Week # 09

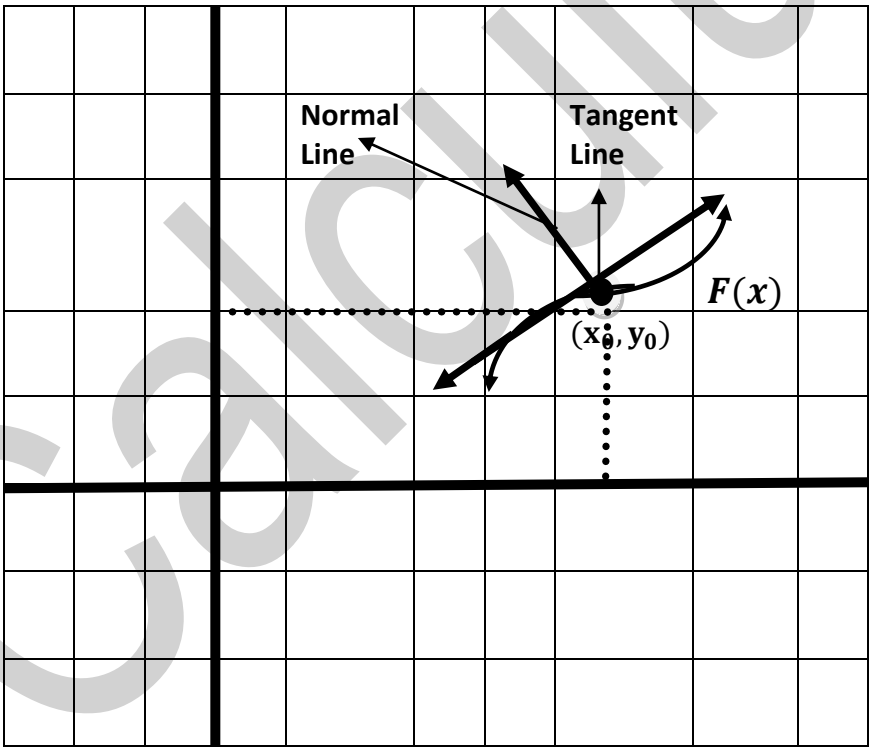
- **Equation Of Tangent**
- **Equation Of Normal**

- **Equation Of Tangent:-**

$$(y - y_0) = m(x - x_0)$$

- **Equation Of Normal:-**

$$(y - y_0) = \frac{-1}{m}(x - x_0)$$



- **Questions:-**

1. Find the equation of Tangent and Normal

   $y = x^2$ At a Point (2, 4).

$Sol: -$

$y = x^2$

Differentiate w-r-t 'x'

