Week # 06

- **Differentiation, Rules of Differentiation**
- **Derivatives of Algebraic Functions**

- **Differentiation:-**

The Process of finding Derivative is called Differentiation.

- **Derivative:-**

The Instantaneous rate of change of dependent variable w-r-t independent variable is called derivative.

OR

To find slope of a curve is called derivative.

- **First Principle Rule:-**

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

- **Questions:-**

i. $f(x) = x$

$Sol:-$

$f(x) = x \longrightarrow$ (i)

$Put\ x = \Delta x$

$f(x + \Delta x) = x + \Delta x \longrightarrow$ (ii)

Using First Principle Rule

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \longrightarrow$ (iii)

Put ⟶ (i) & ⟶ (ii) in ⟶ (iii)

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{(x+\Delta x)-(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{\cancel{x} + \Delta x - \cancel{x}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}}$$

$$\boxed{\frac{dy}{dx} = 1\ Ans.}$$

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ii. $f(x) = x^2$

$Sol:-$

$f(x) = x^2$ ⟶ (i)

$Put\ x = \Delta x$

$f(x + \Delta x) = (x + \Delta x)^2$ ⟶ (ii)

Using First Principle Rule

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \longrightarrow \text{(iii)}$$

Put ⟶ (i) & ⟶ (ii) in ⟶ (iii)

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-(x)^2}{\Delta x} \qquad \because\ (a+b)^2 = a^2 + b^2 + 2ab$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{(x)^2+(\Delta x)^2+2(x\Delta x)-x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{x}^2 + \Delta x^2 + 2x\Delta x - \cancel{x}^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta x^2 + 2x\Delta}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta x(x\Delta + 2x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(\Delta x + 2x)}{\cancel{\Delta x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} x\Delta + 2x$$

Apply the Limit

$$\frac{dy}{dx} = 0 + 2x$$

$$\boxed{\frac{dy}{dx} = 2x\ Ans.}$$

iii. $f(x) = x^3$

$Sol: -$

$f(x) = x^3 \longrightarrow$ (i)

$Put\ x = \Delta x$

$f(x + \Delta x) = (x + \Delta x)^3 \longrightarrow$ (ii)

Using First Principle Rule

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \longrightarrow \text{(iii)}$$

Put $\longrightarrow$ (i) & $\longrightarrow$ (ii) in $\longrightarrow$ (iii)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - (x)^3}{\Delta x} \qquad \because \ (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x)^3 + (\Delta x)^3 + 3x^2\Delta x + 3x\Delta x^2 - x^3}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{x^3} + x\Delta^3 + 3x^2 x\Delta + 3xx\Delta^2 - \cancel{x^3}}{x\Delta}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta x^3 + 3x^2\Delta x + 3x\Delta x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{x\Delta}\,(x\Delta^2 + 3x^2 + 3xx\Delta)}{\cancel{x\Delta}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \Delta x^2 + 3x^2 + 3x\Delta$$

Apply the Limit

$$\frac{dy}{dx} = (0)^2 + 3x^2 + 3(0)$$

$$\boxed{\frac{dy}{dx} = 3x^2 \ Ans.}$$

iv. $f(x) = \sqrt{x}$

$Sol:-$

$f(x) = \sqrt{x}$ ⟶ (i)

$Put\ x = \Delta x$

$f(x + \Delta x) = \sqrt{x + \Delta x}$ ⟶ (ii)

Using First Principle Rule

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ ⟶ (iii)

Put ⟶ (i) & ⟶ (ii) in ⟶ (iii)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt{x} + \sqrt{\Delta x} - \sqrt{x}}{\Delta x} \Rightarrow \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt{x} + \sqrt{x\Delta} - \sqrt{x}}{x\Delta}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt{\Delta x}}{\Delta x} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{0}}{0}$$

It is $\frac{0}{0}$ Form then by conjugate

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \times \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(\sqrt{x + \Delta x})^2 - (\sqrt{x})^2}{(x\Delta)(\sqrt{x + \Delta x}) + \sqrt{x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{x + x\Delta - x}{x\Delta(\sqrt{x + \Delta x}) + \sqrt{x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

Now Apply the Limit

$$\frac{dy}{dx} = \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \ Ans.}$$

v. $f(x) = 2 - 4x^2$

$Sol: -$

$f(x) = 2 - 4x^2$ ⟶ (i)

