Digital Image Processing
CS-601, IT-613

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Lecture 3(week 7 & 8)

Lecture # 3
Basic Concepts in Digital Image Processing

Spatial and gray level resolution

- Resolution refer to the smallest discernible change
- M x N…spatial resolution
- L…gray level resolution

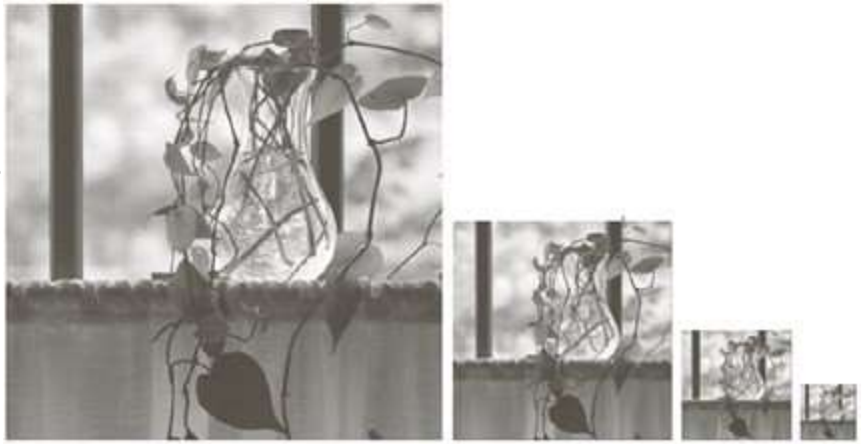
The number of selected values in the sampling process is known as the image spatial resolution. This is simply the number of pixels relative to the given image area. The number of selected values in the quantization process is called the grey-level (color level) resolution.

Sampling is the principal factor determining the spatial resolution of an image. ... A widely used definition of resolution is simply the smallest number of discernible line pairs per unit distance; for es 100 line pairs/mm. Gray level resolution: This refers to the smallest discernible change in gray level.

Image Resampling & Interpolation

Need to resample the image when

- Rescaling
- Geometrical transformation
- The output image coordinates are not discrete



Interpolation methods:

- Nearest neighbor
  - Fast and simple
  - Loss of sharpness
  - Artifacts (checkerboard)
- Bilinear
- Bicubic
  - Images are sharpest
  - Fine details are preserved
  - Slow

Zooming and shrinking

- Closest neighbor interpolation
- Pixel replication
- Bilinear interpolation

Image interpolation occurs when you resize or distort your image from one pixel grid to another. Image resizing is necessary when you need to increase or decrease the total number of pixels, whereas remapping can occur when you are correcting for lens distortion or rotating an image

Interpolation is the process of transferring image from one resolution to another without losing image quality. In Image processing field, image interpolation is very important function for doing zooming, enhancement of image, resizing and many more.

Image interpolation is generally achieved through one of three methods: nearest neighbor, bilinear interpolation, or bicubic interpolation.

Closest neighbor interpolation

Nearest neighbour interpolation is the simplest approach to interpolation. Rather than calculate an average value by some weighting criteria or generate an intermediate value based on complicated rules, this method simply determines the “nearest” neighbouring pixel, and assumes the intensity value of it.

Nearest neighbor interpolation method is a technique of blind pixel replication

10	4	22
2	18	7
9	14	25

10	10	4	4	22	22
10	10	4	4	22	22
2	2	18	18	7	7
2	2	18	18	7	7
9	9	14	14	25	25
9	9	14	14	25	25

Bilinear interpolation

Bilinear Interpolation : is a resampling method that uses the distance weighted average of the four nearest pixel values to estimate a new pixel value.

