Week # 01

- **Complex Numbers**
- **Real numbers**
- **Imaginary Numbers**

- **Real Numbers :-**

The set of those numbers whose square is always non - negative is called Real Numbers.

- Examples :-
  - $(1)^2 = 1$
  - $(2)^2 = 4$
  - $(-2)^2 = 4$
  - $(-3)^2 = 9$

- **Imaginary Numbers :-**

The set of those numbers whose square is always negative is called Imaginary Numbers.

- **Examples :-**
  - $(\sqrt{-3})^2 = -3$
  - $(\sqrt{-5}\,)^2 = -5$
  - $(\sqrt{-10})^2 = -10$

- **Conversation of negative to positive by the use of iota (i) :-**

  - $\sqrt{-3} = \sqrt{-1 \times 3} = \sqrt{-1} \times \sqrt{3} = \sqrt{3i}$ $\quad\quad \therefore \sqrt{-1} = i$
  - $\sqrt{-5} = \sqrt{-1 \times 5} = \sqrt{-1} \times \sqrt{5} = \sqrt{5i}$
  - $\sqrt{-16} = \sqrt{-1 \times 16} = \sqrt{-1} \times \sqrt{16} = \sqrt{16i} = 4i$

- **Equations :-**

(1). $x^2 + 1 = 0$

$Sol: -$

- $x^2 + 1 = 0$

- $x^2 + \cancel{1} - \cancel{1} = 0 - 1$
- $x^2 = -1$

Taking "$\sqrt{\ \ }$" on b/s

- $\sqrt{x^2} = \sqrt{-1}$ $\qquad\qquad \therefore \sqrt{-1} = i$
- $x = \sqrt{-1}$

$\boxed{x = \pm i \text{ } Ans.}$

(2). $x^2 + 4 = 0$

Sol:−

- $x^2 + 4 = 0$
- $x^2 + \cancel{4} - \cancel{4} = 0 - 4$
- $x^2 = -4$

Taking "$\sqrt{\ \ }$" on b/s

- $\sqrt{x^2} = \sqrt{-4}$
- $x = \sqrt{-1 \times 4}$
- $x = \sqrt{-1} \times \sqrt{4}$
- $x = i \times 2$

$\boxed{x = 2i \text{ } Ans.}$

(3). $x^2 + 5 = 0$

Sol:−

- $x^2 + 5 = 0$
- $x^2 + \cancel{5} - \cancel{5} = 0 - 5$
- $x^2 = -5$

Taking "$\sqrt{\ \ }$" on b/s

- $\sqrt{x^2} = \sqrt{-5}$
- $x = \sqrt{-1 \times 5}$
- $x = \sqrt{-1} \times \sqrt{5}$
- $x = i \times \sqrt{5}$

$\boxed{x = \sqrt{5}\, i \text{ } Ans.}$

(4). $x^2 + 9 = 0$

Sol:−

- $x^2 + 9 = 0$

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- $x^2 + \cancel{9} - \cancel{9} = 0 - 9$
- $x^2 = -9$

$Taking" \sqrt{\ } " \ on\ b/s$

- $\sqrt{x^2} = \sqrt{-9}$
- $x = \sqrt{-1 \times 9}$
- $x = \sqrt{-1} \times \sqrt{9}$
- $x = i \times 3$

  $\boxed{x = 3\,i\ Ans.}$

(5). $x^2 + 11 = 0$

$Sol :-$

- $x^2 + 11 = 0$
- $x^2 + \cancel{11} - \cancel{11} = 0 - 11$
- $x^2 = -11$
- $Taking \text{ “}\sqrt{\ }\text{” } on\ b/s$
- $x = \sqrt{-1 \times 11}$
- $x = \sqrt{-1} \times \sqrt{11}$
- $x = i \times \sqrt{11}$

  $\boxed{x = \sqrt{11}\ i\ Ans.}$

- **Complex numbers :-**

  The combination of Real and Imaginary numbers is called complex numbers .It is represented by Ƶ.

- Examples **:-**

  Ƶ $= x + yi$ or Ƶ $= a + bi$ ⟶ (1)

  In ⟶ (1) “a” represents real Part,” bi" represents imaginary Part and “Ƶ” complex Numbers.