$Put\ x = \Delta x$

$f(x + \Delta x) = 2 - 4(x + \Delta x)^2$ ⟶ (ii)

Using First Principle Rule

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ ⟶ (iii)

Put ⟶ (i) & ⟶ (ii) in ⟶ (iii)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2-4(x+\Delta x)^2-(2-4x^2)}{\Delta x} \qquad \because (a+b)^2 = a^2 + b^2 + 2ab$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2-4(\ x^2+\Delta x^2+2x\Delta x)-2+4x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2-4\,x^2+4\Delta x^2+8x\Delta x-2+4x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{2} - \cancel{4x^2} - 4\Delta x^2 - 8x\Delta x - \cancel{2} + \cancel{4x^2}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{-4\Delta x^2-8x\Delta x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(-4\Delta x - 8x)}{\cancel{\Delta x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} (-4\Delta x - 8x)$$

Apply the Limit

$$\frac{dy}{dx} = (-4(0) - 8x)$$

$$\boxed{\frac{dy}{dx} = -8x \; Ans.}$$

vi. $f(x) = \frac{1}{x}$

$Sol:-$

$f(x) = \frac{1}{x}$ ⟶ (i)

$Put\ x = \Delta x$

$f(x+\Delta x) = \frac{1}{x+\Delta x}$ ⟶ (ii)

Using First Principle Rule

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \longrightarrow \text{(iii)}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{\cancel{x} - \cancel{x} - \cancel{\Delta x}}{x(x+\Delta x)}}{\cancel{\Delta x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)}$$

Apply the Limit

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{-1}{x(x+0)}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{-1}{x(x)}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{x^2}\ Ans.}$$

vii. $f(x) = x + \sqrt{x}$

$Sol: -$

$$f(x) = x + \sqrt{x} \longrightarrow \text{(i)}$$

$Put\ x = \Delta x$

$$f(x+\Delta x) = (x+\Delta x) + \sqrt{x+\Delta x} \longrightarrow \text{(ii)}$$

Using First Principle Rule

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \longrightarrow \text{(iii)}$$

Put $\longrightarrow$ (i) & $\longrightarrow$ (ii) in $\longrightarrow$ (iii)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)+\sqrt{x+\Delta x} - (x+\sqrt{x})}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{x} + \Delta x + \sqrt{x+\Delta x} - \cancel{x} - \sqrt{x}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta x + \sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

Apply the Limit

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{0+\sqrt{x+0} - \sqrt{x}}{0} => \frac{\cancel{\sqrt{x}} - \cancel{\sqrt{x}}}{0} => \frac{0}{0}$$

It is $\frac{0}{0}$ Form then by conjugate

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta x + \sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \times \frac{\Delta x + \sqrt{x+\Delta x} + \sqrt{x}}{\Delta x + \sqrt{x+\Delta x} + \sqrt{x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(\Delta x)^2 + (\cancel{\sqrt{x+\Delta x}})^{\cancel{2}} - (\cancel{\sqrt{x}})^{\cancel{2}}}{(\Delta x)(\Delta x + \sqrt{x+\Delta x} + \sqrt{x})}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta x^2 + \cancel{x} + \Delta x - \cancel{x}}{(\Delta x)(\Delta x + \sqrt{x+\Delta x} + \sqrt{x})}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(\Delta x + 1)}{\cancel{\Delta x}\,(\Delta x + \sqrt{x+\Delta x} + \sqrt{x})}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}} + \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} 1 + \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

Apply the Limit

$$\frac{dy}{dx} = 1 + \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$\frac{dy}{dx} = 1 + \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}} \; Ans.}$$

- **Rules Of Differentiation:-**

1. **Power Rule:-**

$$\frac{\mathrm{d}}{\mathrm{dx}} = x^n = nx^{n-1}$$

Example:

$$\frac{d}{dx} = \sqrt{x}$$

$$\frac{d}{dx} = x^{\frac{1}{2}} \quad => \quad \frac{d}{dx} = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$\frac{d}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \quad => \frac{d}{dx} = \frac{1}{2x^{\frac{1}{2}}} \qquad \because \; x^{\frac{1}{2}} = \sqrt{x}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \; Ans.}$$

2. **Sum Rule or Additional Rule:-**

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

3. **Product Rule:-**

$$\frac{d}{dx}[f(x).g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

4. **Quotient Rule:-**

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

5. **Constant Rule:-**

$$\frac{d}{dx}(c) = 0$$

- **Questions:-**

Find the derivative of the following functions.

