4}{4}\Big|_{-1}^{1} - \frac{x^2}{2}\Big|_{-1}^{1} + \frac{3}{2}x\Big|_{-1}^{1}$$

$$-\frac{3}{10}\left((1)^5-(-1)^5\right) - \frac{1}{4}\left((1)^4-(-1)^4\right) - \frac{1}{2}\left((1)^2-(-1)^2\right) + \frac{3}{2}\left(1-(-1)\right)$$

$$-\frac{3}{10}(1+1) - \frac{1}{4}(1-1) - \frac{1}{2}(1-1) + \frac{3}{2}(1+1)$$

$$-\frac{3}{10}(2) - \cancel{\frac{1}{4}(0)} - \cancel{\frac{1}{2}(0)} + \frac{3}{2}(2)$$

$$-\frac{6}{10} + \frac{6}{2}$$

$$= \frac{-12+60}{20}$$

$$= \frac{\cancel{48}^{\,12}}{\cancel{20}_{\,5}} = \boxed{\frac{12}{5}}$$ Ans

Q # 03 Using Green theorem the Double integral Find

$$F(x,y) = -y\hat{i} + x\hat{j}$$ with boundary condition.

$$1 \le x \le 2 \qquad x^2 \le y \le x$$

Sol:-

$$F(x,y) = P(x,y)\hat{i} + Q(x,y)\hat{j}$$

$$F(x,y) = -y\hat{i} + x\hat{j}$$

$$P = -y \qquad Q = x$$

$$\frac{\partial P}{\partial y} = \frac{\partial y}{\partial y} = -1 \qquad \frac{\partial Q}{\partial x} = \frac{\partial x}{\partial x} = 1$$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_1^2 \int_{x^2}^{x} (1-(-1))\, dy\, dx$$

$$= \int_1^2 \left[\int_{x^2}^{x} 2\, dy\right] dx$$

$$= \int_1^2 \left[2y \Big|_{x^2}^{x}\right] dx$$

$$= \int_1^2 (2x - 2x^2)\, dx$$

$$\int_1^2 2x\,dx - \int_1^2 2x^2\,dx$$

$$\left[\frac{\cancel{2}x^2}{\cancel{2}}\right]_1^2 - \left.\frac{2x^3}{3}\right|_1^2$$

$$x^2\Big|_1^2 - \frac{2}{3}x^3\Big|_1^2$$

$$= \left((2)^2 - (1)^2\right) - \frac{2}{3}\left((2)^3 - (1)^3\right)$$

$$= (4-1) - \frac{2}{3}(8-1)$$

$$= (4-1) - 2/3\,(7)$$

$$= 3 - \frac{14}{3}$$

$$= \frac{9-14}{3}$$

$$= \boxed{\frac{-5}{3}}.$$ Ans

Q # 04 Using Green Theorem Find double integral

$$F(x,y) = x^4\hat{i} + xy\hat{j}$$

$$0 \le x \le 1, \quad 0 \le y \le 1-x$$

Sol:-

$$F(x,y) = P(x,y)\hat{i} + Q(x,y)\hat{j}$$

$P = x^4$	$Q = xy$
$\frac{\partial P}{\partial y} = \frac{\partial x^4}{\partial y} = 0$	$\frac{\partial Q}{\partial x} = \frac{\partial xy}{\partial x} = y$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_0^1 \int_0^{1-x} y - 0 \, dy \, dx$$

$$\int_0^1 \left[\int_0^{1-x} y \, dy \right] dx$$

$$\int_0^1 \left[\frac{y^2}{2} \Big|_0^{1-x} \right] dx$$

$$\int_0^1 \left[\frac{1}{2} \left((1-x)^2 - (0)^2 \right) \right] dx$$

$$= \int_0^1 \left[\frac{1}{2}\left(x^2+1-2x\right)\right] dx$$

$$= \frac{1}{2}\int_0^1 \left(x^2+1-2x\right) dx$$

$$= \frac{1}{2}\left[\int_0^1 x^2\,dx + \int_0^1 1\,dx - \int_0^1 2x\,dx\right]$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\Big|_0^1 + x\Big|_0^1 - \frac{\cancel{2}x^2}{\cancel{2}}\Big|_0^1\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}\left((1)^3-(0)^3\right) + (1-0) - \left((1)^2-(0)^2\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}(1-0) + (1) - (1-0)\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + \cancel{1} - \cancel{1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}\right]$$

$$= \boxed{\frac{1}{6}}$$ Ans

Q # 5

$$F(x,y) = (3y - e^{\sin x})\hat{i} + 7x + \sqrt{y^4+1}\,\hat{j}$$

$$0 \le x \le (2x) \text{ or } (2\pi) \rightarrow (36\pi)$$
$$0 \le y \le 3$$ ans

Sol:-

$P = 3y - e^{\sin x}$	$Q = 7x + \sqrt{y^4+1}$
$\frac{dP}{dy} = 3 - 0$	$\frac{dQ}{dx} = 7 + 0$
$\frac{dP}{dy} = 3$	$\frac{dQ}{dx} = 7$

$$\iint_R \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA = \int_0^3 \int_0^{2x} (7 - 3)\, dx\, dy$$

$$= \int_0^3 \int_0^{2x} 4 \, dx \, dy \rightarrow (A)$$

$$= \int_0^{2x} 4 \, dx$$

$$= 4x \Big|_0^{2x}$$

$= 4x(2x - 0)$

$= 8x$ put in (A)

$= \int_0^3 8x \, dy$

$= 8xy \Big|_0^3$

$= 8x(3 - 0)$

$= \boxed{24x}$ Ans

Stokes theorem :- Let S be a piecewise - smooth surface that is bounded by a simple closed piecewise - smooth boundry curve C. Let F be a vector field whose components have continuous partial Derivatives on an open region in $\mathbb{R}^3$ (three dimension) that contain S

$$\int_c F \cdot dr = \iint_c \text{curl } F \cdot ds$$

CUr

ds =

Q #

F

Sol

$$\text{curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P & Q & R \end{vmatrix}$$

$$ds = \left(\frac{dz}{dx} \hat{i} + \frac{dz}{dy} j \right).$$

Q # 1: By using Stoke's theorem. Find Double Integral.