  (1). Ƶ $= 3 + 4i \Rightarrow (3,4)$ $(order\ pair\ form)$
  (2). Ƶ $= 5 - 7i \Rightarrow (5, -7)$ $(order\ pair\ form)$
  (3). Ƶ $= -2 + 5i \Rightarrow (-2,5)$ $(order\ pair\ form)$

OR

(1). Ƶ $= a + bi \Rightarrow (a, b)$ $(order\ pair\ form)$

(2). Ƶ $= 6 + 0i \Rightarrow (6,0)$ $(order\ pair\ form)$

(3). Ƶ $= 0 + 6\,i \Rightarrow (0,6)$ $(order\ pair\ form)$

- **Operations:-**

- Addition of complex Numbers

- General Form :-

$$Let$$

- $Ƶ1 = a + bi \quad \forall\ a, b \in IR$
- $Ƶ2 = c + di \quad \forall\ c, d \in IR$

$$Add\ Ƶ1\ and\ Ƶ2$$

- $Ƶ1 + Ƶ2 = (a + c) + (bi + di)$
- $Ƶ1 + Ƶ2 = (a + c) + (bi + di)$

$$\boxed{Ƶ1 + Ƶ2 = (a + c) + (b + d)i\ Ans.}$$

- **Questions:-**

Add the following complex numbers.

(1). $Z_1 = 2+3i$ and $Z_2 = 3+4i$

$Sol:-$

- $Z1 = 2 + 3i$
- $Z2 = 3 + 4i$

Add Z_1+Z_2

- $Z1 + Z2 = (2 + 3i) + (3 + 4i)$
- $Z1 + Z2 = (2 + 3) + (3i + 4i)$

$$\boxed{Z_1+Z_2 = 5+7i\ \text{Ans.}}$$

(2). $Z1 = 3 + 6i\ and\ Z2 = 10 - 7i$

$Sol:-$

- $Z1 = 3 + 6i$
- $Z2 = 10 - 7i$

$$Add\ Z1\ \ and\ \ Z2$$

- $Z1 + Z2 = (3 + 6i) + (10 - 7i)$
- $Z1 + Z2 = (3 + 10) + (6i - 7i)$
- $Z1 + Z2 = 13 + (-i)$

$Z_1 + Z_2 = 13 - i$ Ans.

(3). $Z1 = 3a - 5bi$ and $Z2 = 4a + 11bi$

Sol :-

- $Z1 = 3a - 5bi$
- $Z2 = 4a + 11bi$

Add Z1 and Z2

- $Z1 + Z2 = (3a - 5bi) + (4a + 11bi)$
- $Z1 + Z2 = (3a + 4a) + (-5bi + 11bi)$
- $Z1 + Z = 7a + 6i$

$Z1 + Z2 = 7a + 6i$ Ans.

(4). $Z1 = \sqrt{3} - di$ and $Z2 = a - 5di$

Sol :-

- $Z1 = \sqrt{3} - di$
- $Z2 = a - 5di$

Add Z1 and Z2

- $Z1 + Z2 = (\sqrt{3} - di) + (a - 5di)$
- $Z1 + Z2 = (\sqrt{3} + a) + (-1di - 5di)$

$Z1 + Z2 = (\sqrt{3} + a) - 6di$ Ans.

- **Subtraction of complex Numbers**
- General Form :-

Let

- $Z1 = a + bi \quad \forall\ a, b \in IR$
- $Z2 = c + di \quad \forall\ c, d \in IR$

Subtract Z1 from Z2

- $Z1 - Z2 = (a + bi) - (c + di)$
- $Z1 + Z2 = a + bi - c - di$
- $Z1 + Z2 = a - c + bi - di$

$$\boxed{Z1 + Z2 = (a - c) + (b - d)i \; Ans.}$$

- **Questions :-**

  Subtract the following complex numbers.

(1). $a = 2 + i \; and \; b = 8 - 6i$

$Sol: -$

- $a = 2 + i$
- $b = 8 - 6i$

Subtract “b” from “a”

- $b - a = (8 - 6i) - (2 + i)$
- $b - a = 8 - 6i - 2 - i$
- $b - a = 8 - 2 - 6i - i$
- $b - a = 6 - 7i$

$$\boxed{b - a = 6 - 7i \; Ans.}$$

(2). $Z1 = 2a - 3bi \; and \; Z2 = 3a + 12bi$

$Sol: -$

- $Z1 = 2a - 3bi$
- $Z2 = 3a + 12bi$

Subtract "$Z2$" from "$Z1$"