$\frac{d}{dx}y = \frac{d}{dx}x^2$

$\frac{dy}{dx} = 2x \qquad \because \frac{dy}{dx} = m$

$Put\ x = 2$

$\frac{dy}{dx} = 2(2)$

$\boxed{m = 4}$

Equation of Tangent line

$(y - y_0) = m(x - x_0)$

$\because x_0 = 2\ ,\ y_0 = 4\ ,\ \ m = 4$

$(y - 4) = 4(x - 2)$

$y - 4 = 4x - 8$

$y = 4x - 8 + 4$

$\boxed{y = 4x - 4}$

Equation of Normal Line

$(y - y_0) = \frac{-1}{m}(x - x_0)$

$\because x_0 = 2\ ,\ y_0 = 4\ ,\ \ m = 4$

$(y - 4) = \frac{-1}{4}(x - 2)$

$4(y - 4) = -1(x - 2)$

$4y - 16 = -x + 2$

$4y = -x + 2 + 16$

$4y = -x + 18$

$\boxed{y = \frac{-x+18}{4}}$

2. Find the equation of Tangent and Normal
   $y = x^3$ At a Point $(-\frac{1}{2}, -\frac{1}{8})$.

$Sol: -$

$y = x^3$

Differentiate w-r-t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}x^3$$

$$\frac{dy}{dx} = 3x^2 \qquad \because \frac{dy}{dx} = m$$

$$Put\ x = -\frac{1}{2}$$

$$\frac{dy}{dx} = 3(-\frac{1}{2})^2$$

$$\frac{dy}{dx} = 3(\frac{1}{4})$$

$$\boxed{m = \frac{3}{4}}$$

Equation of Tangent line

$$(y - y_0) = m(x - x_0)$$

$$\because\ x_0 = -\tfrac{1}{2}\ ,\ y_0 = -\tfrac{1}{8}\ , m = \tfrac{3}{4}$$

$$(y - (-\tfrac{1}{8})) = \tfrac{3}{4}(x - (-\tfrac{1}{2}))$$

$$4(y + \tfrac{1}{8}) = 3(x + \tfrac{1}{2})$$

$$4y + \frac{4}{8} = 3x + \frac{3}{2}$$

$$4y + \frac{\cancel{4}^{1}}{\cancel{8}^{2}} = 3x + \frac{3}{2}$$

$$4y + \frac{1}{2} = 3x + \frac{3}{2}$$

$$4y = 3x + \frac{3}{2} - \frac{1}{2}$$

Equation of Normal Line

$$(y - y_0) = \tfrac{-1}{m}(x - x_0)$$

$$\because\ x_0 = -\tfrac{1}{2}\ ,\ y_0 = -\tfrac{1}{8}\ , m = \tfrac{3}{4}$$

$$(y - (-\tfrac{1}{8})) = \frac{-1}{\frac{3}{4}}(x - (-\tfrac{1}{2}))$$

$$\left(y + \tfrac{1}{8}\right) = -\tfrac{4}{3}(x + \tfrac{1}{2})$$

$$y + \frac{1}{8} = -\frac{4x}{3} - \frac{4}{6}$$

$$y = -\frac{4x}{3} - \frac{4}{6} - \frac{1}{8}$$

$$y = -\frac{4x}{3} - \frac{4-1}{24} \qquad By\ \ L - C - M$$

$$y = -\frac{4x}{3} - \frac{\cancel{3}^{1}}{\cancel{24}^{8}}$$

$$4y = 3x + \frac{3-1}{2}$$

$$4y = 3x + \frac{\cancel{2}}{\cancel{2}}$$

$$4y = \frac{3x+1}{4}$$

$$\boxed{y = \frac{3x}{4} + \frac{1}{4} \ \ Ans.}$$

$$y = -\frac{4x}{3} - \frac{1}{8}$$

$$\boxed{y = -\frac{4x}{3} - \frac{1}{8} \quad Ans.}$$

3. Find the equation of Tangent and Normal

$y = xe^x$ At a Point (1, e).

$Sol: -$

$y = xe^x$

Differentiate w-r-t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}xe^x$$

$$\frac{dy}{dx} = x\frac{d}{dx}e^x + e^x\frac{d}{dx}x \qquad \because \frac{dy}{dx} = m$$

$$m = x.e^x + e^x(1)$$

$$m = x.e^x + e^x$$

$Put\ x = 1$

$$m = 1.e^1 + e^1$$

$$m = e + e$$

$$\boxed{m = 2e}$$

Equation of Tangent line

Equation of Normal Line

$(y - y_0) = m(x - x_0)$

$\because\ x_0 = 1\ ,\ y_0 = e\ ,\ \ m = 2e$

$(y - e) = 2e(x - 1)$

$y - e = 2ex - 2e$

$y = 2ex - 2e + e$

$\boxed{y = 2ex - e}$

$(y - y_0) = \frac{-1}{m}(x - x_0)$

$\because\ x_0 = 1\ ,\ y_0 = e\ ,\ \ m = 2e$

$(y - e) = \frac{-1}{2e}(x - 1)$

$2e(y - e) = -1(x - 1)$

$2ey - 2e^2 = -x + 1$

$\boxed{2ey - 2e^2 = -x + 1}$

4. Find the equation of Tangent and Normal

$y = sin\ x$ At a Point $(\frac{\pi}{6}, \frac{1}{2})$.

$Sol: -$

$y = sin\ x$

Differentiate w-r-t 'x'

$\frac{d}{dx}y = \frac{d}{dx} sin\ x$

$\frac{dy}{dx} = cos\ x \qquad \because\ \frac{dy}{dx} = m$

$Put\ x = \frac{\pi}{6}$

$m = cos\ \frac{\pi}{6}$ or $m = cos 30^0$

$\boxed{m = \frac{\sqrt{3}}{2}}$

Equation of Tangent line

$(y - y_0) = m(x - x_0)$

Equation of Normal Line

$(y - y_0) = \frac{-1}{m}(x - x_0)$

$\because x_0 = \frac{\pi}{6}\ ,\ y_0 = \frac{1}{2},\ m = \frac{\sqrt{3}}{2}$

$(y - \frac{1}{2}) = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

$2(y - \frac{1}{2}) = \sqrt{3}(x - \frac{\pi}{6})$

$2y - \cancel{2}/\cancel{2} = \sqrt{3}x - \sqrt{3}\frac{\pi}{6}$

$2y - 1 = \sqrt{3}x - \sqrt{3}\frac{\pi}{6}$

$2y = \sqrt{3}x - \sqrt{3}\frac{\pi}{6} + 1$

$\boxed{2y = \sqrt{3}x - \sqrt{3}\frac{\pi}{6} + 1}$

$\because x_0 = \frac{\pi}{6}\ ,\ y_0 = \frac{1}{2},\ m = \frac{\sqrt{3}}{2}$

$(y - \frac{1}{2}) = \frac{-1}{\frac{\sqrt{3}}{2}}(x - \frac{\pi}{6})$

$\left(y - \frac{1}{2}\right) = -\frac{2}{\sqrt{3}}(x - \frac{\pi}{6})$

$\sqrt{3}\left(y - \frac{1}{2}\right) = -2(x - \frac{\pi}{6})$

$\sqrt{3}y - \frac{\sqrt{3}}{2} = -2x + \frac{2\pi}{6})$

$\sqrt{3}y - \frac{\sqrt{3}}{2} = -2x + \frac{\cancel{2}\,\pi}{\cancel{6}^{3}}$

$\boxed{\sqrt{3}y = -2x + \frac{\pi}{3} + \frac{\sqrt{3}}{2}}$

5. Find the equation of Tangent and Normal

$y = xsin\ hx$ At a Point $(0, 0)$.