Zooming and shrinking

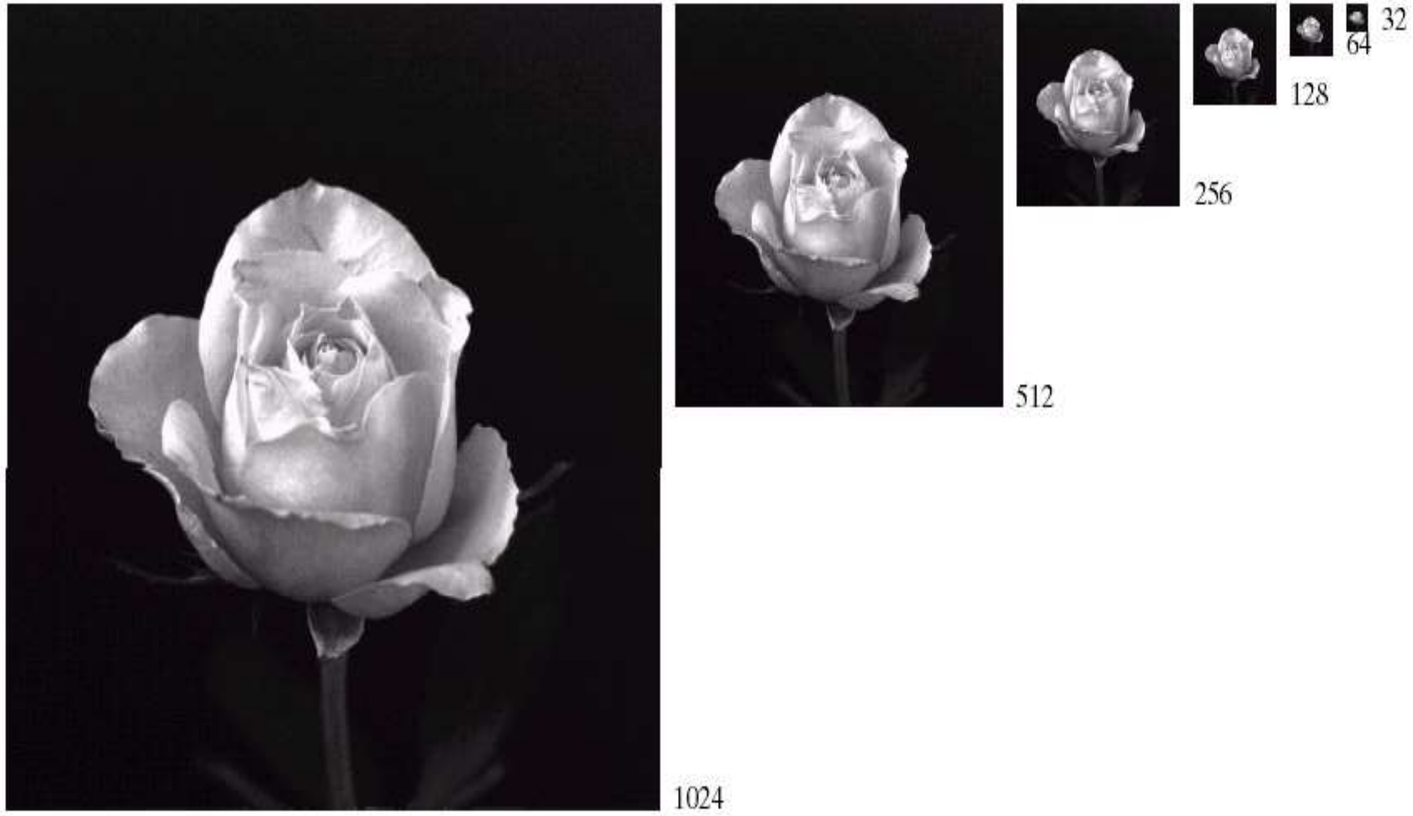


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

Zooming by row and column duplication

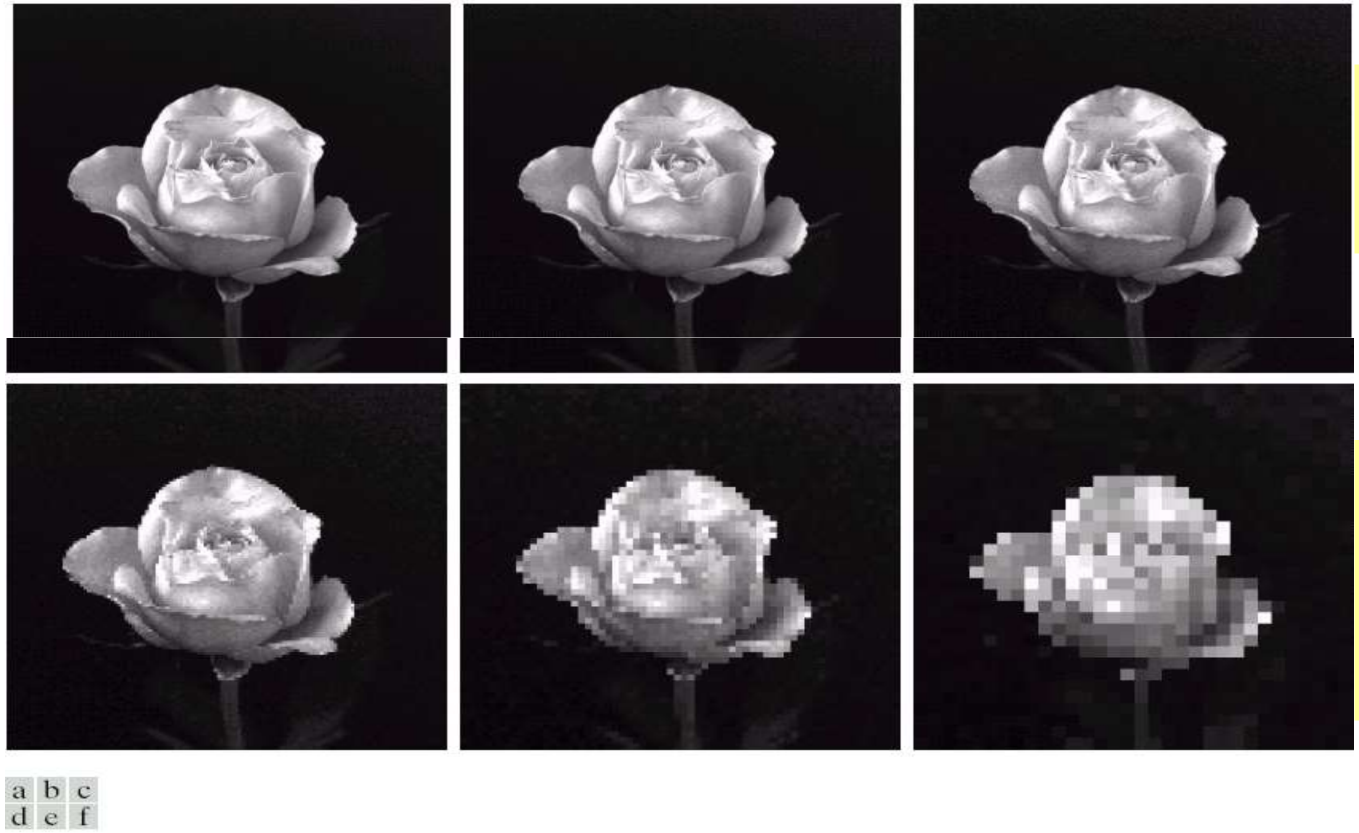
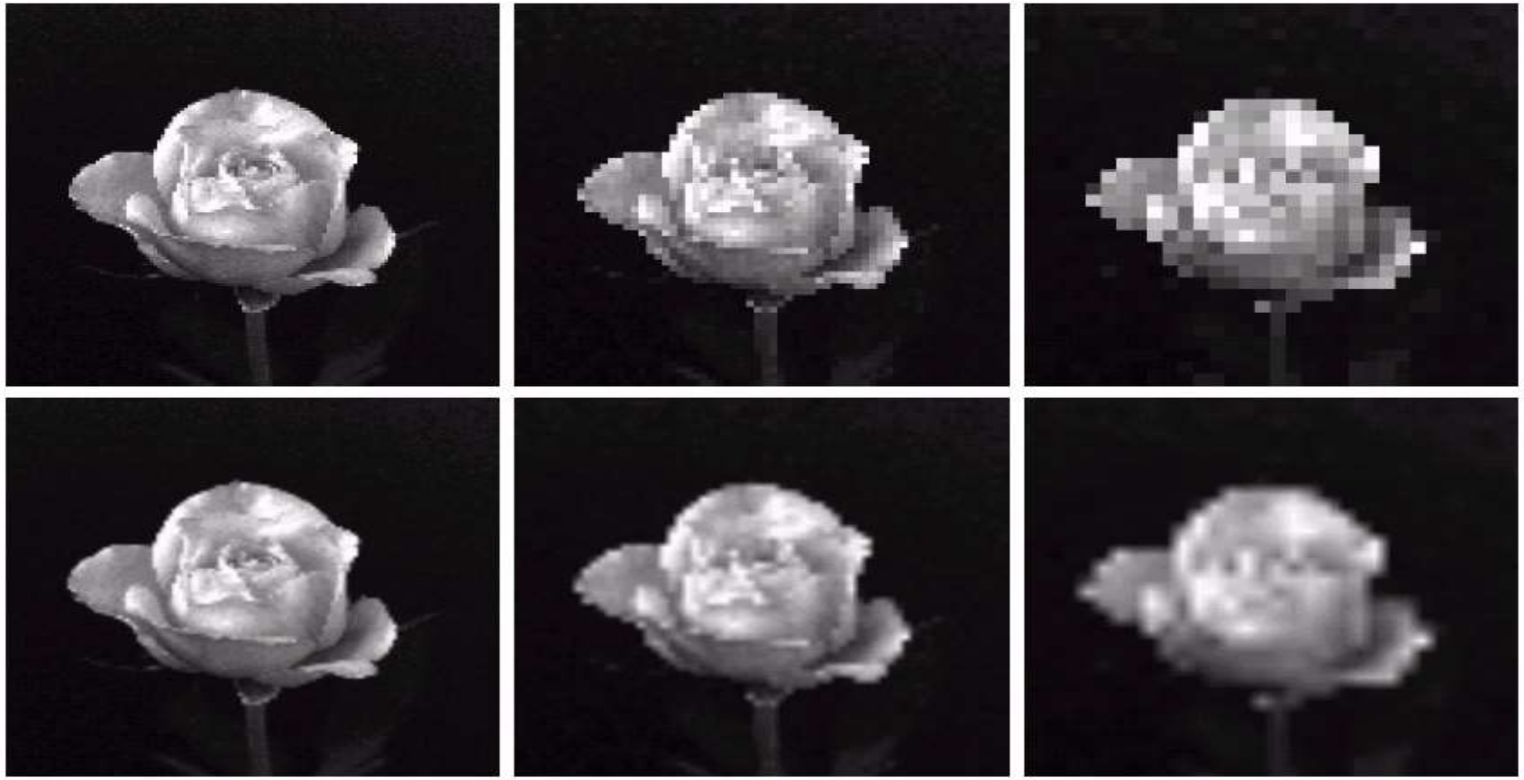


FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

A slight checkerboard pattern in the border area

The graininess is increasingly visible with the decreasing number of samples

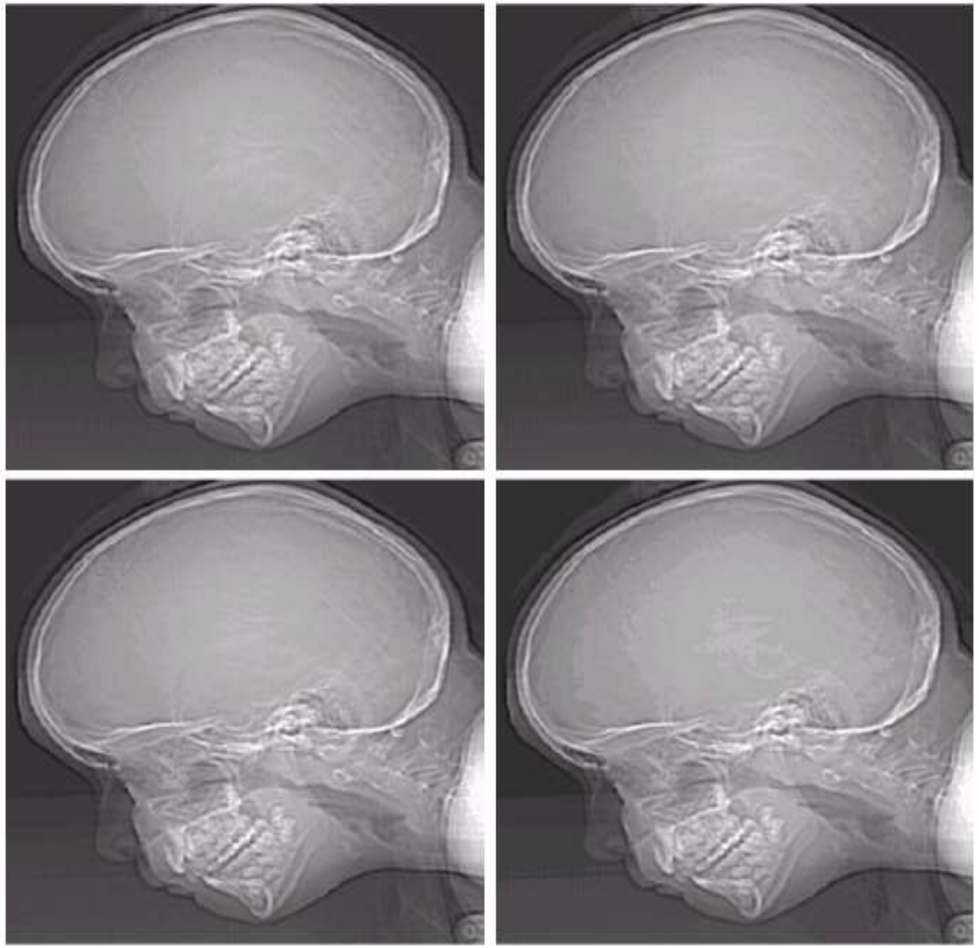
Zooming by nearest neighbor/bilinear interpolation



a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Effect of reducing gray levels



a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Note the contours that are appearing in areas of smooth gray levels

