Week # 05

## Continuity of Different Functions

- **Continuity of different functions :-**

A function "$f$" is said to be continuous at a point "c" if the following conditions are satisfied.

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x) = f(c)$

- **Condition for Continuity :-**

If $f(x)$ is any given function and we have to find the continuity at $x = c$ then,

$$L - H - L = R - H - L = f(c)$$

If

$$\lim_{h \to 0^-} f(c - h) = \lim_{h \to 0^+} f(c + h)$$

Then the function will be continuous otherwise not.

- **Questions :-**

- Discuss the continuity.

1. $f(x) = \frac{1}{x^2+3}$ , $-4 < x < 7$.

$Sol :-$

$$f(x) = \frac{1}{x^2+3}$$

Condition 1:-

$f(c)$ Is defined.

$Suppose\ x = c = 0\ then,$

$$f(x) = \frac{1}{(0)^2+3}$$

$f(0) = \frac{1}{3}$

Condition (1) is satisfied.

Condition 2:-

$$\lim_{x \to c} f(x)\ is\ defined/\ exist.$$

Now Apply the Limit

$\lim_{x \to 0} f(x) = \frac{1}{(0)^2+3}$ => $\frac{1}{0+3}$

$= \frac{1}{3}$

Condition 3:-

$$\lim_{x \to c} f(x) = f(c)$$

$$\frac{1}{3} = \frac{1}{3}$$

$$L - H - S = R - H - S$$

Hence it is Continuous.

2. $f(x) = \frac{x+3}{x^2-2x-15}$ , $-7 < x < 4$, $x \neq -3$

$f(-3) = 2$

$Sol: -$

$f(x) = \frac{x+3}{x^2-2x-15}$

Condition 1:-

$f(c)$ is defined

Already given that $f(-3) = 2$

Condition 2:-

$$\lim_{x \to c} f(x)\ is\ defined/\ exist.$$

Now Apply the Limit

$\lim_{x \to -3} f(x) = \frac{x+3}{x^2-2x-15}$ => $\frac{-3+3}{(-3)^2-2(-3)-15}$ => $\frac{-\cancel{3}+\cancel{3}}{-\cancel{3}^2 \cancel{-2(-3)-15}}$ => $\frac{0}{0}$ form

It $\frac{0}{0}$ form then by factorization

$$\lim_{x\to-3} f(x) = \frac{x+3}{x^2-5x+3x-15}$$

$$\lim_{x\to-3} f(x) = \frac{x+3}{x(x-5)+3(x-5)}$$

$$\lim_{x\to-3} f(x) = \frac{x+3}{(x+3)(x-5)}$$

$$\lim_{x\to-3} f(x) = \frac{\cancel{(x+3)}}{\cancel{(x+3)}\,(x-5)}$$

$$\lim_{x\to-3} f(x) = \frac{1}{(x-5)}$$

Apply the Limit

$$= \frac{1}{(-3-5)}$$

$$= \frac{1}{(-8)}$$

Condition 3:-

$$\lim_{x\to c} f(x) = f(c)$$

$$\frac{1}{3} \neq 2$$

$$L-H-S \neq R-H-S$$

Hence it is discontinuous.

3. $f(x) = \frac{x^2-4}{x^2-8x+12}$ , $0 < x < 5,\quad x \neq 2$

$f(2) = 1$

$Sol: -$

$$f(x) = \frac{x^2-4}{x^2-8x+12}$$

Condition 1:-

$f(c)$ is defined

Already given that $f(2) = 1$

Condition 2:-

$$\lim_{x \to c} f(x) \; is \; defined / \; exist.$$

Now Apply the Limit

$$\lim_{x \to 2} f(x) = \frac{x^2-4}{x^2-8x+12} \quad => \frac{(2)^2-4}{(2)^2-8(2)+12} => \frac{\cancel{4} - \cancel{4}}{\cancel{2^2 - 8(2) + 12}} \quad => \frac{0}{0} \text{ form}$$

It $\frac{0}{0}$ form then by factorization

$$\lim_{x \to 2} f(x) = \frac{x^2-4}{x^2-6x-2x+12}$$

$$\lim_{x \to 2} f(x) = \frac{x^2-2^2}{x(x-6)-2(x-6)}$$

$$\lim_{x \to 2} f(x) = \frac{\cancel{(x-2)}\,(x+2)}{\cancel{(x-2)}\,(x-6)}$$

$$\lim_{x \to 2} f(x) = \frac{(x+2)}{(x-6)}$$

Apply the Limit

$$= \frac{(2+2)}{(2-6)} \quad => \frac{4}{(-4)}$$

$$= \frac{\cancel{4}}{-\cancel{4}}$$

$$= -1$$

Condition 3:-

$$\lim_{x \to c} f(x) = f(c)$$

$$-1 \neq 1$$

$$L-H-S \neq R-H-S$$

Hence it is discontinuous.