$$F(x,y,z) = -y^2 \hat{i} + x \hat{j} + z^2 \hat{k}$$

$$z = 2 - y \text{ with a unit Circle}$$

$$x^2 + y^2 = 1$$

$$x = r\cos\theta \quad , \quad y = r\sin\theta .$$

Boundry condition: $0 \leq \theta \leq 2\pi$

$0 \leq r \leq 1$

Sol:-

$$\text{Curl } F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y^2 & x & z^2 \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(z^2) - \frac{\partial}{\partial z}x\right) - j\left(\frac{\partial}{\partial x}(z^2) - \frac{\partial}{\partial z}(-y^2)\right)$$

$$+k\left(\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y^2)\right)$$

$$i(0-0) - j(0-0) + k(1+2y)$$

$$\text{curl} F = (0 - 0 + k(1+2y))$$

$$\boxed{\text{curl} F = 1+2y}$$

$$\text{curl} F = 1+2(r\sin\theta)$$

$$\int_C \text{curl} F \cdot ds = \int_0^{2\pi}\int_0^1 (1+2r\sin\theta)\, dr\, d\theta$$

$$= \int_0^{2\pi}\left[\int_0^1 (1+2r\sin\theta)\, dr\right] d\theta \rightarrow (A)$$

$$= \int_0^1 1+2r\sin\theta\, dr$$

$$r\Big|_0^1 + \frac{\cancel{2}r^2\sin\theta}{\cancel{2}}\Big|_0^1$$

$-y^2)]$ $\quad (1-0) + \left((1)^2 - (0)^2\right) \sin\alpha$

$-y^2)]$ $= 1 + \sin\alpha$ put in A

$$\int_0^{2\pi} (1 + \sin\alpha)\, d\alpha$$

$$= \int_0^{2\pi} 1\, d\alpha + \int_0^{2\pi} \sin\alpha\, d\alpha$$

$$= \alpha \Big|_0^{2\pi} + (-\cos\alpha \Big|_0^{2\pi}$$

$) d\nu d\alpha$ $$= \alpha \Big|_0^{2\pi} - \cos\alpha \Big|_0^{2\pi}$$

$$= (2\pi - 0) - (\cos 2\pi - \cos 0)$$

$$= 2\pi - (1 - 1)$$

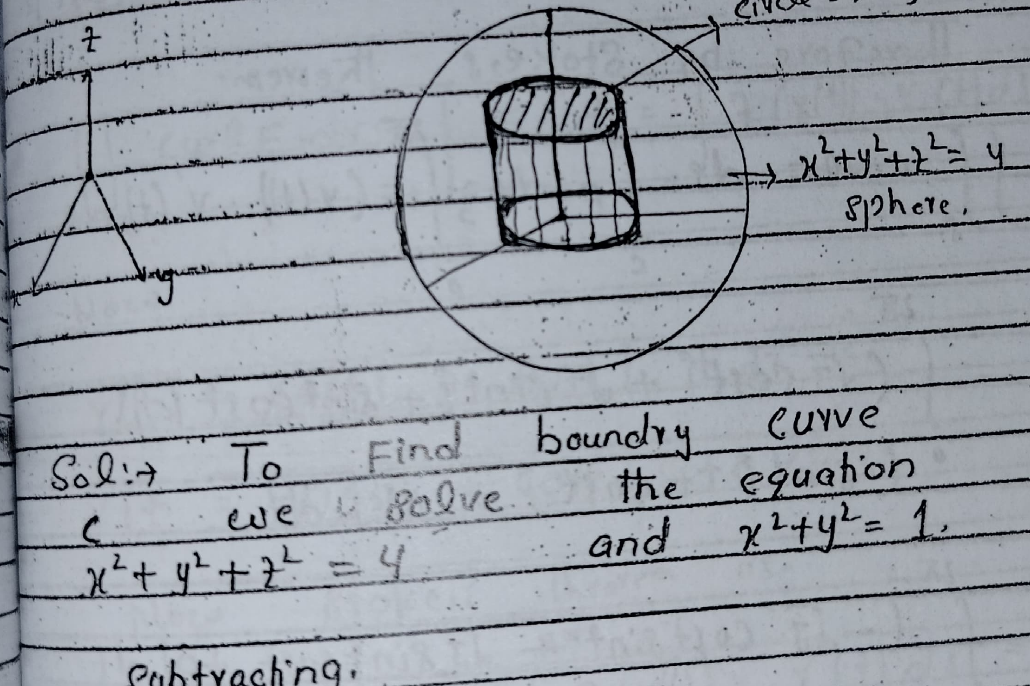
$$= 2\pi - (0)$$

$$= \boxed{2\pi} \text{ Ans}$$

Q# 2:→ Use Stoke,s theorem to compute the Integral $\iint_S \text{curl} F \cdot dS$ where

$F(x,y,z) = xz\hat{i} + yz\hat{j} + xy\hat{k}$

and S is the part of the sphere $x^2+y^2+z^2 = 4$; that lies inside the cylinder $x^2+y^2 = 1$ and above the xy plane.



Sol:→ To Find boundry curve C we solve the equation $x^2+y^2+z^2 = 4$ and $x^2+y^2 = 1$.

Subtracting.

$$\cancel{x^2} + \cancel{y^2} + z^2 = 4$$
$$+\cancel{x^2} \pm \cancel{y^2} = -1$$

$$z^2 = 3$$

Taking square root

$$\sqrt{z^2} = +\sqrt{3}$$

$z = \sqrt{3}, \quad z > 0$

Thus c is the circle given by the equation $x^2+y^2 = 1, \quad z = \sqrt{3}$

A vector equation of c is

$$r(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \sqrt{3}\,\hat{k}$$

$$0 \le t \le 2\pi$$

$$r(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$$

Also we have

$$F(x,y,z) = xz\hat{i} + yz\hat{j} + xy\hat{k}$$

$$x = \cos t, \quad y = \sin t, \quad z = \sqrt{3}$$

$$F(r(t)) = \cos t\sqrt{3}\,\hat{i} + \sin t\sqrt{3}\,\hat{j} + \sin t\cos t\,\hat{k}$$

Therefore by Stoke,s therem.

$$\iint_S \operatorname{curl} F \cdot dS = \int_C F \cdot dr = \int_0^{2\pi} F(r(t)) \cdot r'(t)\, ds$$

$$= \int_0^{2\pi} (\sqrt{3}\cos t\,\hat{i} + \sqrt{3}\sin t\,\hat{j} + \sin t\cos t\,\hat{k}) \cdot (-\sin t\,\hat{i} + \cos t\,\hat{j} + 0\hat{k})\, dt$$

$$= \int_0^{2\pi} (-\sqrt{3}\cos t\sin t + \sqrt{3}\sin t\cos t + 0)\, dt$$

$$= \int_0^{2\pi} 0\, dt = \text{Constant} = 0 \text{ Ans}$$

~~Fourier~~ ~~Series.~~ Home work

Q#3: Using Stoke's theorem.

$$F(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$x = \cos t, \quad y = \sin t, \quad z = \sqrt{2}$$

$$r(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \sqrt{2}\,k$$

$$0 \le t \le 2\pi$$

In This case stock theorem as,

$$\iint_D \text{curl} F \cdot dr = \int_c F \cdot dr = \int F(r(t)) \cdot r'(t)\, ds.$$