Effect of reducing gray levels

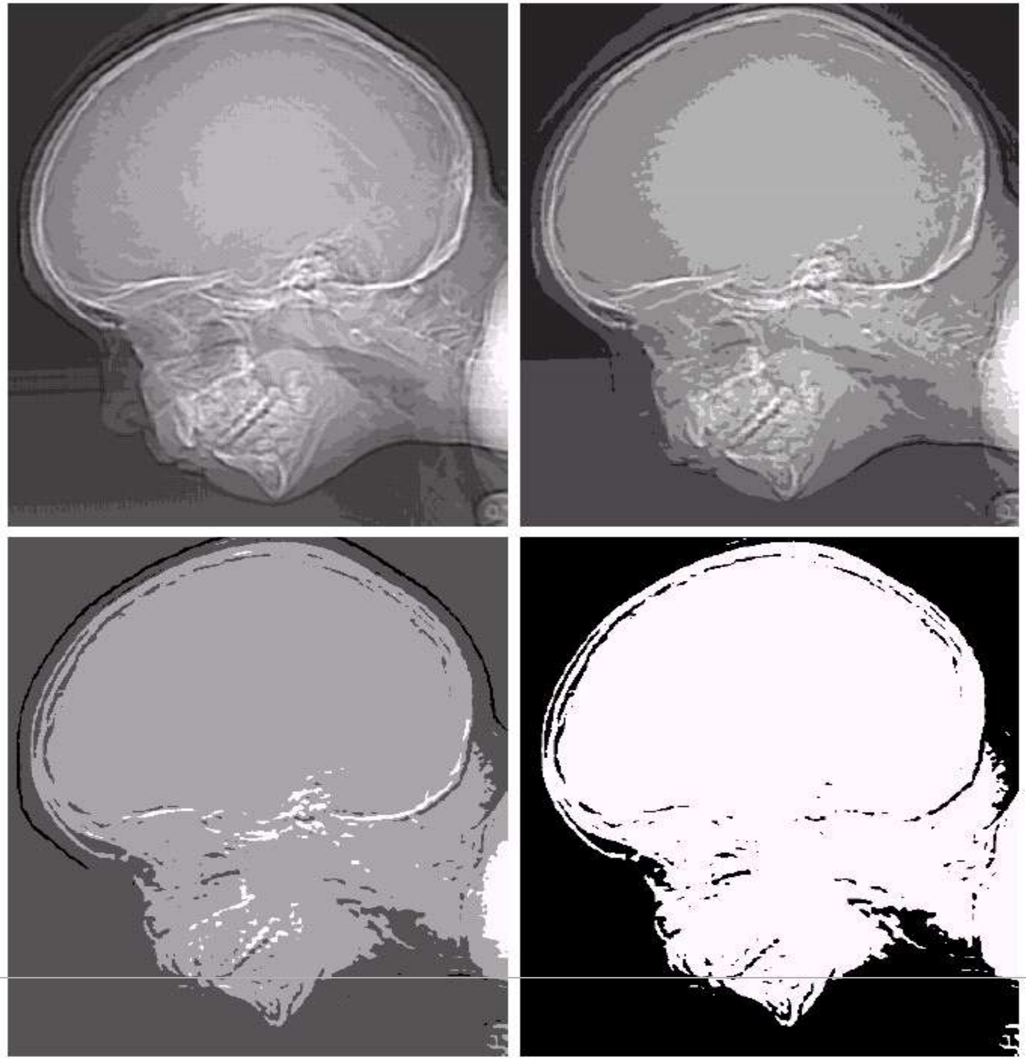


FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

The effect is getting more pronounced with the decreasing number of gray levels

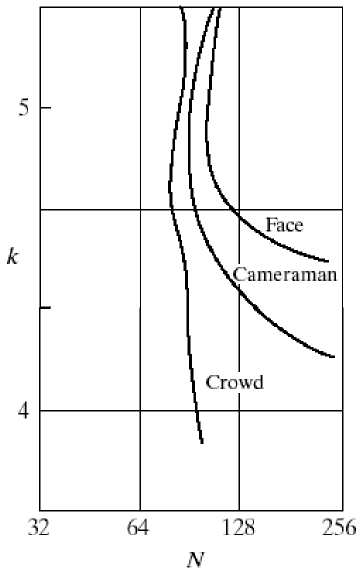
Combined effect of resolution/gray levels

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Isopreference curves

FIGURE 2.23 Representative isopreference curves for the three types of images in Fig. 2.22.



Images with a large amount of details may be represented with only a few gray levels

Pixel neighborhood

- 4-neighbors (N_4)
- Diagonal neighbors
- 8-neighbors

		(x-1 ,y)		
	(x,y-1)	(x, y)	(x,y+1)	
		(x+1,y)		

Pixel neighborhood

- 4-neighbors
- Diagonal neighbors (N_D)
- 8-neighbors

	(x-1, y-1)		(x-1, y+1)	
		(x, y)		
	(x+1, y1)		(x+1, y+1)	

Pixel neighborhood

- 4-neighbors
- Diagonal neighbors
- 8-neighbors (N_8)

	(x-1, y-1)	(x-1 ,y)	(x-1, y+1)	
	(x,y-1)	(x, y)	(x,y+1)	
	(x+1, y-1)	(x+1,y)	(x+1, y+1)	

Connectivity of pixels

- Two pixels are connected if they are neighbors and their gray levels satisfy a specified criterion of similarity

	0	1	1	
	1	1	0	
	0	1	0	

Review of Adjacency

Adjacency is the relationship between two pixels *p* and *q*

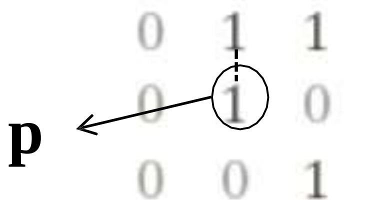
***V* is a set of intensity values used to define adjacency**

- **Binary image: V={1} or V={0}**
- **Gray level image:** $\boldsymbol{V \sqsubseteq \{0, 1, \ldots, 255\}}$

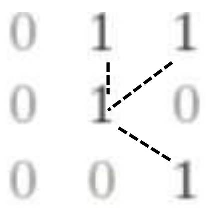
$\boldsymbol{f(p) \in V}$ and $\boldsymbol{f(q) \in V}$ ⇨ **Intensity constraints**

Three types of adjacency:

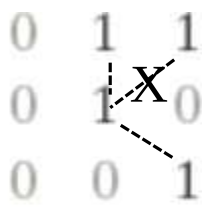
4-adjacency **8-adjacency** **m-adjacency**



$\boldsymbol{q \in N_4(p)}$



$\boldsymbol{q \in N_8(p)}$



$\boldsymbol{q \in N_D(p)}$ **and** $N_4(q) \cap N_4(p) = \emptyset$

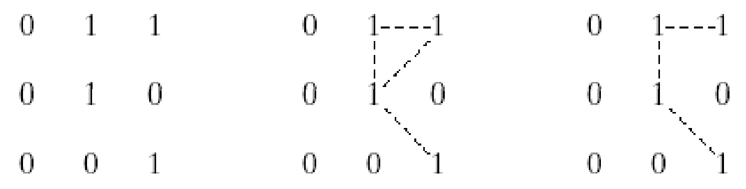
$or\ q \in N_4(p)$

Pixel adjacency

Mixed (m) adjacency is modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used. For example, consider
the pixel arrangement shown in Fig. 2.26(a) for V={1}.The three pixels at the top of Fig. 2.26(b) show multiple (ambiguous) 8-adjacency, as indicated by the dashed lines (diagonal and 8). This ambiguity is removed by using m-adjacency, as shown in Fig. 2.26(c).

- 4-adjacency
- 8-adjacency
- m-adjacency

Two pixels p and q with values from a set V are 4-adjacent if q is in the set $N_4(p)$



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Two pixels p and q with values from a set V are said to be mixed (m) adjacent if
(1) if q is in the set $N_4(p)$ or
(2)q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V

Adjacency of Image subsets

Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2

Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2. It is understood here and in the following definitions that *adjacent* means 4-, 8-, or m-adjacent.

Q. Consider two image subsets S_1 and S_2. For $V = [0]$, determine whether the regions are:

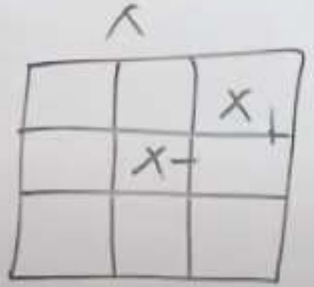
1. 4-adjacent ✗
2. 8-adjacent ✓
3. m-adjacent ✓

S_1				S_2			
1	1	1	1	1	1	1	0
1	1	0	1	1	0	1	1
1	1	0	1	0	0	1	1
1	0	0	0	1	1	1	1

4-adj

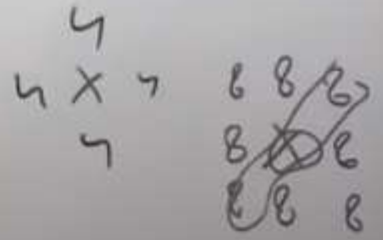
✗

$V = [0]$ ✓



1) D-adj ✓

2) ✓



8. p, q, D and $N_4(p) \cap N_4(q) = \phi$

Adjacency of Image subsets

Consider the two image subsets, S_1 and S_2
Shown in the following figure.