1. $y = x^2 - 5$

$Sol:-$

$y = x^2 - 5$

Differentiate $w - r - t$ 'x'

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - 5)$$

$$\frac{dy}{dx} = \frac{d}{dx}x^2 - \frac{d}{dx}5$$

$$\frac{dy}{dx} = 2x - 0$$

$$\boxed{\frac{dy}{dx} = 2x \ Ans.}$$

2. $y = 2x^3 + 3x^2 - 12x + 4$

$Sol: -$

$$y = 2x^3 + 3x^2 - 12x + 4$$

Differentiate $w - r - t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}(y) = \frac{\mathrm{d}}{\mathrm{dx}}(2x^3 + 3x^2 - 12x + 4)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}}2x^3 + \frac{\mathrm{d}}{\mathrm{dx}}3x^2 - \frac{\mathrm{d}}{\mathrm{dx}}12x + \frac{\mathrm{d}}{\mathrm{dx}}4$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = 6x^2 + 6x - 12 + 0$$

$$\boxed{\frac{dy}{dx} = 6x^2 + 6x - 12 \ Ans.}$$

3. $y = x^3 + ax^2 + 3x - 1$

$Sol: -$

$$y = x^3 + ax^2 + 3x - 1$$

Differentiate $w - r - t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}(y) = \frac{\mathrm{d}}{\mathrm{dx}}(x^3 + ax^2 + 3x - 1)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}}x^3 + \frac{\mathrm{d}}{\mathrm{dx}}ax^2 + \frac{\mathrm{d}}{\mathrm{dx}}3x - \frac{\mathrm{d}}{\mathrm{dx}}1$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = 3x^2 + 2ax + 3 - 0$$

$$\boxed{\frac{dy}{dx} = 3x^2 + 2ax + 3 \ Ans.}$$

4. $y = (x^2 - 5)(x^4 + 4)$

$Sol: -$

$$y = (x^2 - 5)(x^4 + 4)$$

Differentiate $w-r-t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}(y)=\frac{\mathrm{d}}{\mathrm{dx}}(x^2-5)(\,x^4+4)$$

Using Product Rule

$$\frac{\mathrm{dy}}{\mathrm{dx}}=(x^2-5)\frac{\mathrm{d}}{\mathrm{dx}}(\,x^4+4)+(\,x^4+4)\frac{\mathrm{d}}{\mathrm{dx}}(x^2-5)$$

$$\frac{dy}{dx}=(x^2-5)(4x^3)+\ (\,x^4+4)(2x)$$

$$\frac{dy}{dx}=4x^5-20x^3+2x^5+8x$$

$$\frac{dy}{dx}=4x^5+2x^5-20x^3+8x$$

$$\frac{dy}{dx}=6x^5-20x^3+8x$$

$$\boxed{\frac{dy}{dx}\ =6x^5-20x^3+8xAns.}$$

5. $f(x)=\ x^{\frac{3}{2}}+\sqrt{x}$

$Sol:-$

$$f(x)=\ x^{\frac{3}{2}}+\sqrt{x}$$

Differentiate $w-r-t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}f(x)=\ \frac{\mathrm{d}}{\mathrm{dx}}(\,x^{\frac{3}{2}}+\sqrt{x}\,) \qquad \because \quad x^{\frac{1}{2}}=\ \sqrt{x}$$

$$\mathrm{f}\,'(x)=\ \frac{\mathrm{d}}{\mathrm{dx}}\ x^{\frac{3}{2}}\ +\ \frac{\mathrm{d}}{\mathrm{dx}}\ x^{\frac{1}{2}}$$

$$f'(x) = \frac{3}{2}\, x^{\frac{3}{2}-1} + \frac{1}{2}x^{\frac{1}{2}}$$

$$f'(x) = \frac{3}{2}\, x^{\frac{3-2}{2}} + \frac{1}{2}\, x^{\frac{1-2}{2}}$$

$$f'(x) = \frac{3}{2}\, x^{\frac{1}{2}} + \frac{1}{2}\, x^{\frac{-1}{2}}$$

$$f'(x) = \frac{3}{2}\sqrt{2} + \frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{3}{2}\sqrt{2} + \frac{1}{2\sqrt{x}}\ Ans.}$$