Now

$$r(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \sqrt{2}\,\hat{k}.$$

$$r'(t = -\sin t\,\hat{i} + \cos t\,\hat{j} + 0\,\hat{k}$$

Now Stoke's theorem as.

$$\iint_D \text{curl} F\, dr = \int_0^{2\pi} F(r(t)) \cdot r'(t)\, ds$$

$$\int_0^{2\pi} (\cos t\,\hat{i} + \sin t\,\hat{j} + \sqrt{2}) \cdot (-\sin t\,\hat{i} + \cos t\,\hat{j} + 0\,\hat{k})$$

$$= \int_0^{2\pi} -\cancel{\cos t}\sin t + \cancel{\cos t}\sin t + 0\, dt$$

$$\int_0^{2\pi} o \, dt = \text{constant}$$ Answer

Fourier Series:

Fourier series periodic function:

Periodic Functions:→

A function $f(x)$ is said to be periodic if there exists a number $p > 0$ such that.

(t) ds.

$$f(x+p) = f(x)$$

for all x belonging to the domain of difinition of the function.
The least value of $p > 0$ is called The periodic of the function $f(x)$.
For example.

The function $\sin x$ posses the property

$$\sin x = \sin(x+2\pi) = \sin(x+4\pi) \cdots$$

However 2π is the least of the periodic $2\pi, 4\pi, 6\pi$ etc
$\sin x$ is a periodic function with period 2π.

ok]

$$f(x) = \sin x$$

$$f(x+2\pi) = \sin(x+2\pi)$$
$$= \sin x \cos 2\pi + \cos x \sin 2\pi$$

$= \sin x \cdot 1 + \cos x \cdot (0)$

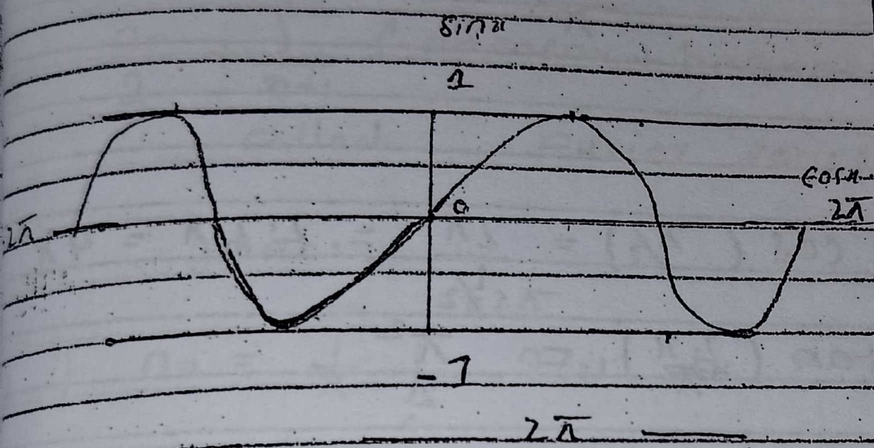
$f(x) = \sin x$. proof.

$f(x+P) = f(x)$

Note
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

$\cos 2\pi = \cos 360 = 1$

$\sin 2\pi = \sin 360^\circ = 0$



Similarly the period of $\cos x$ is 2π and that of $\tan x$ is π

$f(x) = \cos x$

$f(x+2\pi) = \cos(x+2\pi)$

$= \cos x \, \cancel{\sin}\cos 2\pi - \sin x \sin 2\pi$

$= \cos x \cdot 1 - \sin x \, (0)$

$\boxed{f(x+2\pi) = \cos x}$

~~moreover, the period of sin nx~~ or cos nx, where n is a positive integer is $2\pi/n$, that is, the period of sin 2x or cos 2x is π

period

$f(x) = \sin x$, $= 2\pi$.

$f(x) = \sin nx = \frac{2\pi}{n}$

$f(x) = \sin 2x = \frac{2\pi}{2} = \pi$

$f(x) = \sin 10x = \frac{2\pi}{10} = \pi/5$

$f(x) = \sin \pi x = \frac{2\pi}{\pi} = 2$.

cos x :-

$f(x) = 5\cos(x/2) = \frac{2\pi}{1/2} = 2\cdot 2\pi = 4\pi$

$f(x) = \tan\left(\frac{2x}{\pi}\right) = \frac{\pi^2}{2}$

$f(x) = \sin x = 2\pi$

period

$\operatorname{cosec} x = 2\pi$

$\cos x = 2\pi$

$\sec x = 2\pi$

$\tan x = \pi$

$\cot x = \pi$

B) =
B +
B.
cos 760 = 1
sin 760 = 0

Fourier Series:→

Definition:→ A trigonometric infinite series of the form

$$\frac{a_0}{2} + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \cdots a_n\cos nx + \cdots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \cdots b_n\sin nx + \cdots$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$ is called Furier series.

where:

i) $a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx$

ii) $a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx$

iii) $b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx.$

Q # 1

obtain the fourier series representing $f(x) = x$, $-\pi \leq x \leq \pi$

Sol:→

we shall find a_0, a_n, b_n for the fourier series.

$$a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cdot dx$$

te

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \, dx.$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \, dx.$$

$$a_0 = \frac{1}{\pi}\left(\frac{x^2}{2}\Big|_{-\pi}^{\pi}\right)$$

$$a_0 = \frac{1}{2\pi}(x^2)\Big|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left((\pi)^2 - (-\pi)^2\right)$$

$$a_0 = \frac{1}{2\pi}\left(\cancel{\pi^2} - \cancel{\pi^2}\right)$$

$$a_0 = \frac{1}{2\pi}(0)$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx \, dx.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{-\pi+2\pi} x \cdot \cos nx \, dx.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cos nx \, dx$$ (integration by parts)

$$a_n = \frac{1}{\pi}\left[x \int_{-\pi}^{\pi} \cos nx \, dx - \int \left(\frac{d(x)}{dx} \cdot \int \cos nx \, dx \right) dx \right]$$

$$a_n = \frac{1}{\pi}\left[\frac{x \sin nx}{n} \Big|_{-\pi}^{\pi} - \int 1 \cdot \frac{\sin nx}{n} dx \right.$$

$$a_n = \frac{1}{\pi}\left[\frac{x \sin nx}{n} \Big|_{-\pi}^{\pi} \right] - \int_{-\pi}^{\pi} \frac{\sin nx}{n} dx$$