- $Z2 - Z1 = (3a + 12bi) - (2a - 3bi)$
- $Z2 - Z1 = 3a + 12bi - 2a + 3bi$
- $Z2 - Z1 = 3a - 2a + 12bi + 3bi$
- $Z2 - Z1 = a + 15bi$

$$\boxed{Z2 - Z1 = a + 15bi \; Ans.}$$

(3). $Z1 = 3(2+5i)$ *and* $Z2 = 5(1-2i)$

Sol:−

- $Z1 = 3(2+5i) \Rightarrow Z1 = (6+15i)$
- $Z2 = 5(1-2i) \Rightarrow Z2 = (5-10i)$

Subtract "Z1" From" Z2"

- $Z1 - Z2 = (6+15i) - (5-10i)$
- $Z1 - Z2 = 6 + 15i - 5 + 10i$
- $Z1 - Z2 = 6 - 5 + 15i + 10i$
- $Z1 - Z2 = 1 + 25i$

$\boxed{Z1 - Z2 = 1 + 25i \ \ Ans.}$

- **Multiplication of Complex Numbers**
- General form :-

Let

- $Z1 = a + bi \quad \forall\ a, b \in IR$
- $Z2 = c + di \quad \forall\ c, d \in IR$

Multiply "Z1" and "Z2

As $i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2$

$\therefore i^2 = -1$

- $Z1.Z2 = (a + bi).(c + di)$
- $Z1.Z2 = a(c + di) + bi(c + di)$
- $Z1.Z2 = ac + adi + bci + bd\,(i)^2$
- $Z1.Z2 = ac + (ad + bc)i + bd(-1)$
- $Z1.Z2 = ac + (ad + bc)i - bd$

$\boxed{Z1.Z2 = ac + (ad + bc)i - bd \ \ Ans.}$

- **Questions :-**

Multiply the following complex numbers.

(1) . $a = -3 + 6i$ *and* $b = 10 - 7i$

Sol:−

- $a = -3 + 6i$

- $b = 10 - 7i$
  $Mltiply ""a" " \ with "b"$
- $a\,.\,b = (-3+6i)\,.\,(10-7i)$
- $a\,.\,b = -3(10-7i) + 6i(10-7i)$
- $a\,.\,b = -30 + 21i + 60i - 42\,(i)^2$ ⟶ $\therefore i^2 = -1$
- $a\,.\,b = -30 + (21+60)i - 42(-1)$
- $a\,.\,b = -30 + 42 + 81i$
- $a\,.\,b = 12 + 81i$

  Real part = 12 Ans.

  Imaginary part =81 Ans.

(2) . $Z1 = 3a - 5bi \ and \ Z2 = 4a + 11\,bi$

$Sol: -$

- $Z1 = 3a - 5bi$
- $Z2 = 4a + 11\,bi$

  Multiply “Z1” with “Z2”

- $Z1.Z2 = (3a - 5bi)\,.\,(4a + 11bi)$
- $Z1.Z2 = 3a(4a + 11bi) - 5bi(4a + 11bi)$
- $Z1.Z2 = 12a^2 + 33abi - 20abi - 55b^2\,(i)^2$ ⟶ $\therefore i^2 = -1$
- $Z1.Z2 = 12a^2 + (33ab - 20ab)i - 55b^2\,(-1)$
- $Z1.Z2 = 12a^2 + 55b^2 + 12abi$

$Real\ part = 12a^2 + 55b^2\ Ans.$

$Imaginary\ part = 12abi\ Ans.$

(3) . $Z1 = 2(1+2i) \ and \ Z2 = -3(2-i)$

$Sol: -$

- $Z1 = 2(1+2i) \quad => (2+4i)$
- $Z2 = -3(2-i) \quad => (-6+3i)$

  Multiply “Z1” with “Z2”

- $Z1.Z2 = (2+4i).(-6+3i)$
- $Z1.Z2 = 2(-6+3i) + 4i(-6+3i)$
- $Z1.Z2 = -12 + 6i - 24i + 12(i)^2$ ⟶ $\therefore i^2 = -1$
- $Z1.Z2 = -12 + (6-24)i + 12(-1)$
- $Z1.Z2 = -12 + (-18)i - 12$
- $Z1.Z2 = -12 - 12 - 18i$
- $Z1.Z2 = -24 - 18i$

Real part = −24 Ans.