4. $f(x) = \begin{cases} x - 4\ , & -1 < x \le 2 \\ f(2) = -2\ , & x = 2 \\ x^2 - 6\ , & 2 < x < 5 \end{cases}$

$Sol: -$

$This\ function\ is\ called\ piece\ wise\ function.$

Condition 1:-

$f(c)$ Is defined

Already given that $f(2) = -2$

Condition 2:-

For Left hand Limit

$$\lim_{h\to 2^-} f(x-4) \ => 2-4 => -2$$

For Right Hand Limit

$$\lim_{h\to 2^+} f(x^2-6) \ => 2^2-6 => -2$$

$$L-H-L = R-H-L$$

Condition 3:-

$$\lim_{x\to c} f(x) = f(c)$$

$$-2 = -2$$

$$L-H-S = R-H-S$$

Hence it is continuous.

5. $f(x) = \frac{x^3-1}{x^2+x-2}$ , $0 < x < 2,\quad x \ne 1$

$f(1) = 1$

$Sol: -$

$f(x) = \frac{x^3-1}{x^2+x-2}$

Condition 1:-

$f(c)$ Is defined

Already given that $f(1) = 1$

Condition 2:-

$$\lim_{x \to c} f(x)\ is\ defined/\ exist.$$

Now Apply the Limit

$$\lim_{x \to 1} f(x) = \frac{x^3-1}{x^2+x-2} \quad => \frac{(1)^3-1}{(1)^2+1-2} => \frac{\cancel{1} - \cancel{1}}{\cancel{1^3} \cancel{+1} - 2} \quad => \frac{0}{0} \text{ form}$$

It $\frac{0}{0}$ form then by factorization

$$\lim_{x \to 1} f(x) = \frac{x^3-1}{x^2-x+2x-2}$$

$$\lim_{x \to 1} f(x) = \frac{(x-1)(x^2+(x)(1)+1^2)}{x(x-1)+2(x-1)}$$

$$\lim_{x \to 1} f(x) = \frac{\cancel{(x-1)}\,(x^2 + x + 1)}{\cancel{(x-1)}\,(x + 2)} \quad => \lim_{x \to 1} f(x) = \frac{(x^2+(x)+1)}{(x+2)}$$

Apply the Limit

$$= \frac{(1^2 + 1 + 1)}{(1 + 2)} \quad => \frac{3}{3}$$

$$= \frac{\cancel{3}}{\cancel{3}}$$

$$= 1$$

Condition 3:-

$$\lim_{x \to c} f(x) = f(c)$$

1= 1

$$L - H - S = R - H - S$$

Hence it is continuous

6. $f(x) = \begin{cases} x - 4 \ , & -1 < x \le 2 \\ f(2) = -2 \ , & x = 2 \\ x^2 - 6 \ , & 2 < x < 5 \end{cases}$

$Sol: -$

Condition 1:-

$f(c)$ Is defined

Already given that $f\left(\frac{1}{2}\right) = 1$

Condition 2:-

For Left hand Limit

$$\lim_{x \to \frac{1}{2}^-} f(x) \ => \ \lim_{x \to \frac{1}{2}^-} (x) => \frac{1}{2}$$

For Right Hand Limit

$$\lim_{x \to \frac{1}{2}^+} f(x) \ => \ \lim_{x \to \frac{1}{2}^+} (1 - x) => 1 - \frac{1}{2} \ => \ \frac{2-1}{2} \ => \frac{1}{2}$$

$$L - H - L = R - H - L$$

Condition 3:-

$$\lim_{x \to c} f(x) = f(c)$$

$$\frac{1}{2} \neq 1$$

$$L - H - S \neq R - H - S$$

Hence it is discontinuous.

7. $f(x) = \begin{cases} 2 - x \ , & x < 0 \\ 2 \ , & x = 0 \\ 2 + x \ , & x > 0 \end{cases}$

$Sol:$

Condition 1:-

$f(c)$ Is defined

Already given that $f(0) = 2$

Condition 2:-

For Left hand Limit

$$\lim_{x\to 0^-} f(x) \;=>\; \lim_{x\to 0^-}(2-x) => 2-0 => 2$$

For Right Hand Limit

$$\lim_{x\to 0^+} f(x) \;=>\; \lim_{x\to 0^+}(2+x) => 2+0 => \;2$$

$$L-H-L = R-H-L$$

Condition 3:-

$$\lim_{x\to c} f(x) = f(c)$$

$$2 = 2$$

$$L-H-S = R-H-S$$

Hence it is continuous.

8. $\mathrm{f(x)} = \begin{cases} x^2-1\ , & x > 1 \\ 0\ , & \mathrm{x} = 1 \\ 1-\mathrm{x}\ , & x < 1 \end{cases}$

*Sol*:

Condition 1:-

$f(c)$ Is defined

Already given that $f(1) = 0$

Condition 2:-

For Left hand Limit

$$\lim_{x\to 1^-} f(x) \;=>\; \lim_{x\to 1^-}(x^2-1) => 1^2-1 => 0$$

For Right Hand Limit

$$\lim_{x \to 1^+} f(x) \;=>\; \lim_{x \to 1^+} (1 - x) => 1 - 1 => \;0$$

$$L - H - L = R - H - L$$

Condition 3:-

$$\lim_{x \to c} f(x) = f(c)$$

$$0 = 0$$

$$L - H - S = R - H - S$$

Hence it is continuous.


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**The End Of Week # 05**