$$a_n = \frac{1}{\pi}\left[\frac{x \sin nx}{n} \Big|_{-\pi}^{\pi} \right] + \frac{\cos nx}{n^2} \Big|_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\pi \sin n\pi}{n} - \left(\frac{-\pi \sin n(-\pi)}{n} \right) \right] + \frac{\cos n\pi}{n^2} - \frac{\cos n(-\pi)}{n^2}$$

$$a_n = \frac{1}{\pi}\left[0 - 0 \right] + \left[\cancel{\frac{\cos n\pi}{n^2}} - \cancel{\frac{\cos n\pi}{n^2}} \right]$$

$$a_n = \frac{1}{\pi}[0] + \boxed{0}$$

$$a_n = \frac{1}{\pi}[0]$$

$$\boxed{a_n = 0}$$

Note

$\sin n\pi = 0$

$\sin \pi = 0$

$\cos(-x) = x$

$\sin(-x) = -x$

$\int \cos x \, dx = \sin x$

$\int \sin x \, dx = -\cos x$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin nx \, dx.$$ (integration by part)

$$b_n = \frac{1}{\pi}\left[x \int_{-\pi}^{\pi} \sin nx \, dx - \int \left(\frac{d(x)}{dx} \cdot \int \sin nx \, dx \right) dx \right.$$

$$b_n = \frac{1}{\pi}\left[x \left(-\frac{\cos nx}{n}\right) \Big|_{-\pi}^{\pi} - \int 1 \cdot \left(-\frac{\cos nx}{n}\right) dx \right.$$

$$b_n = \frac{1}{\pi}\left[-\frac{x\cos nx}{n} \Big|_{-\pi}^{\pi} \right] + \int \frac{\cos nx}{n}\, dx$$

$$b_n = \frac{1}{\pi}\left[-\frac{x\cos nx}{n} \Big|_{-\pi}^{\pi} \right] + \left[\frac{\sin nx}{n^2} \Big|_{-\pi}^{\pi} \right]$$

$$b_n = -\frac{1}{\pi n}\left[\pi\cos n\pi - (-\pi\cos(-n\pi)) \right] + \frac{\sin n\pi}{n^2} - \frac{\sin n(-\pi)}{n^2}$$

$$b_n = -\frac{1}{\pi n}\left[\pi\cos n\pi + \pi\cos n\pi \right] + \left[0 + 0 \right]$$

$$b_n = -\frac{1}{\cancel{\pi} n}\left[2\cancel{\pi}\cos n\pi \right]$$

$$b_n = -\frac{1}{n}(2\cos n\pi)$$

$$b_n = -\frac{1}{n}\left(2(-1)^n\right)$$

$$\boxed{b_n = -\frac{2}{n}(-1)^n}$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$\frac{0}{2} + \sum_{n=1}^{\infty} \left(0 \cos nx + \frac{-2}{n}(-1)^n \sin nx\right)$$

$$= \sum_{n=1}^{\infty} \frac{-2}{n}(-1)^n \sin nx$$

$$= \frac{-2}{1}(-1)^1 \sin 1x + \frac{-\cancel{2}^{1}}{\cancel{2}_{1}}(-1)^2 \sin 2x + \frac{-2}{3}(-1)^3 \sin 3x$$

$$= \boxed{2 \sin x - \sin 2x + \tfrac{2}{3} \sin 3x + \cdots}$$

Ans

Q # 02

Find Fourier series for the function:

$$f(x) = 1 \qquad 0 \le x \le \pi$$

Sol:- $a_0 = ?$ $a_n = ?$ & $b_n = ?$

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)\, dx.$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\ dx$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} 1\ dx$$

$$a_0 = \frac{1}{\pi}\, x \Big|_0^{\pi}$$

$$a_0 = \frac{1}{\cancel{\pi}}(\cancel{\pi} - 0)$$

$$\boxed{a_0 = 1}$$

$(-1)^3 \sin 3x$

$$a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cdot \cos nx\ dx$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} 1\cdot \cos nx\ dx$$

$$a_n = \frac{1}{\pi}\ \frac{\sin nx}{n}\Big|_0^{\pi}$$

$$a_n = \frac{1}{n\pi}(\sin nx)\Big|_0^{\pi}$$

$$a_n = \frac{1}{n\pi}(\sin n(\pi) - \sin n(0))$$

$$a_n = \frac{1}{n\pi}(0 - 0)$$

$$a_n = \frac{1}{n\pi}(0)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \cdot \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} 1 \cdot \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left(\frac{-\cos nx}{n} \right) \Big|_0^{\pi}$$

$$b_n = \frac{-1}{n\pi} \left(\cos nx \right) \Big|_0^{\pi}$$

$$b_n = -\frac{1}{n\pi} \left(\cos n(\pi) - \cos n(0) \right)$$

$$\boxed{b_n = -\frac{1}{n\pi} \left((-1)^n - 1 \right)}$$

Formula for Furier series.

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$\frac{1}{2} + \sum_{n=1}^{\infty} 0 \cos nx + \left(-\frac{1}{\pi n} \left((-1)^n - 1 \right) \sin \right.$$

$$\frac{1}{2} + \sum_{n=1}^{\infty} -\frac{1}{\pi n} \left((-1)^n - 1 \right) \sin nx$$

$\frac{1}{2} + \left(\frac{-1}{\pi(1)} \left((-1)^{1} - 1 \right) \sin 1x + \frac{-1}{2\pi} \left((-1)^{2} - 1 \right) \sin 2x + \right.$

$\left(-\frac{1}{3\pi} \left((-1)^{3} - 1 \right) \sin 3x + \left(\frac{-1}{4\pi} \left((-1)^{4} - 1 \right) \sin 4x \right.\right.$

$\frac{1}{2} + \frac{2}{\pi} \sin x + 0 + \frac{2}{3\pi} \sin 3x + 0 + \frac{2}{5\pi} \sin 5x + \ldots$

$$\boxed{\frac{1}{2} + \frac{2}{\pi} \sin x + \frac{2}{3\pi} \sin 3x + \frac{2}{5\pi} \sin 5x + \cdots}$$

Ans

$\sin nx$

wee#08

Function of any period p-2L

$f(x) = x \quad 0 \leq x \leq 2\pi$

$f(x) = x, \quad 0 \leq x \leq 2$

Let $f(x)$ be a function with a periodic $2l$.
defined on the interval $-l \leq x \leq l$, where l is any positive number and

$$x = \frac{lt}{\pi} \Rightarrow t = \frac{x\pi}{l}$$

Then $g(t+2\pi) = f[\frac{l}{\pi}(t+2\pi)] =$

$f(\frac{lt}{\pi}) = g(t)$ is called period function. Hence the fourier series becomes as.