$Imaginary\ part = -18i\ Ans.$

- **Division of Complex Numbers**
  - General Form :-

Let

- $Z1 = a + bi \quad \forall\ a, b \in IR$
- $Z2 = c + di \quad \forall\ c, d \in IR$

Divide "Z1" by "Z2"

- $\frac{Z1}{Z2} = \frac{a+bi}{c+di}$

Multiplying and dividing $c - di$

- $\frac{Z1}{Z2} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$ ⟶ $\therefore a^{2-} b^2 = (a-b)(a+b)$
- $\frac{Z1}{Z2} = \frac{(a+bi)(c-di)}{(c)^2-(d)^2(i)^2}$ $\quad \therefore i^2 = -1$
- $\frac{Z1}{Z2} = \frac{a(c-di)+bi(c-di)}{c^2 - d^2(-1)}$
- $\frac{Z1}{Z2} = \frac{ac-adi+bci-bdi^2}{c^2+d^2}$
- $\frac{Z1}{Z2} = \frac{ac-(ad+bc)i-bd(-1)}{c^2+d^2}$

- $\frac{Z1}{Z2} = \frac{ac+bd+(bc-ad)i}{c^2+d^2}$

$$\boxed{Real\ \ part\ = \frac{ac+bd}{c^2+d^2} Ans.}$$

$$\boxed{Imaginary\ \ part\ = \frac{bc-ad}{c^{2+}d^2} i\ \ Ans}$$

- **Questions :-**

  Divide the following Complex numbers.

(1) . $a\ =\ 2+i\ and\ b\ =\ 8-6i$

$Sol:-$

- $a = 2 + i$
- $b = 8 - 6i$

Divide "a" by "b"

- $\frac{a}{b} = \frac{2+i}{8-6i}$

$Multiplying\ and\ dividing\ 8 + 6i$

- $\frac{a}{b} = \frac{2+i}{8-6i} \times \frac{8+6i}{8+6i}$ → $\therefore a^{2-} b^2 = (a-b)(a+b)$
- $\frac{a}{b} = \frac{(2+6i)(8+6i)}{(8)^2-6^2(i)^2}$
- $\frac{a}{b} = \frac{2(8+6i)+i(8+6i)}{64-36(-1)}$
- $\frac{a}{b} = \frac{16+12i+8i+6(i)^2}{64+36}$ → $\therefore i^2 = -1$
- $\frac{a}{b} = \frac{16+(12+8)i+6(-1)}{100}$

$$\frac{a}{b} = \frac{16+20i-6}{100}$$

$$\frac{a}{b} = \frac{16-6+20i}{100}$$

$$\frac{a}{b} = \frac{10+20i}{100}$$

$$\frac{a}{b} = \frac{10}{100} + \frac{20}{100} i$$

$$\frac{a}{b} = \frac{1}{10} + \frac{1}{5} i$$

$Real\ \ part\ = \frac{1}{10} Ans.$

$Imaginary\ \ part\ = \frac{1}{5} i\ \ Ans$

(2) $. Z1 = 2a - 3bi$ and $Z2 = 12a + 3bi$

Sol:-

- Z1 $=2a - 3bi$
- Z2 $=12a + 3bi$

Divide "Z1" by "Z2"

$$\frac{z1}{z2} = \frac{2a-3bi}{12a+3bi}$$

Multiplying and Dividing $12a - 3bi$

$$\frac{Z1}{Z2} = \frac{2a-3bi}{12a+3bi} \times \frac{12a-3bi}{12a-3bi}$$

$$\frac{Z1}{Z2} = \frac{(2a-3bi)(12a-3bi)}{(12a)^2-(3b)^2(i)^2}$$ $\therefore a^{2-} b^2 = (a-b)(a+b)$

$$\frac{Z1}{Z2} = \frac{2a(12a-3bi)-3bi(12a-3bi)}{144a^2-9b^2(-1)}$$ $\therefore i^2 = -1$

$$\frac{Z1}{Z2} = \frac{24a^2-6abi-36abi+9b^2(i)^2}{144a^2+9b^2}$$

- $\frac{Z1}{Z2} = \frac{24a^2 - 42abi + 9b^2(-1)}{144a^2 + 9b^2}$
- $\frac{Z1}{Z2} = \frac{24a^2 - 9b^2 - 42abi}{144a^2 + 9b^2}$