$Sol: -$

$y = xsin\ hx$

Differentiate w-r-t 'x'

$\frac{d}{dx}y = x\frac{d}{dx}(sin\ hx) + sin\ hx\frac{d}{dx}(\mathrm{x})$

$\frac{dy}{dx} = x.cos\ hx + sin\ hx(1)$ $\quad \because \frac{dy}{dx} = m$

$m = x.cos\ hx + sin\ hx$

$Put\ x = 0$

$m = 0.cos\ h(0) + sin\ h(0)$

$m = 0.(1) + 0)$

$\boxed{m = 0}$

Equation of Tangent line

$(y - y_0) = m(x - x_0)$

$\because\ x_0 = 1\ ,\ y_0 = 0\ ,\ m = 0$

$(y - 0) = 0(x - 0)$

$y = 0$

$\boxed{y = 0}$

Equation of Normal Line

$(y - y_0) = \frac{-1}{m}(x - x_0)$

$\because\ x_0 = 1\ ,\ y_0 = 0\ ,\ m = 0$

$(y - 0) = \frac{-1}{0}(x - 0\ )$

$0(y - 0) = -1(x - 0\ )$

$0 = -x + 0$

$\boxed{x = 0}$

6. Find the equation of Tangent and Normal

$x^2+y^2 = 2$ At a Point $(1,\ 1).$

$Sol: -$

$x^2+y^2 = 2$

Differentiate w-r-t 'x'

$\frac{d}{dx}(x^2+y^2) = 2$

$\frac{d}{dx}x^2+\frac{d}{dx}y^2 = \frac{d}{dx}2$

$2x + 2y\frac{d}{dx} = 0$ $\quad\because\ \frac{dy}{dx} = m$

$2y\frac{d}{dx} = -2x$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$m = \frac{-x}{y}$$

$Put\ x = 1\ \&\ y = 1$

$$m = \frac{-1}{1}$$

$$\boxed{m = -1}$$

Equation of Tangent line

$$(y - y_0) = m(x - x_0)$$

$\because x_0 = 1\ ,\ y_0 = 1,\ m = -1$

$$(y - 1) = -1(x - 1)$$

$$y - 1 = -x + 1$$

$$y = -x + 1 + 1$$

$$\boxed{y = -x + 2}$$

Equation of Normal Line

$$(y - y_0) = \frac{-1}{m}(x - x_0)$$

$\because x_0 = 1\ ,\ y_0 = 1,\ m = -1$

$$(y - 1) = \frac{-1}{-1}(x - 1)$$

$$y - 1 = 1(x - 1)$$

$$y = x - 1 + 1$$

$$\boxed{y = x}$$

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7. Find the equation of Tangent and Normal

$x^2 + xy = 0$ At a Point $(1, -1)$.

$Sol: -$

$x^2 + xy = 0$

Differentiate w-r-t 'x'

$$\frac{d}{dx}(x^2+xy)=\frac{d}{dx}(0)$$

$$\frac{d}{dx}x^2+\left[x\frac{d}{dx}(y)+y\frac{d}{dx}(x)\right]=0$$

$$2x+\left[x\frac{dy}{dx}+y(1)\right]=0 \quad => 2x+x\frac{dy}{dx}=-y$$

$$x\frac{dy}{dx}=-\mathrm{y}-2x$$

$$\frac{dy}{dx}=\frac{-\mathrm{y}-2x}{x} \qquad \because \frac{dy}{dx}=m$$

$$m=\frac{-\mathrm{y}}{x}-\frac{2\cancel{x}}{\cancel{x}}$$

$$m=\frac{-y}{x}-2$$

$Put\ x=1\ \&\ y=-1$

$$m=\frac{-(-\cancel{1})}{\cancel{1}}-2$$

$$m=1-2$$

$$\boxed{m=-1}$$

Equation of Tangent line

$$(y-y_0)=m(x-x_0)$$

$\because x_0=1,\ y_0=-1,\ m=-1$

$$(y-(-1))=-1(x-1)$$

$$y+1=-x+1$$

Equation of Normal Line

$$(y-y_0)=\frac{-1}{m}(x-x_0)$$

$\because x_0=1,\ y_0=-1,\ m=-1$

$$(y-(-1))=\frac{\cancel{-1}}{\cancel{-1}}(x-1)$$

$$y-1=1(x-1)$$

$$y=x-1+1$$

$y = -x + 2$ | $y = x$

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The End of Week # 12

Calculus