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

with the fourier coefficient

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(t).\cos nt \; dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(t).\sin nt \cdot dt$$

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n x \pi}{l}\right) + b_n \sin\left(\frac{n x \pi}{l}\right) \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(t) \cdot \cos nt \cdot dt$$

$$a_n = \frac{1}{\cancel{\pi}} \int_{-l}^{l} g\left(\frac{x\pi}{l}\right) \cdot \cos\left(\frac{n x \pi}{l}\right) \cdot \frac{\cancel{\pi}}{l}\, dx$$

any

$$a_n = \frac{1}{l} \int_{-l}^{l} g\left(\frac{x\pi}{l}\right) \cos\left(\frac{n\pi x}{l}\right) dx$$

=

$$b_n = \frac{1}{l} \int_{-l}^{l} g\left(\frac{x\pi}{l}\right) \sin\left(\frac{n\pi x}{l}\right) dx$$

put l instead of π in boundry.

$$a_n = \frac{1}{l} \int_{0}^{2l} g\left(\frac{x\pi}{l}\right) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l} \int_{0}^{2l} g\left(\frac{x\pi}{l}\right) \cdot \sin\left(\frac{n\pi x}{l}\right) dx.$$

Note.

$$t = \frac{x\pi}{l}$$

$$\frac{dt}{dx} - \frac{x\pi}{l}$$

$$\frac{dt}{dx} = \pi/l$$

$$dt = \frac{\pi}{l} dx$$

Q # 7

Expand into Fourier series.

$f(x) = x \qquad 0 \leq x \leq 1$

$0 \leq x \leq 2l$

$2l = 1$

$l = 1/2$

$$a_0 = \frac{1}{l} \int_0^{2l} f(x)\, dx$$

$$\frac{1}{1/2} \int_0^{2(1/2)} x \, dx$$

$$= 2 \int_0^1 x \, dx$$

$$= \frac{\cancel{2} x^2}{\cancel{2}} \Big|_0^1$$

$$= \left[(1)^2 - (0)^2 \right]$$

$$\boxed{a_0 = 1}$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$$

$$a_n = \frac{1}{1/2} \int_0^1 x \cos\left(\frac{n\pi x}{1/2}\right) dx$$

ies.

$$a_n = 2\int_0^1 x\cos(2n\pi x)\,dx$$

$$a_n = 2\left[x\int_0^1 \cos(2\pi n x)\,dx - \int_0^1\left(\frac{dx}{dx}\cdot\int\cos(2n\pi x)\,dx\right)dx\right]$$

$$a_n = 2\left[\frac{x\sin(2n\pi x)}{2n\pi}\Big|_0^1\right] - \int_0^1 \frac{\sin 2n\pi x}{2n\pi}\,dx$$

$$a_n = 2\left[\frac{x\sin(2n\pi x)}{2\pi n}\Big|_0^1 + \frac{\cos 2n\pi x}{4n^2\pi^2}\Big|_0^1\right]$$

$$a_n = 2\left[\frac{1\sin(2n\pi 1)}{2\pi n} - \frac{(0)\sin(2n\pi 0)}{2\pi n} + \frac{\cos 2n\pi(1)}{4n^2\pi^2} - \frac{\cos 2n\pi(0)}{4n^2\pi^2}\right]$$

$$a_n\ 2\left[0 - 0 + \frac{1}{4n^2\pi^2} - \frac{1}{4n^2\pi^2}\right]$$

$$a_n = 2\left[\cancel{\frac{1}{4n^2\pi^2}} - \cancel{\frac{1}{4n^2\pi^2}}\right]$$

$$a_n = 2\,[\,0\,]$$

$$\boxed{a_n = 0}$$

$$bn = \frac{1}{l}\int_0^{2l} f(x) \cdot \sin\left(\frac{n\pi x}{l}\right) dx$$

$$bn = \frac{1}{1/2}\int_0^{1} x \sin\left(\frac{n\pi x}{l}\right) dx$$

$$bn = 2\int_0^{1} x \sin\left(\frac{2n\pi x}{l}\right) dx.$$

$$= 2\left[x \int_0^1 \sin(2n\pi x)\, dx - \int_0^1 \frac{d(x)}{dx} \cdot \int \sin(2n\pi x)\right.$$

$$= 2\left[x\left(\frac{-\cos(2n\pi x)}{2n\pi x}\right)\Big|_0^1 - \int_0^1 1 \cdot \frac{-\cos 2n\pi x}{2n\pi}\right.$$

$$= -2\left[\frac{x\cos(2n\pi x)}{2n\pi}\Big|_0^1 + \frac{\sin(2n\pi x)}{4n^2\pi^2}\Big|_0^1\right]$$

$$= -2\left[\frac{x\cos(2n\pi x)}{2n\pi}\Big|_0^1 + 0\right]$$

$$= -2\left[\frac{x\cos(2n\pi x)}{2n\pi}\Big|_0^1\right]$$

$$= -2\left[\frac{1\cos 2n\pi \times 1}{2n\pi} - \frac{(0)\cos 2n\pi(0)}{2n\pi}\right]$$

$$= -\cancel{2}\left[\frac{1}{\cancel{2}n\pi} - 0\right]$$

$$\boxed{b_n = \frac{-1}{n\pi}}$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n\left(\sin\left(\frac{n\pi x}{l}\right)\right.$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} (0)\cos\left(\frac{n\pi x}{l}\right) + \frac{-1}{n\pi} \sin(2n\pi x)$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} 0 + -\frac{1}{n\pi} \sin(2n\pi x)$$

$$= \frac{1}{2} - \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin(2n\pi x)$$

$$\boxed{= \frac{1}{2} - \frac{1}{\pi}\sin(2\pi x) - \frac{1}{2\pi}\sin(4\pi x) + \cdots}$$

Ans

week # 9:

Even Function:

A function $f(x)$ is said to be Even function if

$$f(-x) = f(x)$$

For exampl:

1) $f(-x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$ Even function.

2) $f(x) = \cos x$

$f(-x) = \cos(-x)$

$f(-x) = \cos x$

3) $f(x) = x^2 - 1$

4) $f(-x) = \sec x$

$f(-x) = \sec(-x)$

$f(-x) = \sec x$

Odd Function:

A function $f(x)$ is said to be odd function

if $f(-x) = -f(x)$

For example.

1) $f(x) = x$

$f(-x) = -x$ odd function.

2) $f(x) = x^3$

$f(-x) = (-x)^3$

$f(-x) = -x^3$

function. 3) $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$f(-x) = -x^3 - x$

4) $f(x) = x^2 + x$

$f(-x) = (-x)^2 + (-x)$

$f(-x) = +x^2 - x$ neither function.

Fourier series of even and odd function:

we know that Fourier series when $(-l, l)$ is the interval then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$$

Even case

Case: I when $f(x)$ is an even, then.