$$\boxed{Real\ part\ = \frac{24a^2 - 9b^2}{144a^2 + 9b^2}\ Ans.}$$

$$\boxed{Imaginary\ part\ = -\frac{42ab}{144a^2 + 9b^2}i\ \ Ans.}$$

Conjugate and Modulus of Complex Numbers :-

- Conjugate :-

$Z\ =\ x + yi$

$\bar{Z} = \overline{x + yi}$

$\bar{Z} = x - yi$

$\bar{Z} = x - yi$

$\boxed{Z = x - yi\ Ans.}$

- Modulus :-

$|Z| = \sqrt{x^2 + y^2}$

By Pythagoras Theorem

$|Hyp|^2 = (Base)^2 + (Per)^2$

$|Z|^2 = x^2 + y^2$

$|Z| = \sqrt{x^2 + y^2}$

$\boxed{|Z|\ =\ \sqrt{x^2 + y^2}\ Ans.}$

- **Questions:-**

Find the Modulus Of the following

$(1).Z1 = 1 + i \; and \; Z2 = 2 + 3i$

(i). Find $|Z1 + Z2|$

$Sol: -$

- $Z1 = 1 + i$
- $Z2 = 2 + 3i$

Add "Z1" and "Z"

- $Z1 + Z2 = (1 + i) + (2 + 3i)$
- $Z1 + Z2 = (1 + 2) + (i + 3i)$
- $Z1 + Z2 = 3 + 4i$

Now Taking Modulus

- $|Z1 + Z2| = \sqrt{3 + 4i}$
- $|3 + 4i| = \sqrt{(3)^2 + (4)^2}$
- $|3 + 4i| = \sqrt{9 + 16}$
- $|3 + 4i| = \sqrt{25}$
- $|3 + 4i| = 5$

$\boxed{|3 + 4i| = 5 \; Ans.}$

(ii). Find $|Z1.Z2|$

$Sol: -$

- $Z1 = 1 + i$
- $Z2 = 2 + 3i$

Multiply "Z1" with "Z2"

- $Z1 \,.\, Z2 = (1+i)\,.\,(2+3i)$
- $Z1 \,.\, Z2 = 1(2+3i) + i(2+3i)$
- $Z1 \,.\, Z2 = 2+3i+2i+3(i)^2$ ⟶ $\therefore i^2 = -1$
- $Z1 \,.\, Z2 = 2+(3+2)i+3(-1)$
- $Z1 \,.\, Z2 = 2+(5i)-3$
- $Z1 \,.\, Z2 = 2-3+5i$
- $Z1 \,.\, Z2 = -1+5i$

Taking Modulus

- $|\, Z1 \,.\, Z2| = \sqrt{x^2+y^2}$
- $|-1+5i| = \sqrt{(-1)^2+(5)^5}$
- $|-1+5i| = \sqrt{1+25}$
- $|-1+5i| = \sqrt{26}$

$\boxed{|-1+5i| = \sqrt{26}\, Ans.}$

(iii) .Find $|\frac{Z1}{Z2}|$

$Sol:-$

- $Z1 = 1+i$
- $Z2 = 2+3i$

Divide "Z1 by"Z2"

- $\frac{Z1}{Z2} = \frac{1+i}{2+3i}$

$Multiplying\ and\ Dividing\ 2+3i$

- $\frac{Z1}{Z2} = \frac{1+i}{2+3i} \times \frac{2-3i}{2-3i}$ ⟶ $\therefore a^{2-}\, b^2 = (a-b)(a+b)$
- $\frac{Z1}{Z2} = \frac{(1+i)\,(2-3i)}{(2)^2-(3)^2(i)^2}$ $\therefore i^2 = -1$

- $\frac{Z1}{Z2} = \frac{1(2-3i)+i(2-3i)}{4-9(-1)}$

- $\frac{Z1}{Z2} = \frac{2-3i+2i-3(i)^2}{4+9}$

- $\frac{Z1}{Z2} = \frac{2-(3-2)i-3(-1)}{13}$

- $\frac{Z1}{Z2} = \frac{2+3-1i}{13}$

- $\frac{Z1}{Z2} = \frac{5}{13} - \frac{i}{13}$

Taking Modulus

- $|\frac{Z1}{Z2}| = \sqrt{x^2 + y^2}$

- $|\frac{Z1}{Z2}| = \sqrt{(\frac{5}{13})^2 + (\frac{1}{13})^2}$

- $|\frac{Z1}{Z2}| = \sqrt{(\frac{25}{169}) + (\frac{1}{169})}$

- $|\frac{Z1}{Z2}| = \sqrt{(\frac{25+1}{169})}$

- $|\frac{Z1}{Z2}| = \sqrt{(\frac{26}{169})}$

- $|\frac{Z1}{Z2}| = \frac{\sqrt{26}}{13}$

$$\boxed{\left|\frac{Z1}{Z2}\right| = \frac{\sqrt{26}}{23}\ Ans}$$

- **Properties of Complex Numbers:-**
  - Commutative Property w. r. t Addition & Multiplication :-