(1) For V={1,4}, determine whether these two subsets are 4-adjacent, (b) 8-adjacent (c) m-adjacent

(2) Repeat for V={2,7}

1	2	3	8	1	5	2	2	2	2
2	2	1	2	3	4	1	1	1	2
0	1	0	1	4	1	1	7	1	2
0	2	1	2	6	2	1	1	1	2
9	7	4	3	7	2	2	2	2	2

S_1 (columns 3–5, rows 1–4)

S_2 (columns 6–9, rows 1–4)

Some Basic Relationships between Pixels

Neighbors of a pixel

$N_4(p)$

	(x-1,y)	
(x,y-1)	(x,y)	(x,y+1)
	(x+1,y)	

$N_D(p)$

(x-1,y-1)		(x-1,y+1)
	(x,y)	
(x-1,y+1)		(x+1,y+1)

$N_8(p)$

(x-1,y-1)	(x-1,y)	(x-1,y+1)
(x,y-1)	(x,y)	(x,y+1)
(x-1,y+1)	(x+1,y)	(x+1,y+1)

Path or curve

A path from pixel p with coordinates (x,y) to pixel q with coordinates (s, t) is a sequence of distinct pixels with coordinates

$(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$ where

$(x_0, y_0) = (x, y)$ and $(x_n, y_n) = (s, t)$

and $(x_i, y_i), (x_{i-1}, y_{i-1})$ are adjacent for $1 \leq i \leq n$

Path length n
Closed path if?

If $(x_0, y_0) = (x_n, y_n)$
the path is a *closed* path

Depending on adjacency the path is a 4, 8, m-path, depending on the adjacency

Connectivity

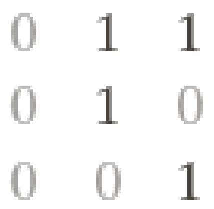
- **Path from p to q: a sequence of distinct and adjacent pixels with coordinates**

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

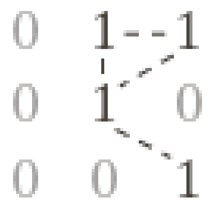
Starting point p ← (x_0, y_0); $(x_0, y_0), (x_1, y_1)$: **adjacent**; (x_n, y_n) → **ending point q**

- ***Closed* path: if the starting point is the same as the ending point**
- **p and q are *connected*: if there is a path from p to q in S**
- ***Connected component*: all the pixels in S connected to p**
- ***Connected set*: S has only one connected component**

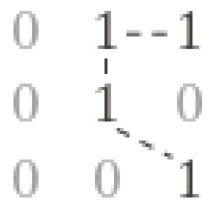
Are they connected sets?



4-adjacency: No



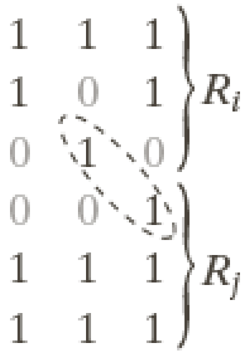
8-adjacency: Yes



m-adjacency: Yes

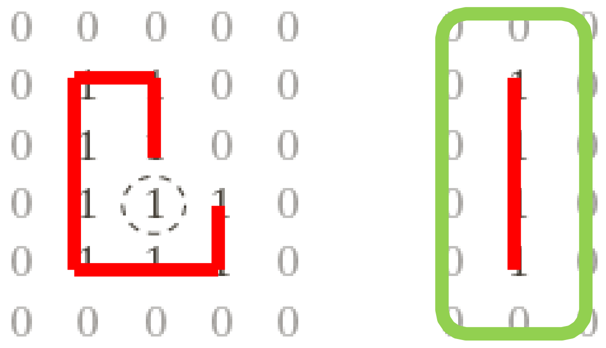
Regions

- R is a region if R is a connected set
- R_i and R_j are adjacent if $R_i \cup R_j$ is a connected set



Boundaries

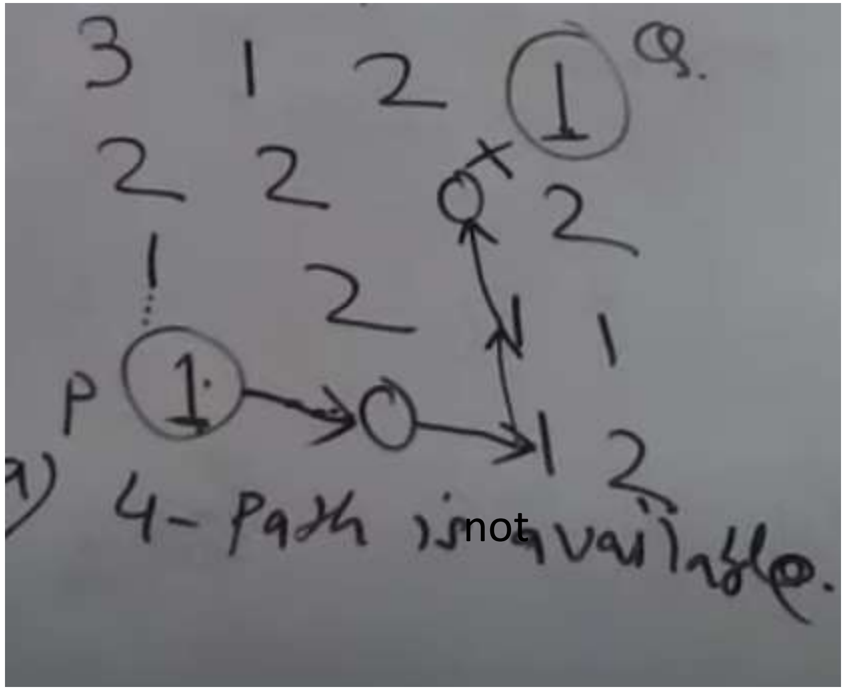
- **Inner boundary (boundary) -- the set of pixels each of which has at least one background neighbor**
- **Outer boundary – the boundary pixels in the background**