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx = \frac{1}{l}\int_{0}^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{0}^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{0}^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx = 0$$

Odd case:

when $f(x)$ is an odd, then

$$a_0 = \frac{1}{l}\int_{0}^{2l} f(x) = 0$$

$$a_n = \frac{1}{l}\int_{0}^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx = 0$$

$$b_n = \frac{1}{l}\int_{0}^{2l} f(x)\cdot\sin\left(\frac{n\pi x}{l}\right)dx$$

Even & odd Function:

Express the Function:

$f(x) = x, \quad -\pi \le x \le \pi$

Sol:→

$f(x) = x$

$f(-x) = -x$

$f(-x) = -f(x)$ Here it is odd function

Condition:

$a_0 = 0$

$a_n = 0$

$b_n \neq 0$

$$b_n = \frac{1}{l}\int_0^{2l} f(x).\sin\left(\frac{n\pi x}{l}\right) dx.$$

$$b_n = \frac{1}{\pi}\int_0^{2l} x.\sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right) dx$$

$$b_n = \frac{1}{\pi}\int_0^{2l} x.\sin nx \, dx. \rightarrow (A)$$

$\frac{d}{dx}$	$\int$	
x	$\sin nx$	
1	$-\frac{\cos nx}{n}$	$(+)$
0	$-\frac{\sin nx}{n^2}$	$(-)$

put in (A)

$$bn = \frac{1}{\pi}\left[-\frac{x\cos nx}{n}\Big|_0^{2\pi} + \frac{\sin nx}{n}\Big|_0^{2\pi} \right]$$

$$\frac{1}{\pi}\left[-\frac{2\cancel{\pi}\cos 2n\pi}{n} - 0 \right] + 0$$

$$\left[-\frac{2(1)}{n} - 0 \right]$$

$$\boxed{bn = -\frac{2}{n}}$$

put in a Fourier Series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} bn \sin\left(\frac{n\pi x}{l}\right)$$

$$= 0 + \sum_{n=1}^{\infty} -\frac{2}{n} \sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right)$$

$$= \sum_{n=1}^{\infty} -\frac{2}{n} \sin(nx)$$

$$\boxed{= -2\sin x - \sin 2x - \frac{2}{3}\sin 3x + \cdots}$$

Ans

Now ODD Function:

$$f(x) = x^3 \qquad -\pi \le x \le \pi$$

Sol:-

$$f(-x) = (-x)^3$$

$$f(-x) = -x^3 \quad \text{odd Function}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\,\sin\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^3 \cdot \sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right)dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^3 \cdot \sin(nx)\;dx \rightarrow (A)$$

$\frac{d}{dx}$	$\int$
x^3	$\sin nx$
$3x^2$	$-\frac{\cos nx}{n}$
$6x$	$-\frac{\sin nx}{n^2}$
6	$\frac{\cos nx}{n^3}$
0	$\frac{\sin nx}{n^4}$

put in (A)

(excellent)$^{\infty}$ ⓚ

$$b_n = \frac{1}{\pi}\left[-\frac{x^3 \cos nx}{n}\Big|_0^{2\pi} - \frac{3x^2 \sin nx}{n^2}\Big|_0^{2\pi} - \frac{6x \cos nx}{n^3}\Big|_0^{2\pi} + \frac{6 \sin nx}{n^4}\Big|_0^{2\pi} \right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{x^3 \cos nx}{n}\Big|_0^{2\pi} - 0 - \frac{6x \cos nx}{n^3}\Big|_0^{2\pi} + 0 \right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{x^3 \cos nx}{n}\Big|_0^{2\pi} - \frac{6x \cos nx}{n^3}\Big|_0^{2\pi} \right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{(2\pi)^3 \cos n(2\pi)}{n} - 0 - \frac{6(2\pi) \cos n(2\pi)}{n^3} - 0 \right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{8\pi^3 (1)}{n} - \frac{12\pi (1)}{n^3} \right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{8\pi^3}{n} - \frac{12\pi}{n^3} \right]$$

$$\frac{1}{\pi}\left[\frac{-8n^2\pi^3 - 12\pi}{n^3} \right]$$

$$b_n = \frac{1}{\cancel{\pi} n^3}\left[\cancel{\pi}\left(-8n^2\pi^2 - 12 \right) \right]$$

$$bn = \frac{1}{n^3}\left[-8n^2\pi^2 - 12\right]$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} bn \sin\left(\frac{n\pi x}{l}\right)$$

$$= 0 + \sum_{n=1}^{\infty} bn \sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right)$$

$$+ \sum_{n=1}^{\infty} bn \sin(nx)$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3}(-8n^2\pi^2 - 12) \sin(nx)$$

$$\frac{1}{(1)^3}(-8(1)^2\pi^2 - 12) \sin(1)x + \frac{1}{2^3}(-8(2)^2\pi^2 - 12) \sin 2x$$

$$(-8\pi^2 - 12) \sin x + \frac{1}{8}(-32\pi^2 - 12) \sin 2x + \dots$$

$$\boxed{(-8\pi^2 - 12) \sin x + \frac{1}{8}(-32\pi^2 - 12) \sin 2x + \cdots}$$

For Even Function:

$f(x) = \cos x \rightarrow$ Even Function:

$-\pi/2 < x < \pi/2$

$-l < x < l$

$$\boxed{l = \pi/2}$$

$a_0 = ?$
$a_n = ?$

Sol:

$$a_0 = 1/l \int_0^{2l} f(x)\, dx$$

$$a_0 = \frac{1}{\pi/2} \int_{-\pi/2}^{\pi/2} \cos x \, dx$$

$$a_0 = \frac{2}{\pi} \cdot \sin x \Big|_{-\pi/2}^{\pi/2}$$

$$a_0 = \frac{2}{\pi} \left[\sin(\pi/2) - \sin(-\pi/2) \right]$$

Note

$\sin(-x) = -\sin x$

$$a_0 = \frac{2}{\pi} [1 + 1]$$

$$a_0 = \frac{2}{\pi} [2]$$

$$\boxed{a_0 = 4/\pi}$$

an
an =
an =
an =
an =
an =

$$a_n = \frac{1}{l} \int_{l}^{l} f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$$

n:

$$a_n = \frac{1}{\pi/2} \int_{-\pi/2}^{\pi/2} \cos x \cdot \cos\left(\frac{n\cancel{\pi} x}{\cancel{\pi}/2}\right) dx$$

$$a_n = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \cos x \cdot \cos(2nx) \, dx$$

$$a_n = \frac{\cancel{2}}{\pi} \cdot \frac{1}{\cancel{2}} \int_{\pi/2}^{\pi/2} \cos(x + 2nx) + \cos(x - 2nx) \, dx$$