6. $$f(x) = x^{\frac{5}{3}} + 2x^{\frac{4}{3}} - 3x^{\frac{-2}{3}}$$

$Sol: -$

$$f(x) = x^{\frac{5}{3}} + 2x^{\frac{4}{3}} - 3x^{\frac{-2}{3}}$$

Differentiate $w - r - t$ 'x'

$$\frac{d}{dx}f(x) = \frac{d}{dx}\left(x^{\frac{5}{3}} + 2\, x^{\frac{4}{3}} - 3\, x^{\frac{-2}{3}}\right)$$

$$f'(x) = \frac{d}{dx}\, x^{\frac{5}{3}} + \frac{d}{dx}\, 2\, x^{\frac{4}{3}} - \frac{d}{dx} 3\, x^{\frac{-2}{3}}$$

$$f'(x) = \frac{5}{3}\, x^{\frac{5}{3}-1} + 2\left(\frac{4}{3}\right)\, x^{\frac{4}{3}-1} - 3\left(\frac{-2}{3}\right) x^{\frac{-2}{3}-1}$$

$$f'(x) = \frac{5}{3}x^{\frac{5-3}{3}} + \frac{8}{3}x^{\frac{4-3}{3}} + \frac{\cancel{6}^{\,2}}{\cancel{3}}x^{\frac{-2-3}{3}}$$

$$f'(x) = \frac{5}{3} x^{\frac{2}{3}} + \frac{8}{3} x^{\frac{1}{3}} + 2x^{\frac{-5}{3}}$$

$$\boxed{f'(x) = \frac{5}{3} x^{\frac{2}{3}} + \frac{8}{3} x^{\frac{1}{3}} + 2x^{\frac{-5}{3}} \quad Ans.}$$

7. $y = (3x-2)^{\frac{4}{3}}$

$Sol: -$

$$y = (3x-2)^{\frac{4}{3}}$$

Differentiate $w-r-t$ 'x'

$$\frac{d}{dx}(y) = \frac{d}{dx}(3x-2)^{\frac{4}{3}}$$

Using Power Rule

$$\frac{dy}{dx} = \frac{4}{3}(3x-2)^{\frac{4}{3}-1}\frac{d}{dx}(3x-2)$$

$$\frac{dy}{dx} = \frac{4}{3}(3x-2)^{\frac{4-3}{3}}(3-0)$$

$$\frac{dy}{dx} = \frac{4}{\cancel{3}}(\cancel{3})(3x-2)^{\frac{4-3}{3}}$$

$$\frac{dy}{dx} = 4(3x-2)^{\frac{1}{3}}$$

$$\boxed{\frac{dy}{dx} = 4(3x-2)^{\frac{1}{3}} \; Ans.}$$

8. $y = (2x+3)^{\frac{-10}{3}}$

$Sol:-$

$$y=(2x+3)^{\frac{-10}{3}}$$

Differentiate $w-r-t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}(y)=\frac{\mathrm{d}}{\mathrm{dx}}(2x+3)^{\frac{-10}{3}}$$

Using Power Rule

$$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-10}{3}\ (2x+3)^{\frac{-10}{3}-1}\ \frac{\mathrm{d}}{\mathrm{dx}}(2x+3)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-10}{3}\ (2x+3)^{\frac{-10-3}{3}}\ (2+0)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}}=(2)\frac{-10}{3}\ (2x+3)^{\frac{-13}{3}}$$

$$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-20}{3}\ (2x+3)^{\frac{-13}{3}}$$

$$\boxed{\frac{dy}{dx}=\frac{-20}{3}\ (2x+3)^{\frac{-13}{3}}\ Ans.}$$

9. $y=(x^2+2x+3)^{\frac{3}{2}}$

$Sol:-$

$$y=(x^2+2x+3)^{\frac{3}{2}}$$

Differentiate $w-r-t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}(y)=\frac{\mathrm{d}}{\mathrm{dx}}(x^2+2x+3)^{\frac{3}{2}}$$