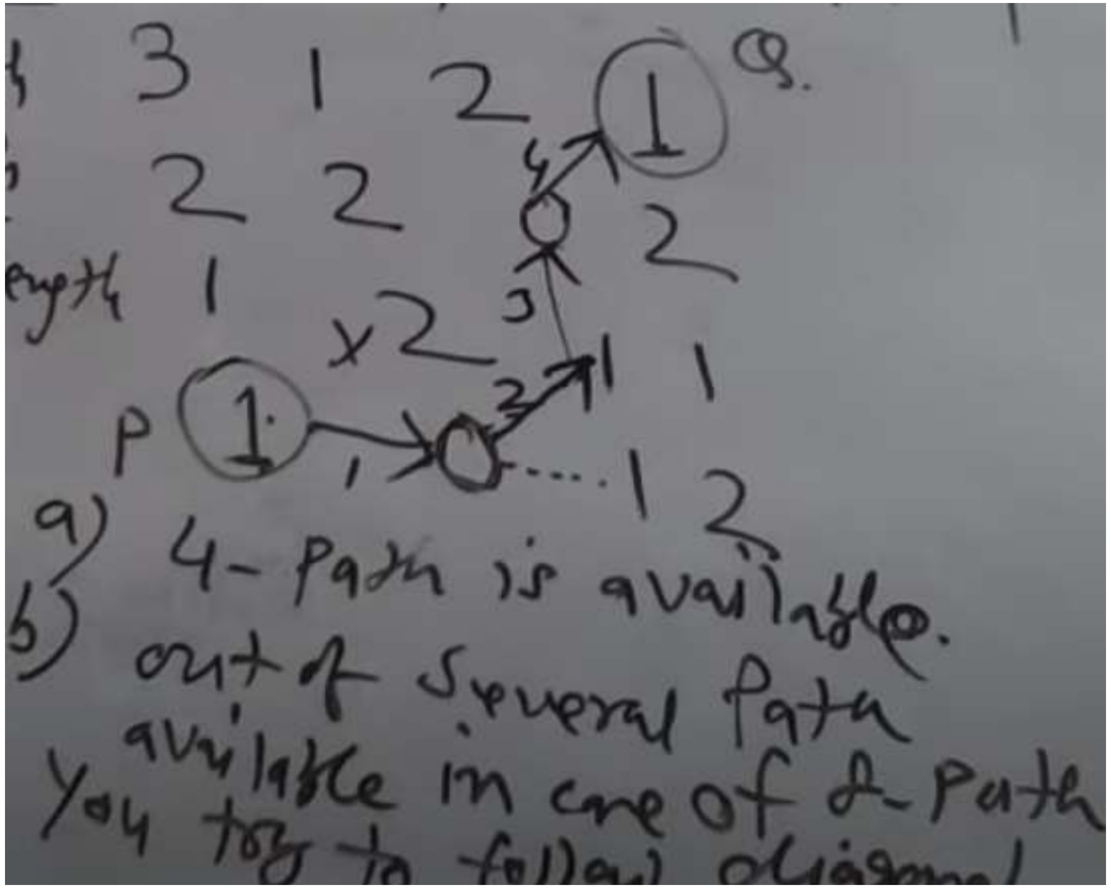


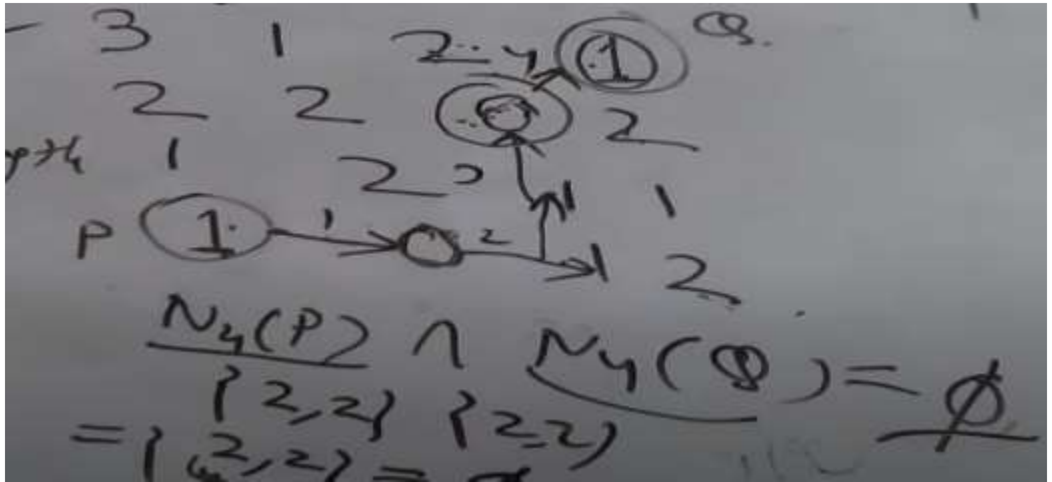
Path or curve

- Find the length of shortest 4, 8, m-path between p and q if V={0,1}
- Repeat for V={1,2}

3	1	2	1(q)
2	2	0	2
1	2	1	1
1(p)	0	1	2


P
Q
a) 4- Path is not available.


P
Q
a) 4- Path is available.
b) out of several Path
available in one of 8- Path


P
Q
N4(P) ∩ N4(Q) = ∅
{2,2} {2,2}

Connected component / set

- Two pixels p and q in S (S is a subset of pixels in an image) are connected if there exist a path between them consisting entirely of pixels from S
- For any pixel p in S, the set pixels that are connected to it in S is called a connected component
- If S has one connected component then set S is called connected set

Regions and boundary

- A subset R of an image is called a region if R is a connected set
- A border or boundary of a region R is the set of pixels in the region that have one or more neighbors that are not in R
- The border of a region is a closed path! WHY?

The boundary of a finite region forms a closed path and is thus a "global" concept

What if the region R happens to be the whole image???

If *R* happens to be an entire image (which we recall is a rectangular set of pixels), then its boundary is defined as the set of pixels in the first and last rows and columns of the image. This extra definition is required because an image has no neighbors beyond its border

Distance measure

- Euclidean distance
- D_4 distance
- D_8 distance
- D_m distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

$$D_4(p,q) = |x-s| + |y-t|$$

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

The shortest m-path between two pixels

	2	1	2	
	1	0	1	
	2	1	2	

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance Measures

For pixels p, q, and z, with coordinates (x,y), (s,t) and (v,w), D is a distance function or metric if

$$(a)\ D(p,q) \geq 0 \quad D(p,q) = 0 \ iff \ \ p = q$$

$$(b)\ D(p,q) = D(q,p), \quad and$$

$$(c)\ D(p,z) \leq D(p,q) + D(q,z)$$

Distance Measures

Euclidean distance $$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

City-block (D4) distance $$D_4(p,q) = |x-s| + |y-t|$$

Chessboard (D8) distance (Chebyshev distance)