Note

$\cos A \cdot \cos B =$

$\frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$

$$a_n = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(x + 2nx) + \cos(x - 2nx) \, dx$$

Take common x

$$a_n = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(1+2n)x + \cos(1-2n)x \, dx$$

n

$$a_n = \frac{1}{\pi}\left[\frac{\sin(1+2n)x}{1+2n}\Bigg|_{-\pi/2}^{\pi/2} - \frac{\sin(1-2n)x}{1-2n}\Bigg|_{-\pi/2}^{\pi/2} \right]$$

$$a_n = \frac{1}{\pi}\left[\frac{\sin(1+2n)(\pi/2)}{1+2n} - \frac{\sin(1+2n)(-\pi/2)}{1+2n} + \frac{\sin(1-2n)(\pi/2)}{1-2n} - \frac{\sin(1-2n)(-\pi/2)}{(1-2n)} \right]$$

<u>Note</u> $1+2n = \text{odd}$ $\quad 1-2n = -\text{odd}$

$$a_n = \frac{1}{\pi}\left(-\frac{1}{1+2n} - \frac{1}{1+2n} + \cancel{\frac{1}{1-2n}} - \cancel{\frac{1}{1-2n}} \right)$$

$$a_n = \frac{1}{\pi}\left(\frac{-2}{1+2n} \right)$$

$$\boxed{a_n = \frac{-2}{\pi(1+2n)}}$$ <u>Ans</u>

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$\frac{4/\pi}{2} + \sum_{n=1}^{\infty} \frac{-2}{\pi(1+2n)} \cos\left(\frac{n\cancel{\pi} x}{\cancel{\pi}/2}\right)$$

$$\frac{8}{\pi} + \sum_{n=1}^{\infty} \frac{-2}{\pi(1+2n)} \cos(2nx)$$

$$\frac{8}{\pi} + \sum_{n=1}^{\infty} \frac{-2}{\pi(1+2(1))} \cos(2x) + \frac{2}{\pi(1+2(2))} \cos(2(2)x) + \cdots$$

$$\frac{8}{\pi} - \frac{2}{\pi(3)} \cos 2x - \frac{2}{\pi(5)} \cos 4x + \cdots$$

$$\boxed{\frac{8}{\pi} - \frac{2}{3\pi} \cos 2x - \frac{2}{5\pi} \cos 4x + \cdots}$$

Half Range Series

Fourier Sine Series

1) The series $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

defined in the half Range interval $0 < x < l$ is called (x, y).

Here $b_n = \frac{2}{l}\int_0^l f(x) \cdot \sin\left(\frac{n\pi x}{l}\right) dx$

Fourier Cosine Series :-

The Series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$

defined in the half Range interval $0 < x < l$ is called Fourier Cosine Series

Here

$a_n = \frac{2}{l}\int_0^l f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$

By Using half Range:-

Fourier Sine Series is

$$f(x) = k \quad , \quad 0 \leq x \leq \pi$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} k \cdot \sin\left(\frac{n\cancel{\pi} x}{\cancel{l}}\right) dx$$

$$= \frac{2k}{\pi}\int_0^{\pi} \sin(nx)\, dx$$

$$= \frac{2k}{\pi}\left[-\frac{\cos nx}{n}\Big|_0^{\pi} \right]$$

$$= -\frac{2k}{n\pi}\left[\cos(n\pi) - \cos(0)\right]$$

$$= \begin{cases} \dfrac{4k}{\pi n} & n = \text{odd} \\ 0 & n = \text{even} \end{cases}$$

OR

$$= -\frac{2k}{\pi n}\left((-1)^n - 1\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{4k}{\pi n} \sin(nx)$$

$$\boxed{= \frac{4k}{\pi} \sin x + \frac{4k}{3\pi} \sin 3x + \cdots}$$

Q # 2

obtain a half Range

cosine series for

$f(x) = 2x - 1, \quad 0 \leq x \leq 1$

Sol:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{l} \int_0^l f(x)\, dx$$

$$a_0 = \frac{2}{1} \int_0^1 (2x - 1)\, dx$$

$$a_0 = 2 \left[\frac{\cancel{2}x^2}{\cancel{2}} \Big|_0^1 - x \Big|_0^1 \right]$$

$$a_0 = 2 \left[x^2 \Big|_0^1 - x \Big|_0^1 \right]$$

$$a_0 = 2\left[1^2 - 0^2 - 1 - 0\right]$$

$$a_0 = 2\left[1 - 0 - 1 - 0\right]$$

$$a_0 = 2\left[\cancel{1} - \cancel{1}\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cdot\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{1}\int_0^1 (2x-1)\cdot\cos\left(\frac{n\pi x}{1}\right)dx$$

$$a_n = 2\int_0^1 (2x-1)\cdot\cos(n\pi x)\,dx$$

Second method for Integration by part

	$\frac{d}{dx}$	$\int$
+	$2x-1$	$\cos(n\pi x)$
−	$2 - 0$	$\frac{\sin(n\pi x)}{n\pi}$
+	0	$\frac{-\cos(n\pi x)}{n^2\pi^2}$

$$a_n = 2\left[(2x-1)\left(\frac{\sin(n\pi x)}{n\pi}\right)\Big|_0^1 + \frac{2\cos(n\pi x)}{n^2\pi^2}\Big|_0^1 \right]$$

$$a_n = 2\left[0 + 2\left(\frac{\cos(\pi n)}{n^2\pi^2} - \frac{\cos(0)}{n^2\pi^2} \right) \right]$$

$$a_n = \frac{2}{n^2\pi^2}\left[2\left(\cos(n\pi) - 1 \right) \right]$$

$$a_n = \frac{4}{n^2\pi^2}\left[\cos(n\pi) - 1 \right]$$

$$a_n = \begin{cases} -\frac{8}{n^2\pi^2}, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{0}{2} + \sum_{n=1}^{\infty} -\frac{8}{n^2\pi^2} \cos\left(\frac{n\pi x}{1}\right)$$

$$\boxed{-\frac{8}{n^2\pi^2}\cos\pi x - \frac{8}{9\pi^2}\cos(3\pi x) + \cdots}$$

Ans

Q # 4 $\pi - x \quad 0 < x < \pi$

Half Range Sine Series.