$Z1 + Z2 = Z1 + Z2$ ⟶ W. r. t Addition

$Z1.Z2 = Z1.Z2$ ⟶ W. r. t Multiplication

- **Associate property w .r. t Addition & Multiplication :-**

$Z1 + (Z1 + Z3) = (Z1 + Z2) + Z3$ ⟶ w.r.t Addition

$Z1(Z2.Z3) = (Z1.Z2).Z3$ ⟶ w.r.t Multiplication

- **Distribute property of Multiplication over Addition :-**

$Z1.(Z2 + Z3) = Z1.Z2 + Z1.Z3$

$(Z1 + Z2).Z3 = Z1.Z3 + Z2.Z3$

- **Conjugate property :-**

$\overline{Z1 + Z2} = \overline{Z1} + \overline{Z2}$ ⟶ Addition

$\overline{Z1.Z2} = \overline{Z1}\,.\overline{Z2}$ ⟶ Multiplication

$\left[\frac{\overline{Z1}}{Z2}\right] = \left[\frac{\overline{\overline{Z1}}}{\overline{Z2}}\right]$ ⟶ Division

- **Questions:-**

- $If\ Z1 = 2x - 3i\ \ and\ \ Z2 = 4x + 5yi\ then\ show\ that\ \overline{Z1 + Z2} = \overline{Z1} + \overline{Z2}$

$Sol: -$

Taking L-H-S

- $\overline{Z1 + Z2} = (2x - 3yi) + (4x + 5yi)$
- $\overline{Z1 + Z2} = 2x + 4x - 3yi + 5yi$

- $\overline{Z1+Z2} = 6x + 2yi$

  Apply conjugate

- $\overline{Z1+Z2} = \overline{6x+2yi}$

$$\boxed{\overline{Z1+Z2} = \overline{6x-2yi}} \longrightarrow (A)$$

Taking R-H-S

- $\overline{Z1} = \overline{2x-3yi} \quad => \quad \overline{Z1} = 2x + 3yi$
- $\overline{Z2} = \overline{4x+5yi} \quad => \quad \overline{Z2} = 4x - 5yi$
- $\overline{Z1} + \overline{Z2} = (2x+3yi) + (4x-5yi)$
- $\overline{Z1} + \overline{Z2} = 2x + 4x + 3yi - 5yi$
- $\overline{Z1} + \overline{Z2} = 6x - 2yi$

$$\boxed{\overline{Z1+Z2} = \overline{6x-2yi}} \longrightarrow (B)$$

$From \longrightarrow (A)\ and \longrightarrow (B)$

$$\boxed{R-H-S = L-H-S}$$

- $If\ Z1 = 1-4i \quad and\ \ Z2 = -2+5i\ then\ show\ that\ \overline{Z1.Z2} = \overline{Z1}\,.\overline{Z2}$

  $Sol:-$

  Taking L-H-R

- $\overline{Z1.Z2} = (1-4i\,).(-2+5i)$
- $\overline{Z1.Z2} = 1(-2+5i) - 4i(-2+5i)$
- $\overline{Z1.Z2} = -2 + 5i + 8i - 20(i)^2 \longrightarrow \therefore i^2 = -1$
- $\overline{Z1.Z2} = -2 + (5+8)i - 20(-1)$
- $\overline{Z1.Z2} = -2 + 20 + 13i$
- $\overline{Z1.Z2} = 18 + 13i$

Apply Conjugate

- $\overline{Z1.Z2} = \overline{18 + 13\imath}$

$\boxed{\overline{Z1.Z2} = \ 18 - 13i}$ ⟶ (A)