Using Power Rule

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{3}{2}\ (x^2 + 2x + 3)^{\frac{3}{2}-1}\ \frac{\mathrm{d}}{\mathrm{dx}}(x^2 + 2x + 3)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{3}{2}\ (x^2 + 2x + 3)^{\frac{3-2}{2}}\ (2x + 2 + 0)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{3}{2}\ (x^2 + 2x + 3)^{\frac{1}{2}}\ (2x + 2)$$

$$\boxed{\frac{dy}{dx} = \frac{3}{2}\ (x^2 + 2x + 3)^{\frac{1}{2}}\ (2x + 2)\ Ans.}$$

10. $y = \frac{x}{1+\sqrt{x}}$

$Sol: -$

$$y = \frac{x}{1 + \sqrt{x}}$$

Differentiate $w - r - t$ 'x'

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{x}{1+\sqrt{x}}\right)$$

Using Quotient Rule

$$\frac{dy}{dx} = \frac{\left(1 + \sqrt{x}\,\right)\frac{d}{dx}(x) - (x)\frac{d}{dx}\left(1 + \sqrt{x}\,\right)}{(1 + \sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{\left(1 + \sqrt{x}\,\right)1 - x\frac{d}{dx}\left(1 +\ (x)^{\frac{1}{2}}\right)}{(1 + \sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{\left(1 + \sqrt{x}\,\right) - x\left(0 + \frac{1}{2}\ (x)^{\frac{1}{2}-1}\right)}{(1 + \sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{\left(1+\sqrt{x}\,\right) - x\left(\frac{1}{2}\,(x)^{\frac{1-2}{2}}\,\right)}{(1+\sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{\left(1+\sqrt{x}\,\right) - x\left(\frac{1}{2}\,(x)^{-\frac{1}{2}}\,\right)}{(1+\sqrt{\mathrm{x}})^2} \qquad \because\ \ x^1.x^{-\frac{1}{2}} = x^{1-\frac{1}{2}} = x^{\frac{2-1}{2}} = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1+\sqrt{x} - \frac{1}{2}\,x^{\frac{1}{2}}}{(1+\sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{1+(\sqrt{x} - \frac{1}{2}\sqrt{x}\ )}{(1+\sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{1+\sqrt{x}\,(1-\frac{1}{2})}{(1+\sqrt{\mathrm{x}})^2} \qquad \because\ \ 1-\frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1+\sqrt{x}\,(\frac{1}{2})}{(1+\sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{1+\ \frac{\sqrt{x}}{2}}{(1+\sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{1}+\ \frac{\sqrt{x}}{2}}{(1+\sqrt{\mathrm{x}})^2}$$

$$\frac{dy}{dx} = \frac{\frac{2+\sqrt{x}}{2}}{(1+\sqrt{\mathrm{x}})^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2+\sqrt{x}}{2(1+\sqrt{\mathrm{x}})^2}\quad Ans.}$$

11. $f(x) = px^{2n} + qx^n + \mathrm{r}$

$Sol:-$

$f(x) = px^{2n} + qx^n + \mathrm{r}$

Differentiate $w-r-t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}f(x) = \frac{\mathrm{d}}{\mathrm{dx}}(px^{2n} + qx^n + \mathrm{r})$$

$$\mathrm{f}'(x) = \frac{\mathrm{d}}{\mathrm{dx}}px^{2n} + \frac{\mathrm{d}}{\mathrm{dx}}\,qx^n + \frac{\mathrm{d}}{\mathrm{dx}}\mathrm{r}$$

$$\mathrm{f}'(x) = p\frac{\mathrm{d}}{\mathrm{dx}}x^{2n} + q\frac{\mathrm{d}}{\mathrm{dx}}\,x^n + 0$$

$$\mathrm{f}'(x) = p(2\mathrm{n}x^{2n-1}) + q(\mathrm{n}\,x^{n-1})$$

$$\boxed{\mathrm{f}'(x) = p2\mathrm{n}x^{2n-1} + q\mathrm{n}\,x^{n-1} \quad Ans.}$$