Sol:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{l} \int_0^l f(x) \cdot \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{2}{\pi} \int_0^{\pi} (\pi - x) \cdot \sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right) dx$$

$$= \frac{2}{\pi} \int_0^{\pi} (\pi - x) \sin(nx)\, dx$$

2nd method used for Intg by par

	$\frac{d}{dx}$	$\int$
+	$\pi - x$	$\sin(nx)$
−	-1	$-\frac{\cos(nx)}{n}$
+	0	$-\frac{\sin(nx)}{n^2}$

$$\frac{2}{\pi}\left[-(\pi-x)\frac{\cos(nx)}{n}\Big|_0^{\pi} - \frac{\sin(nx)}{n^2}\Big|_0^{\pi}\right]$$

$$\frac{2}{\pi}\left[-(\pi-\pi)\frac{\cos\pi n}{n} + \frac{(\pi-0)\cos(0)}{n} - 0\right]$$

$$=\left[-(0)\frac{\cos(\pi n)}{n} + \frac{(\pi)(1)}{n} - 0\right]$$

$$\frac{2}{\cancel{\pi}}\left[0 + \cancel{\pi}/n\right]$$

$$\boxed{b_n = 2/n}$$

$$f(x) = \sum_{n=1}^{\infty} 2/n \sin\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)$$

$$= \sum_{n=1}^{\infty} 2/1 \sin(1x) + 2/2 \sin(2)x$$

$$= \boxed{2 \sin x + \cancel{2}/\cancel{2} \sin 2x + \cdots}$$

OR.

Ans

$$\boxed{2 \sin x + \sin 2x + \cdots}$$

Q # 3 $\quad f(x) = \pi - x$

$-\pi < x < \pi$

Half Range cosine series.

Sol:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) \, dx.$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} (\pi - x) \, dx.$$

$$a_0 = \frac{2}{\pi} \left[\int_0^{\pi} \pi \, dx - \int_0^{\pi} x \, dx \right]$$

$$= \frac{2}{\pi} \left[\pi x \Big|_0^{\pi} - \frac{x^2}{2} \Big|_0^{\pi} \right]$$

$$= \frac{2}{\pi} \left[\pi(\pi - 0) - \frac{1}{2}(\pi^2 - 0^2) \right]$$

$$= \frac{2}{\pi} \left[(\pi^2) - \frac{\pi^2}{2} \right]$$

$$\frac{2}{\pi} \left[\frac{2\pi^2 - \pi^2}{2} \right]$$

$$\frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}}\right]$$

$$\boxed{a_0 = \pi}$$

$$a_n = \frac{2}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{\pi}\int_{0}^{\pi} (\pi - x)\cos\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)dx$$

$$a_n = \frac{2}{\pi}\int_{0}^{\pi} (\pi - x)\cos(nx)\,dx.$$

	$\frac{d}{dx}$	$\int$
(+)	$\pi - x$	$\cos(nx)$
(−)	-1	$\frac{\sin nx}{n}$
(+)	0	$\frac{-\cos(nx)}{n^2}$

$$= \frac{2}{\pi}\left[(\pi - x)\left(\frac{\sin nx}{n}\right)\Big|_0^{\pi} - \frac{\cos(nx)}{n^2}\Big|_0^{\pi} \right]$$

$$\frac{2}{\pi}\left[0 - \frac{\cos \pi x}{n^2} + \frac{\cos(0)}{n^2} \right]$$

$$\frac{2}{\pi}\left[- \frac{(-1)^n}{n^2} + \frac{1}{n^2} \right]$$

$$\frac{2}{\pi}\left(\frac{-(-1)^n + 1}{n^2} \right)$$

$$\boxed{a_n = -\frac{2}{n^2 \pi}\left((-1)^n + 1 \right)}$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{\pi}\right)$$

$$\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{-2}{n^2\pi} (-1)^n + 1 \cdot \cos(nx)$$

$$\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{-2}{(1)^2\pi} (-1)^1 + 1 \cdot \cos(1)x + \cdots$$

$$\frac{\pi}{2} + \quad + \frac{2}{\pi} + 1 \cdot \cos x + \cdots$$

Q # 5 Find half Range Expension cosine series.

$$f(x) = 1 - x^2, \quad 0 \le x \le 1$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{l} \int_0^l f(x)\, dx$$

$$= \frac{2}{1} \int_0^1 (1-x^2)\, dx$$

$$= 2\left[\int_0^1 1\, dx - \int_0^1 x^2\, dx \right]$$

$$= 2\left[x \Big|_0^1 - \frac{x^{2+1}}{2+1} \Big|_0^1 \right]$$

$$= 2\left[x \Big|_0^1 - \frac{x^3}{3} \Big|_0^1 \right]$$

$$= 2\left[1 - 0 - \tfrac{1}{3}\left((1)^3 - (0)^3 \right) \right]$$

$$= 2\left[1 - \tfrac{1}{3}(1-0) \right]$$

$$= 2\left[1 - \tfrac{1}{3} \right]$$

Expension

$$2\left(\frac{2}{3}\right)$$

1

$$\boxed{a_0 = \frac{4}{3}}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos(n\pi x)\,dx.$$

$$a_n = \frac{2}{1}\int_0^1 (1-x^2)\cdot\cos(n\pi x)\,dx.$$

	$\frac{d}{dx}$	$\int$
$+$	$(1-x^2)$	$\cos(n\pi x)$
$-$	$-2x$	$\frac{\sin(n\pi x)}{n\pi}$
$+$	-2	$-\frac{\cos(n\pi x)}{n^2\pi^2}$
$-$	0	$-\frac{\sin(n\pi x)}{n^3\pi^3}$

$$(1-x^2)\frac{\sin(n\pi x)}{n\pi}\Big|_0^1 - \frac{2x\cos(n\pi x)}{n^2\pi^2}\Big|_0^1 + \frac{2\sin(n\pi x)}{n^3\pi^3}\Big|_0^1$$

$$2\left[0 - \frac{2x\cos(n\pi x)}{n^2\pi^2}\Big|_0^1 + 0\right]$$

$$\left[-\frac{2(1)\cos(n\pi 1)}{n^2\pi^2} + \frac{2(0)\cos(n\pi(0))}{n^2\pi^2}\right]$$

$$\left[-\frac{2\cos n\pi}{n^2\pi^2} + 0\right]$$

$$a_n = 2\left[-\frac{2\cos n\pi}{n^2\pi^2}\right]$$

$$\boxed{a_n = -\frac{4}{n^2\pi^2}(-1)^n}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$\frac{4/3}{2} - \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x)$$

$$\frac{2}{3} - \left[\frac{-4}{\pi^2} \cos \pi x + \frac{4}{4\pi^2} \cos 2\pi x + \cdots \right.$$

$$\frac{2}{3} + \frac{4}{\pi^2} \cos \pi x - \frac{\cos 2\pi x}{\pi^2} + \frac{4}{9\pi^2} \cos 3\pi x$$

$$\boxed{\frac{2}{3} + \frac{4}{\pi^2} \cos \pi x - \frac{\cos 2\pi x}{\pi^2} + \frac{4}{9\pi^2} \cdot \cos 3\pi x + \cdots}$$