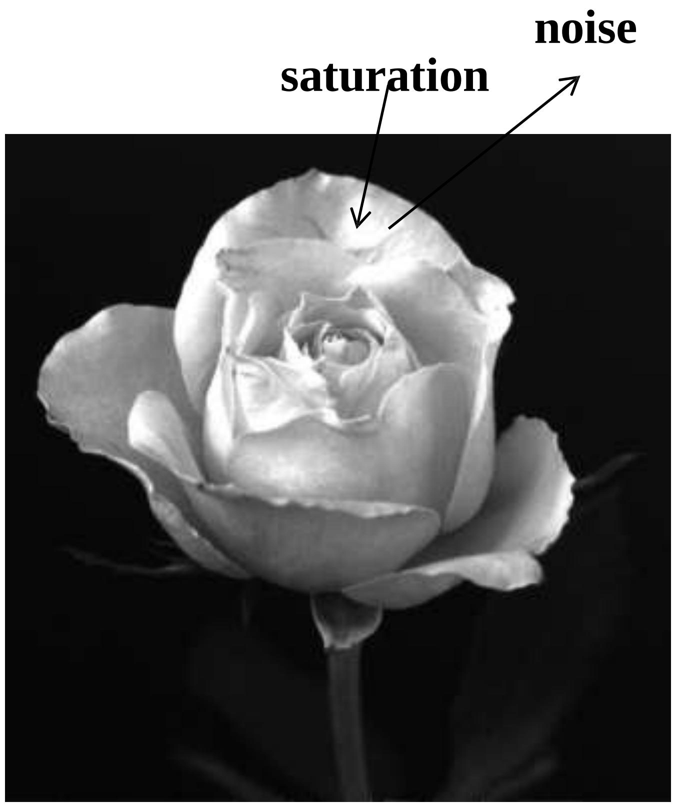
$$D_8(p,q) = \max(|x-s|,\ |y-t|)$$

Dynamic Range

$L_{\min} < f(x,y) < L_{\max}$ **in practice**

where $L_{\min} = i_{\min} r_{\min}$ **and** $L_{\max} = i_{\max} r_{\max}$

$0 \le f(x,y)$ **and** $L = [L_{min}, L_{max}]$



Dynamic range/contrast ratio:

the ratio of the maximum detectable intensity level (saturation) to the minimum detectable intensity level (noise)

$$\frac{I_{max}}{I_{min}}$$

Representing Digital Images

(a)*: f*(*x*,*y*), *x*=0, 1, …, *M*-1, *y*=0,1, …, *N*-1

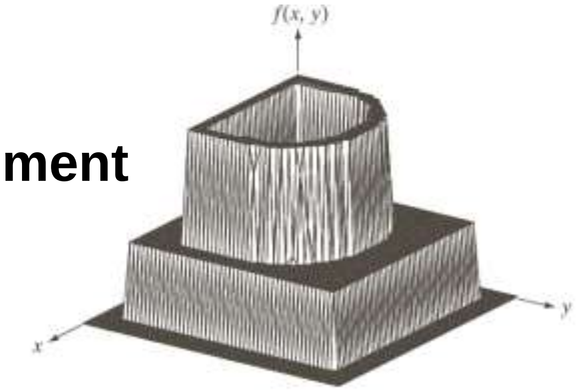
***x*, *y*: spatial coordinates → spatial domain**

(b): suitable for visualization

(c): processing and algorithm development

***x*: extend downward (rows)**

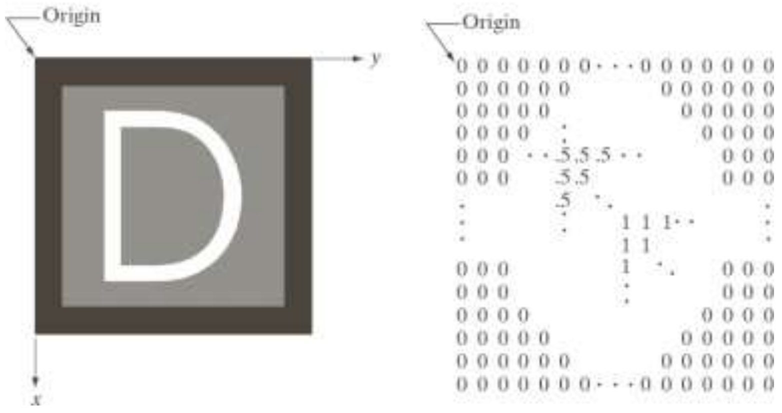
***y*: extend to the right (columns)**



Number of bits storing the image

↑

$$b = M \times N \times k$$



a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

Store an Image

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

For an 8-bit image of size 512×512, determine its gray-scale and storage size.

Solution $\therefore k = 8$, $M = N = 512$

Number of gray levels $L = 2^k = 2^8 = 256$

The gray scale is [0 , 255]

Storage size (b) = $M * N * k = 512 * 512 * 8 = 2{,}097{,}152$ bits

Spatial Resolution

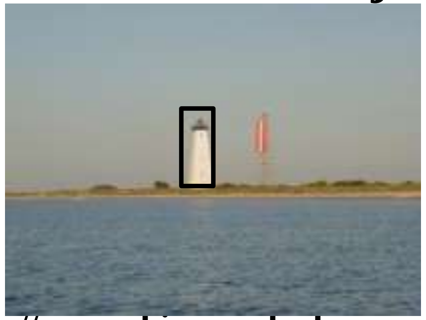
Spatial resolution: **smallest discernible details**

- # of line pairs per unit distance
- # of dots (pixels) per unit distance
  - Printing and publishing
  - In US, dots per inch (dpi)

Newspaper → magazines → book

Large image size itself does not mean high spatial resolution!

→ **Scene/object size in the image**



1280*960

http://www.shimanodealer.com/fishing_reports.htm

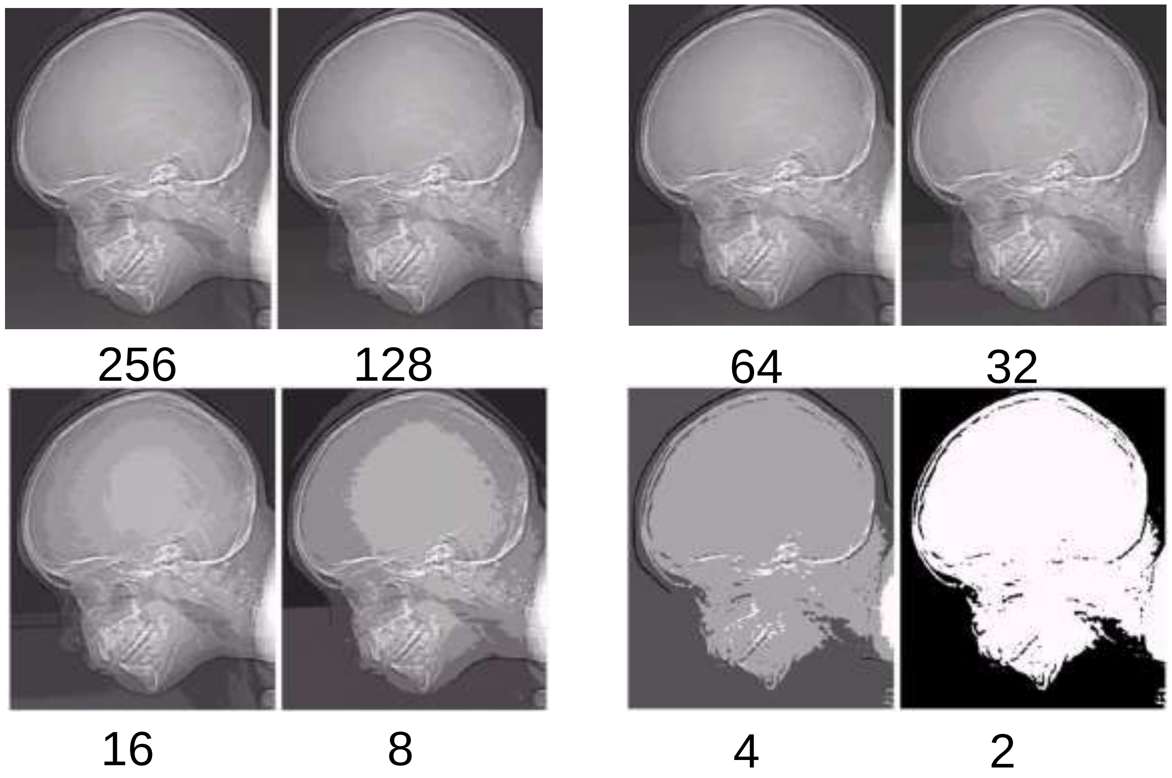
a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Intensity Resolution