- **Derivative of Implicit Function:-**

1. $x^2+y^2 = 25$

$Sol:-$

$x^2+y^2 = 25$

Differentiate $w-r-t$ 'x'

$$\frac{\mathrm{d}}{\mathrm{dx}}(x^2)+\frac{\mathrm{d}}{\mathrm{dx}}(y^2) = \frac{\mathrm{d}}{\mathrm{dx}}(25)$$

$$2x + 2y\frac{d}{dx}(y) = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-\cancel{2}\,x}{\cancel{2}\,y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y} \quad Ans.}$$

2. $x^3 + 5x^2y + 6y^2 = 0$

$Sol: -$

$x^3 + 5x^2y + 6y^2 = 0$

Differentiate $w - r - t$ 'x'

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(5x^2y) + \frac{d}{dx}(6y^2) = \frac{d}{dx}(0)$$

$$3x^2 + 5\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] + 6(2y\frac{d}{dx}(y) = 0$$

$$3x^2 + 5\left[x^2\frac{dy}{dx} + y(2x)\right] + 12y\frac{dy}{dx} = 0$$

$$3x^2 + 5x^2\frac{dy}{dx} + 10xy + 12y\frac{dy}{dx} = 0$$

$$(3x^2 + 10xy) + \left(5x^2\frac{dy}{dx} + 12y\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}(5x^2 + 12y) = -(3x^2 + 10xy)$$

$$\frac{dy}{dx} = \frac{-(3x^2 + 10xy)}{(5x^2 + 12y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-(3x^2 + 10xy)}{(5x^2 + 12y)} \quad Ans.}$$

3. $\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$

a. $\frac{du}{dv}$

b. $\frac{dv}{du}$

$$Sol{:}-$$

$$\frac{u^2}{a^2}+\frac{v^2}{b^2}=1$$

Multiply by a^2b^2 *both sides*

$$b^2u^2+a^2v^2=a^2b^2$$

a. Differentiate $w-r-t$ ‘v’

$$\frac{d}{dv}\left(b^2u^2+a^2v^2\right)=\frac{d}{dv}\left(a^2b^2\right)$$

$$b^2\frac{d}{dv}u^2+a^2\frac{d}{dv}v^2=0$$

$$b^2\left(2u\frac{du}{dv}\right)+a^2(2v)=0$$

$$2ub^2\frac{du}{dv}+2a^2v=0$$

$$\cancel{2}ub^2\frac{du}{dv}=-\cancel{2}a^2v$$

$$\frac{du}{dv}=\frac{-a^2v}{b^2u}$$

$$\boxed{\frac{du}{dv}=\frac{-a^2v}{b^2u}\quad Ans.}$$

b. Differentiate $w-r-t$ ‘u’

$$\frac{d}{du}\left(b^2u^2+a^2v^2\right)=\frac{d}{du}\left(a^2b^2\right)$$

$$b^2\frac{d}{du}u^2+a^2\frac{d}{du}v^2=0$$

$$b^2(2u)+a^2\left(2v\frac{d}{du}(v)\right)=0$$

$$2ub^2+2a^2v\frac{dv}{du}+=0$$

$$\cancel{2}a^2v\frac{dv}{du}=-\cancel{2}b^2u$$

$$\frac{dv}{du}=\frac{-b^2u}{a^2v}$$

$$\boxed{\frac{dv}{du}=\frac{-a^2v}{b^2u}\quad Ans.}$$

4. $\frac{1}{x}+\frac{1}{y}=7$

$Sol:-$

$$\frac{1}{x}+\frac{1}{y}=7$$

$$x^{-1}+y^{-1}=7$$

Differentiate $w-r-t$ 'x'

$$\frac{d}{dx}x^{-1}+\frac{d}{dx}y^{-1}=\frac{d}{dx}7$$

$$-1x^{-1-1}+\left(-1y^{-1-1}\frac{d}{dx}(y)\right)=0$$

$$-1x^{-2}-1y^{-2}\frac{dy}{dx}=0$$

$$\frac{-1}{x^2}-\frac{1}{y^2}\frac{dy}{dx}=0$$

$$\frac{-1}{y^2}\frac{dy}{dx}=\frac{1}{x^2}$$

$$\frac{dy}{dx}=\frac{-y^2}{x^2}$$

$$\boxed{\frac{dy}{dx}=\frac{-y^2}{x^2}\quad Ans.}$$

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The End of Week # 06