Taking R-H-S

- $\overline{Z1} = \overline{1 - 4\imath} \quad => \quad \overline{Z1} = \ 1 + 4i$
- $\overline{Z2} = \overline{-2 + 5\imath} \quad => \quad \overline{Z2} = \ -2 - 5i$
- $\overline{Z1}\,.\overline{Z2} = (1 + 4i)\,.\,(-2 - 5i)$
- $\overline{Z1}\,.\overline{Z2} = 1(-2 - 5i) + 4i(-2 - 5i)$
- $\overline{Z1}\,.\overline{Z2} = -2 - 5i - 8i - 20(i)^2$ ⟶ $\therefore i^2 = -1$
- $\overline{Z1}\,.\overline{Z2} = -2 - (5 + 8)i - 20(-1)$
- $\overline{Z1}\,.\overline{Z2} = -2 + 20 - (13i)$

$\boxed{\overline{Z1}\,.\overline{Z2} = \ 18 - 13i}$ ⟶ (B)

$From$ ⟶ (A) and ⟶ (B)

$\boxed{R - H - S = L - H - S}$

- $If\ Z1 = -a - 3bi \quad and \quad Z2 = 2a - 3bi\ then\ show\ that\ \left[\overline{\frac{Z1}{Z2}}\right] = \left[\frac{\overline{Z1}}{\overline{Z2}}\right]$

$Sol: -$

Taking L-H-S

- $\left[\overline{\frac{Z1}{Z2}}\right] = \frac{-a-3bi}{2a-3bi}$

$Multiplying\ and\ Dividing\ 2a + 3bi$

- $\left[\overline{\frac{Z1}{Z2}}\right] = \frac{-a-3bi}{2a-3bi} \times \frac{2a+3bi}{2a+3bi}$

- $\left[\overline{\frac{Z1}{Z2}}\right] = \frac{(-a-3bi)\,(2a+3bi)}{(2a-3bi)\,(2a+3bi)}$ ⟶ $\therefore a^{2-}\,b^2 = (a - b)(a + b)$

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-a(2a+3bi)-3bi(2a+3bi)}{(2a)^2-(3b)^2(i)^2}$

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2 -3abi-6abi-9b^2(i)^2}{4a^2-9b^2(-1)}$ $\longrightarrow$ $\therefore\ i^2 = -1$

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2-(3abi+6abi)-9b^2(-1)}{4a^2+9b^2}$

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2+9b^2-9abi}{4a^2+9b^2}$

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2+9b^2}{4a^2+9b^2} - \frac{9abi}{4a^2+9b^2}$

Apply Conjugate

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{\overline{-2a^2+9b^2}}{4a^2+9b^2} - \frac{\overline{9abi}}{4a^2+9b^2}$

- $\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{9abi}{4a^2+9b^2}$

$$\boxed{\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{9abi}{4a^2+9b^2}} \longrightarrow \text{(A)}$$

Taking R-H-S

- $\overline{Z1} = \overline{-a - 3bi}$ => $\overline{Z1} = -a + 3bi$
- $\overline{Z2} = \overline{2a - 3bi}$ => $\overline{Z2} = 2a + 3bi$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-a+3bi}{2a+3bi}$

$Multiplying\ and\ Dividing\ 2a - 3bi$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-a+3bi}{2a+3bi} \times \frac{2a-3bi}{2a-3bi}$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-a+3bi}{2a+3bi} \times \frac{2a-3bi}{2a-3bi}$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{(-a+3bi)\,(2a-3bi)}{(2a+3bi)(2a-3bi)}$ $\longrightarrow$ $\therefore a^{2-}\, b^2 = (a-b)(a+b)$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-a(2a-3bi)+3bi(2a-3bi)}{(2a)^2-(3b)^2(i)^2}$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-2a^2+3abi+6abi-9b^2(i)^2}{4a^2-9b^2(-1)}$ $\longrightarrow$ $\therefore\ i^2 = -1$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-2a^2+3abi+6abi-9b^2(-1)}{4a^2+9b^2}$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-2a^2+(3ab+6ab)i+9b^2}{4a^2+9b^2}$

- $\left[\frac{\overline{Z1}}{\overline{Z2}}\right] = \frac{-2a^2+9b^2+9abi}{4a^2+9b^2}$

$$\boxed{\left[\frac{\overline{Z1}}{Z2}\right] = \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{9abi}{4a^2+9b^2}} \longrightarrow \text{(B)}$$

$From \longrightarrow$ (A) $and \longrightarrow$ (B)

$$\boxed{R-H-S = L-H-S}$$

The End of Week # 01