Intensity resolution

- Smallest discernible change in intensity levels
- Using the number of levels of intensities
- False contouring (banding) when *k* is small - undersampling



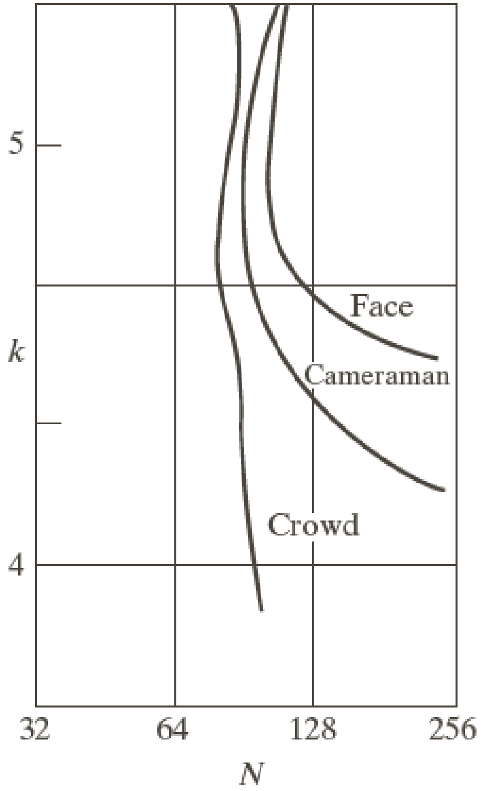
Isopreference Curves

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Vary the spatial and intensity sampling simultaneously:

FIGURE 2.23 Typical isopreference curves for the three types of images in Fig. 2.22.



Basic Set and Logical Operations

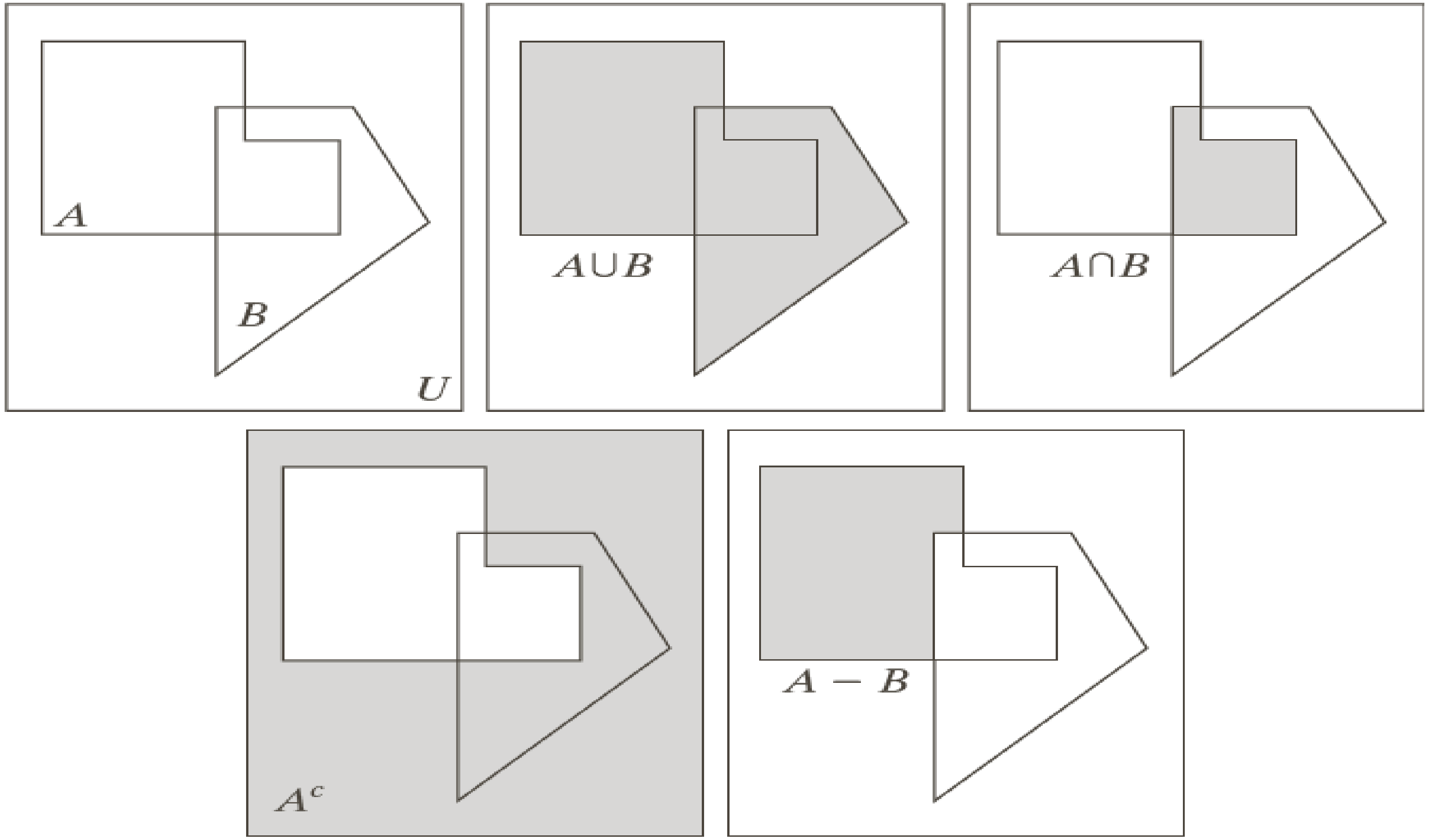
- A is a set: A={.} e.g. A={1,…,255} or $A = \{w|w = 1, \dots, 255\}$

$A = \emptyset$ for empty set

- a is an element of A$(a \in A)$ or a isn't an element of A$(a \notin A)$
- A is a *subset* of B if every element in A also is in B $(A \sqsubseteq B)$
- C is the *union* of two sets A and B $(C = A \cup B)$
- C is the *intersection* of A and B $(C = A \cap B)$
- Disjoint or mutual exclusive sets $(A \cap B = \emptyset)$
- *Set universe* is the set of all elements in an application
- *Set difference* $(A - B = \{w|w \in A, w \notin B\})$

Set Operations Based on Coordinates

A region in an image is represented by a set of coordinates within the region



a b c
d e

FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.